RADIATION CORRECTIONS IN THE DISPERSION RELATIONS FOR $\pi^{\pm} + p \rightarrow \pi^{\pm} + p$

V. K. FEDIANIN

Moscow State University

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PUPPI and Stanghellini[1] again investigated the problem of determining the value of f^2 from the dispersion relations. Analyzing separately $\pi^+ + p$ and $\pi^- + p$ charge states, they reach the conclusion that for energies below resonance* $f^2_{(+)} = 0.08$ and $f^2_{(-)} = 0.04$, and that for energies above resonance $f^2_{(+)} = f^2_{(-)} = 0.08$. Thus, they actually found that f^2 depends on the energy. Yet in earlier works[2] it was found that $f^2_{(+)} = f^2_{(-)} = 0.08$ for the entire energy range. Puppi and Stanghellini attribute this to the fact that the authors of Ref. 2 determine the values of f^2 by using the phases for $T = 3/2$, where $\pi^+ + p \rightarrow \pi^+ + p$ predominates.

One can assume that the difference is due to electromagnetic interaction. In a recent work[3] Agodi and Cini attempted to explain the difference in the presence of the $\pi^{\pm}$ and π^0 masses in the interaction Hamiltonian. This gave a difference on the order of 1% between $f^2_{(+)}$ and $f^2_{(-)}$.

This work is an attempt to determine the difference between $f^2_{(+)}$ and $f^2_{(-)}$ by successively including in the total system of functions also the intermediate states in which both the nucleon and the photon exist.

Using the notation of Ref. 4 for the antihermitian portion of the scattering amplitude, we have

$$A_{\alpha\omega}\left(\frac{q+q'}{2}\right) = \frac{1}{2i}\int \exp\left(i\,\frac{q+q'}{2}\,x\right)\left(F^{(-)}_{\alpha\omega}(x) - \hat{P}_{\rho\rho'}F^{(-)}_{\alpha\omega}(-x)\right)dx, \tag{1}$$

where $F^{(-)}_{\alpha\omega}(x)$ is determined by the following relation:

$$\langle p', s'|\,\hat{j}_{\rho'}(x)\,\hat{j}_{\rho}(y)\,|p,s\rangle = -i\exp\left\{i\,\frac{p-p'}{2}\,(x+y)\right\}F^{(-)}_{\alpha\omega}(x-y), \tag{2}$$

and $\hat{j}_{\rho}(x)$ is the meson-current operator. Inserting (2) into (1) and using the expansion theorem, we get

$$A_{\alpha\omega}(E) = A^{-}_{\alpha\omega}(E) + A^{+}_{\alpha\omega}(E), \tag{3}$$

with $A^{-}_{\alpha\omega}(E) = -\hat{P}_{\rho\rho'}A^{+}_{\alpha\omega}(-E)$, and

$$A^{+}_{\alpha\omega}(E) = -\frac{(2\pi)^4}{2}\sum_{s'',\,\nu}\int d\mathbf{k}\,\langle p', s'|\,j_{\rho}(0)\,|\mathbf{p}'', s'', \mathbf{k}, l_{\nu}\rangle\langle \mathbf{p}'', s'', \mathbf{k}, l_{\nu}\,|\,j_{\rho'}(0)\,|p,s\rangle\,\delta(E + k_0 + p''_0 - E_{\mathbf{p}}). \tag{4}$$

All the calculations are carried out in the system $\mathbf{p} + \mathbf{p}' = 0$, $E_{\mathbf{p}} = \sqrt{M^2 + \mathbf{p}^2}$, $k_0 = |\mathbf{k}|$, $p''_0 = \sqrt{M^2 + (-\lambda\mathbf{e} - \mathbf{k})^2}$, $\mathbf{p}'' = -\lambda\mathbf{e} - \mathbf{k}$, $\lambda\mathbf{e} = (\mathbf{q} + \mathbf{q}')/2$, E is the meson energy, and l_{ν} and $\mathbf{k}$ are the polarization and momentum of the photon in the intermediate state. Using Ref. 4, we reduce $\langle \mathbf{p}'', l_{\nu}, s'', \mathbf{k}\,|\,j_{\rho'}(0)\,|p, s\rangle$ from (4) to the fourth variational derivative of the S matrix.

Estimating the fourth variational derivative by perturbation theory in the lower order relative to e and relative to g with the usual Lagrangian meson-nucleon-photon interaction, we obtain

$$\langle p'', s'', \mathbf{k}, l_{\nu}\,|\,j_{\rho'}(0)\,|\,p, s\rangle$$

$$= -\frac{eg}{(2\pi)^{7/2}\sqrt{2k_0}}\,\bar{u}^{s''}_{+}(p'')\left\{\hat{l}_{\nu}\frac{1+\tau_3}{2}\,S^c(p+q)\,\gamma^5\tau_{\rho'} + \tau_{\rho'}\gamma^5 S^c(p-k)\,\hat{l}_{\nu} + (ql_{\nu})\,\gamma^5\Delta^c(k-q)\,[\tau_{\rho'}\tau_3]\right\}u^{s}_{-}(p), \tag{5}$$

$\hat{a} = a_0\gamma_0 - \mathbf{a}\gamma$, $\gamma_0^+ = \gamma_0$, $\gamma^+ = -\gamma$, and $S^c(k)$, $\Delta^c(k)$ are the Fourier images of the electron and photon propagation functions (see Ref. 4).

For forward scattering, inserting (5) into (4), we obtain for the (+) and (−) processes:

$$A^{+}_{\alpha\omega}(E) = \frac{e^2}{4\pi}\,\frac{g^2}{4\pi}\,\frac{\tau_{\rho}\tau_{\rho'}}{2M}\,F_{+}(E),$$

$$F_{+}(E) = \frac{2M - 5E - 5E_q}{\varkappa}\tan^{-1}\frac{\varkappa}{E-M} + \frac{4E_q}{\varkappa}\tan^{-1}\frac{\varkappa}{E} + \frac{2E}{E+E_q}\left(1 + \frac{E_q}{\varkappa}\tan^{-1}\frac{\varkappa}{E}\right),\quad \varkappa = \sqrt{\mu^2 - E^2};\quad E_q = -\mu^2/2M. \tag{6}$$

From the δ-function (4) we deduce that

$$F_+(E) = \begin{cases} = 0 & \mu^2/2M \leqslant E \leqslant \mu \\ \neq 0 & -\mu \leqslant E \leqslant \mu^2/2M. \end{cases} \qquad (7)$$

An analysis of the isotopic structure of the scattering amplitude[4] yields for $\pi^\pm + p \to \pi^\pm + p$ the following expression:

$$A_\pm(E) = \frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{2}{M}F_\pm(E), \quad F_+(E) = -F_-(-E), \qquad (8)$$

$A_\pm(E)$ is the antihermitian portion of the scattering amplitude for $\pi^\pm + p \to \pi^+ + p$. Taking this into account, the inhomogeneous term in the dispersion relations[4] is rewritten

$$\Delta_\pm = \Delta^0_\pm + \Delta^1_\pm = \pm\frac{2f^2q^2}{\mu^2(E\mp\mu^2/2M)} + \frac{2f^2q^2}{\mu^2}\frac{4M}{\pi}\alpha\int\limits_{-\mu^2/2M}^{\mu}\frac{F_-(E')\,dE'}{(E'\pm E)(E'^2-\mu^2)}, \qquad (9)$$

$\alpha = 1/137$ and q is the meson momentum.

In the calculation of the integral with respect to E we obtain the following singularities: at $E = -\mu^2/2M$ the divergence is of the infrared type; it is eliminated by infinite renormalization of f^2. The divergence at $E = \pm\mu$ is eliminated by extrapolation of $A_\pm(E)$ to the value given by (8) for $E \to \mu$. The terms in $\Delta^1_\pm$, which diminish as the fundamental term of $\Delta^0_\pm$, give a finite charge renormalization; it is the same for (+) and (-). Those terms in $\Delta^1_\pm$, which diminish more rapidly than $\Delta^0_\pm$ as $E \to \infty$, i.e., which give a contribution at small values of E, comprise the sought correction. Finally,

$$\Delta_\pm(E) = \pm\frac{2f^2q^2}{\mu^2(E\mp\mu^2/2M)}\left(1\pm\frac{\alpha}{\pi}I_\pm\right), \qquad (10)$$

where $I_\pm$ is a complicated expression of the form

$$I_\pm \approx \frac{a}{E\pm\mu} + b\frac{\ln(E\mp c)}{E\pm\mu} + \cdots$$

a, b, c are small constants.

The above estimate gave for the energies $(1.5-2)\mu$ of interest to us a difference on the order of 3% between $f^2_{(+)}$ and $f^2_{(-)}$.

I express my deep gratitude to D. V. Shirokov, under whose leadership this work was performed.

Note added in proof (October 16, 1957). It is reported in Ref. 5 that the difference between $f^2_{(+)}$ and $f^2_{(-)}$ has been calculated from the dispersion relations by a method that gives a difference between $f^2_{(+)}$ and $f^2_{(-)}$ on the order of 5% at energies 150 — 200 Mev.

*The (-) sign will be used below to denote the process $\pi^- + p \to \pi^- + p$, while the (+) sign will be used for $\pi^+ + p \to \pi^+ + p$.

[1] G. Puppi and A. Stanghellini, Nuovo cimento 5, No. 5 (1957).

[2] U. Haber-Schaim, Phys. Rev. 104, 1113 (1956); W. C. Davidon and M. L. Goldberger, Phys. Rev. 104, 1119 (1956).

[3] A. Agodi and M. Cini, Nuovo cimento 5, No. 5 (1957).

[4] N. N. Bogoliubov and D. V. Shirkov, Введение в теорию квантованных полей (Introduction into the Theory of Quantized Fields), Gostekhizdat, Moscow 1957.

[5] A. Agodi and M. Cini, Nuovo cimento 6, 3 (1957).

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265