ELECTROMAGNETIC RADIATION IN HIGH-ENERGY NUCLEAR INTERACTIONS

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Bremsstrahlung emitted by charged particles participating in high-energy nuclear interactions is computed on the basis of the hydrodynamical model of an elementary act. The radiated energy is found to be a constant fraction, whereas bremsstrahlung from point charges increases logarithmically with energy.

THE problem of the electromagnetic radiation which accompanies nuclear interactions has arisen frequently. Since the radiation from suddenly stopped charged particles increases logarithmically with the particle energy,[1] attempts were made to explain the soft component of cosmic radiation in this way at a time when the existence of π^0 mesons had not yet been established. The question has appeared again,* but now we have a better understanding of the structure of nuclear particles and of the character of high-energy nuclear interactions (there exists a consistent hydrodynamical theory of multiple particle production). The electromagnetic radiation from real particles participating in an elementary act can therefore be discussed without being confined to estimates[1] which, as we shall see, are correct only when the particles are assumed to be extremely small.

In the present article the electromagnetic radiation which appears in nuclear collisions due to the slowing down of extremely fast charged particles is investigated on the basis of Landau's[3] hydrodynamical model of nuclear interactions.

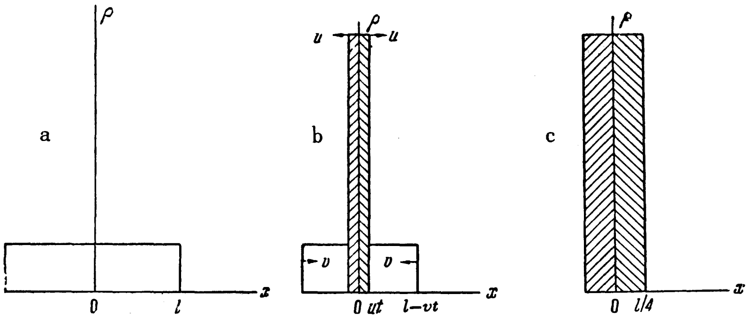

FIG. 1. "Stopping" stage: a — initial instant of collision; b — compression of the nuclei by the shock wave; c — instant when the shock wave has reached the edges of the nuclei.

For simplicity we shall consider the collision of two identical nuclei at high energies. In the center-of-mass system after contact the nuclear matter is symmetrically compressed by a shock wave and a hydrodynamical system is formed. As has been shown by Khalatnikov,[4] after the passage of the shock wave and the complete stopping of the nuclei symmetrical expansion begins which is first described by a traveling wave and then by a general one-dimensional solution. The formation of new particles takes place in the last, three-dimensional stage. The electromagnetic radiation which appears can obviously be estimated classically by assuming the charge to be distributed uniformly over the entire mass, with the charge density thus proportional to the mass density.

The energy radiated in an element of solid angle do and in the frequency interval $d\omega$ is

$$dW_{n\omega} = c\,|\,[\mathbf{k}\times\mathbf{A}'_{\omega}]\,|^2 R_0^2\, do\, d\omega, \qquad (1)$$

where $\mathbf{A}_\omega$ is obtained from the usual formula for retarded potentials:

$$\mathbf{A}_\omega = \frac{e^{ikR_0}}{cR_0}\int \mathbf{j}_\omega e^{-i\mathbf{kr}}\, dV,$$

and $\mathbf{j}_\omega$ is determined from the hydrodynamical solution.

*The author is indebted to V. I. Zatsepin for calling her attention to Friedländer's article.[2]

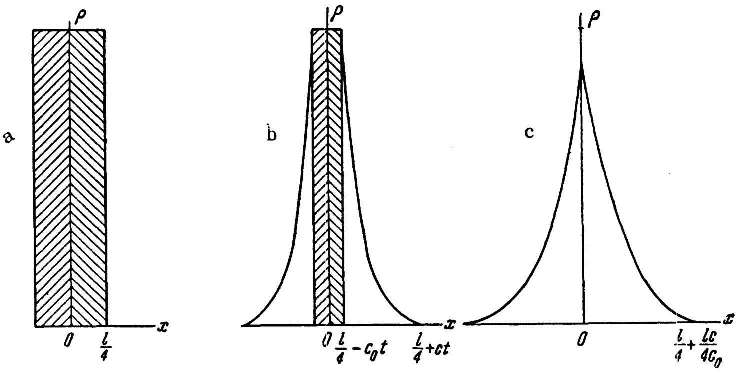

FIG. 2. Stage of "acquiring motion": a — the instant when the shock wave reaches the edges of the nuclei; b — passage of the rarefaction wave; c — the instant when the traveling waves meet.

Earlier calculations[1] show that energy is emitted principally at high frequencies. The characteristic frequency of the first stage (stopping) is determined by the transit time of the shock wave through the nucleus, which is $a\sqrt{1-\beta^2}/u$, where $u = c/3$ is the velocity of the shock wave and a is the diameter of the nucleus. In the expansion all of the matter is set into motion, mainly during the time required for the trailing edge of the traveling wave to reach the plane of symmetry from which expansion takes place. This time is $a\sqrt{1-\beta^2}/4c_0$, where $c_0 = c/\sqrt{3}$ is the velocity of the traveling wave. The time of the remaining stages is about $1/(1-\beta^2)$ greater and the accelerations are correspondingly smaller; it is therefore clear that radiation occurs principally during the first two stages.

We shall assume that the initial charge density ρ is Ze/V_0, where $V_0 = (\pi a^2/4)/a\sqrt{1-\beta^2}$ is the initial volume of the colliding nuclei. The current j differs from zero and is equal to ρv for all $|x|$ in the range $(\ell/4, \infty)$ when $-|x|/v < t < -|x|/v + \ell/v$, and for $\ell/4 > |x| > 0$ when $-|x|/v < t < |x|/u$ ($\ell/4 = a\sqrt{1-\beta^2}/4$ is the longitudinal dimension of the nuclei after compression by the shock wave). Behind the shock front the fluid is at rest and the current vanishes (Fig. 1).

In the traveling (rarefaction) wave

$$j_r = \frac{\rho c^2}{\sqrt{2}c_0}\left(\frac{c-c_0}{c+c_0}\right)^{c/2c_0}\frac{[c_0(t-t_0)+(x-x_0)]\,[c(t-t_0)-(x-x_0)]^{(c-c_0)/2c_0}}{[c(t-t_0)+(x-x_0)]^{(c+c)_0/2c_0}}$$

where $t_0 = \ell/4u$ and $x_0 = \ell/4$ are the initial coordinates of the front of the rarefaction wave. This current differs from zero for

$$l/4 < |x| < 0 \text{ and } |x|/c_0 + l/4u < |t| < l/4c_0 + l/4u$$

and for

$$l/4 < |x| < l/4 + lc/4c_0 \text{ and } |x|/c + l/4u < |t| < l/4c_0 + l/4u$$

(Fig. 2).

From this definition of the current we obtain j_ω and then A_ω:

$$A_\omega = \frac{\rho\beta ac^2}{R_0\omega\sin\vartheta}J_1\left(\frac{a\omega}{2c}\sin\vartheta\right)\left\{\frac{(1+3\beta)}{\omega^2(3+\cos\vartheta)(1-\beta\cos\vartheta)}\left[2\sin^2\frac{a\omega(3+\cos\vartheta)}{8c} - i\sin\frac{a\omega(3+\cos\vartheta)}{4c}\right] + \frac{ce^{i\omega(t_0-x_0/c)}}{2\sqrt{2}c_0}\left(\frac{c-c_0}{c+c_0}\right)^{c/2c_0}\right.$$

$$\left.\times\int_{-c_0/c}^{1}\varphi(\xi)\left[\frac{\exp\{i\omega(1-\xi\cos\vartheta)l/4c_0\}-1}{\omega^2(1-\xi\cos\vartheta)^2} - i\frac{\exp\{i\omega(1-\xi\cos\vartheta)l/4c_0\}}{\omega(1-\xi\cos\vartheta)}\right]d\xi\right\} + A_\omega^-, \qquad (2)$$

$$\varphi(\xi) = \frac{(1-\xi)^{(c-c_0)/2c_0}(c_0/c+\xi)}{(1+\xi)^{(c+c_0)/2c_0}}.$$

In Eq. (2) the expression within the curly brackets corresponds to radiation to the right; the first term within the brackets pertains to radiation from the left-hand shock wave (the first stage; for an undeformable "point" charge we obtain an analogous term); the second term gives the radiation from the right-hand traveling wave. A symmetrical expression for radiation to the left is obtained by changing the sign of $\cos\vartheta$. Equation (2) shows that the radiation is concentrated within small angles [because the denominators contain the factors $(1-\beta\cos\vartheta)$ and $(1-\xi\cos\vartheta)$]. Since we will ultimately be interested in radiation in the laboratory system and the center of mass is regarded as moving from left to right in the laboratory system, we shall neglect radiation to the left and A_ω^- will hereinafter be dropped.

When the traveling wave is taken into account the expression for the energy contains terms of the order of $(1-\beta^2)$. This results from the fact that whereas in the first stage the entire charge is slowed down from its relativistic velocity, in the second stage it acquires motion gradually, and only a small portion (close to the front of the traveling wave) acquires a velocity $\sim c$ during this period. Thus the entire radiation is determined by the stopping stage and we have

$$W \approx \frac{4^5 (Ze)^2 c^3}{2\pi a^4 (1-\beta^2)} \int_0^\infty \int_0^\pi f(\vartheta) \frac{J_1^2 (a\omega \sin\vartheta/2c) \sin^2 (l\omega (3+\cos\vartheta)/8c) \sin\vartheta d\vartheta d\omega}{\omega^4 (1-\beta^2 \cos^2\vartheta)^2}, \quad f(\vartheta) = (1+\cos\vartheta)^2/(1+3\cos\vartheta)^2. \quad (3)$$

Making the substitutions $\sin\vartheta/\sqrt{1-\beta^2} = y$ and $\omega l/2c = z$ together with the assumption $1/\sqrt{1-\beta^2} = \infty$, we find that up to terms containing $(1-\beta^2)$ the radiated fraction of the energy in the laboratory system is

$$\Delta \approx (13\alpha Z^2/\pi A^{4/3}) \mu/M \quad (4)$$

where the factor μ arises from $a \approx (\hbar/\mu c)A^{1/3}$ and M from $\sqrt{1-\beta^2} = MAc^2/E$. For proton collisions $Z = A = 1$ and $\Delta \approx 0.4\%$.

This value of the radiated fraction of the energy is much smaller than was estimated for point particles.[1] The difference is explained by the fact that Eq. (4) does not contain the large logarithmic factor $-\ln\sqrt{1-\beta^2}$. This factor disappears because the integrand in (3) contains the function $J_1^2(\omega a \sin\vartheta/2c)$ which oscillates rapidly at high frequencies and which arose from the extent of the nuclei in the transverse direction. In the center-of-mass system the particles have the shape of discs because of the Lorentz contraction. At high frequencies the radiation from different elements of a slowed-down disc will interfere and give a sharp maximum only at angles $< l/a \approx \sqrt{1-\beta^2}$ around the direction of motion. However, it is easily seen that in the earlier formulas[1] the principal term $-\ln\sqrt{1-\beta^2}$ was derived from the angular range $1 \gg \vartheta \gg \sqrt{1-\beta^2}$.

The former result is obtained from (3) if we make a approach zero. In order to obtain the final result the integral over ω must, as usual, be cut off at a certain ω_{max}:

$$W_{l\to 0} \sim \int_0^{\omega_{max}} d\omega \int_0^\pi \frac{\sin^3\vartheta d\vartheta}{(1-\beta\cos\vartheta)^2} \sim \alpha\omega_{max}\left(\ln\frac{1}{\sqrt{1-\beta^2}} + \text{const}\right). \quad (5)$$

In our expression (3) for the energy the integral converges because of the interference term $J_1^2(\omega a \times \sin\vartheta/2c)$ at frequencies ω_c for which $\omega_c a\sqrt{1-\beta^2}/2c \sim 1$. These frequencies are present in (3) because for $a = \hbar/\mu c$ we have

$$\omega_c \simeq \mu c^2/\sqrt{1-\beta^2} < E,$$

where E is the energy of the colliding particles in the center-of-mass system.

For particles with dimensions much smaller than their Compton wavelength it would be necessary to introduce the cutoff at $\omega_c \sim E$. Then (3) for the given ω_c would contain a logarithmic region of integration for the angles $\sqrt{1-\beta^2} \ll \vartheta \ll (\lambda_0/a)\sqrt{1-\beta^2}$ (λ_0 is the Compton wavelength), which would lead to $\ln(\lambda_0/a)$, thus somewhat increasing the result.

The hydrodynamical procedure developed here is inapplicable for electrons, and evidently also for muons, because of their weak interactions. The radiation must then be determined from the usual formulas,[1] which contain a logarithmic factor. The applicability of these formulas is determined by the possibility of using perturbation theory.

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