*A GROUP-THEORETICAL CONSIDERATION OF THE BASIS OF RELATIVISTIC QUANTUM MECHANICS. III. IRREDUCIBLE REPRESENTATIONS OF THE CLASSES P_0 AND O_0, AND THE NON-COMPLETELY-REDUCIBLE REPRESENTATIONS OF THE INHOMOGENEOUS LORENTZ GROUP**

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All the unitary and non-unitary irreducible representations of class P_0 for the inhomogeneous Lorentz group are obtained in explicit form. The isomorphism of the representations of class O_0 and the representations of the homogeneous Lorentz group is discussed. We indicate certain non-completely-reducible representations of the inhomogeneous Lorentz group which are of importance for physics.

1. IRREDUCIBLE REPRESENTATIONS OF CLASS P_0 FOR THE INHOMOGENEOUS LORENTZ GROUP

THE representations of class P_0 are characterized by a non-vanishing 4-momentum having zero length

$$p_\mu^2 = 0, \quad p_\mu \neq 0.$$

In this case there is no system of coordinates in which all except one of the components of p_μ would vanish. We can, however, choose a system of coordinates so that

$$p_1 = p_2 = 0, \quad p_3 = p_4 / i = p. \tag{1}$$

From (1) and (I.39) we find for the components of the intrinsic angular momentum operator Γ_σ:

$$\Gamma_3 = -i\Gamma_4 = \Gamma_0. \tag{2}$$

With the notation

$$\Gamma_1 + i\Gamma_2 = \Gamma^+, \quad \Gamma_1 - i\Gamma_2 = \Gamma^-, \quad \Gamma_0 = p\Gamma \tag{3}$$

the commutation relations (I.41) between the components Γ_σ, and the invariant Γ_σ^2 become

$$[\Gamma^+, \Gamma] = -\Gamma^+, \quad [\Gamma^-, \Gamma] = \Gamma^-, \tag{4}$$

$$[\Gamma^+, \Gamma^-] = 0, \tag{5}$$

$$\Gamma_\sigma^2 = \Gamma^+\Gamma^-. \tag{6}$$

The relations (4) have the same structure as (II.19), and Γ, like T_0 in (II.19), is the operator of an infinitesimal Euclidean rotation. Repeating word for word the derivation in Sec. 4 of Ref. 2 (formulas II.22 – 32), we find that the admissible wave functions have the form $\psi_{\alpha n}$, where α, n are the eigenvalues of Γ_σ^2, Γ:

$$\Gamma_\sigma^2\psi_{\alpha n} = \alpha\psi_{\alpha n}, \quad \Gamma\psi_{\alpha n} = n\psi_{\alpha n},$$

and n runs through all positive and negative integer (half-integer) values for single-valued (double-valued) representations. The operators Γ^+ and Γ^- respectively increase and decrease the index n by unity. Only those functions can appear in an irreducible representation which belong to the same eigen-

*Notations used without explanation are the same as in Refs. 1 and 2. References like (I.39) and (II.19) are to the corresponding formulas in Refs. 1 and 2.

value α of the invariant. The operator Γ has the form

$$\Gamma_{mn} = \delta_{mn} n, \tag{7}$$

where m, n = ... −2, −1, 0, 1, 2, ... for single-valued representations, and m, n = ... $-3/2$, $-1/2$, $1/2$, $3/2$ ... for double-valued representations.

The operators Γ^+, Γ^- satisfy

$$\Gamma^+_{mn}(n - m + 1) = 0, \quad \Gamma^-_{mn}(n - m - 1) = 0 \tag{8}$$

and have the form

$$\Gamma^+_{mn} = a_n \delta_{m,n+1}, \quad \Gamma^-_{mn} = b_m \delta_{m+1,n}. \tag{9}$$

At this point the analogy with the treatment for class P_{II} ends because of the difference between the commutation relations [(II.20) and (5)] and the expressions for the invariants [(II.33) and (6)]. Using (9) and (6) instead of (II.33) and (II.35), we now get

$$a_n b_n = \alpha. \tag{10}$$

For $\alpha \neq 0$ we can, without loss of generality, set

$$a_n = a, \quad b_n = b = \alpha / a. \tag{11}$$

for all n.

The sign of the energy is the invariant for representations of class P_0, so that to each value of α there correspond two irreducible representations, $P_{+0}(E > 0)$ and $P_{-0}(E < 0)$. For unitary representations,

$$(\Gamma^+)^* = \Gamma^- \tag{12}$$

and, consequently,

$$b = a^*, \quad \alpha = a^* a \geqslant 0, \tag{13}$$

i.e., the representations of class P_0 are unitary for real non-negative $\alpha = c > 0$. We shall designate such single-valued (double-valued) representations by $P^{c}_{\pm 0}$ ($P'^{c}_{\pm 0}$).

In the special case of $\alpha = 0$, we find from (13),

$$a = b = 0, \quad \Gamma^+ = \Gamma^- = 0, \quad \Gamma_1 = \Gamma_2 = 0. \tag{14}$$

From (1), (2), (3), and (14) it follows that for $\alpha = 0$,

$$\Gamma_\sigma = p_\sigma \Gamma, \tag{15}$$

i.e., in this case Γ is the additional invariant which was mentioned earlier in Sec. 12 of Ref. 1 (cf. I.46 and I.47). Because of the invariance of Γ, the representation which was infinite-dimensional in the spin variable splits up into one-dimensional representations. Thus, for $\alpha = 0$, for a given sign of the energy there correspond to each positive integral (half-integral) value Σ of the operator Γ the representations $P^{+\Sigma}_{\pm 0}$ and $P^{-\Sigma}_{\pm 0}$ ($P'^{+\Sigma}_{\pm 0}$ and $P'^{-\Sigma}_{\pm 0}$), where for the representation $P^{+\Sigma}_{\pm 0}$ ($P'^{+\Sigma}_{\pm 0}$)

$$\Gamma_\sigma = \Sigma p_\sigma, \tag{16}$$

while for the representation $P^{-\Sigma}_{\pm 0}$ ($P'^{-\Sigma}_{\pm 0}$)

$$\Gamma_\sigma = -\Sigma p_\sigma, \quad \Sigma = 0, 1, 2, \ldots \quad (1/2, 3/2, 5/2, \ldots). \tag{17}$$

For a representation of this type, the fundamental invariants p^2_μ and Γ^2_σ are equal to zero, but then there are three supplementary invariants: the sign of the energy, the sign of Γ_0, and the factor of proportionality between Γ_σ and p_σ.

All other irreducible representations of class P_0 are non-unitary. We shall denote the single-valued (double-valued) representations, corresponding to real negative $\alpha = -c$ by $P^{-c}_{\pm 0}$ ($P'^{-c}_{\pm 0}$). The remaining complex non-unitary single-valued (double-valued) representations, corresponding to complex α, will be denoted by $P^{\alpha}_{\pm 0}$ ($P'^{\alpha}_{\pm 0}$). By analogy with Sec. 4 of Ref. 2 we may assume that for each non-unitary representation of class P_0 in the neighborhood of the identity we can construct a representation over the whole group, but this point, of course, requires a special proof.

For the representations $P_{+0}^{\pm\Sigma}$ which are one-dimensional in the spin variable, the operator $M_{\mu\nu}$ has the form

$$M_1 = -i(p_2\partial/\partial p_3 - p_3\partial/\partial p_2) \pm \Sigma p p_1/(p_1^2+p_2^2),$$
$$M_2 = -i(p_3\partial/\partial p_1 - p_1\partial/\partial p_3) \mp \Sigma p p_2/(p_1^2+p_2^2), \quad M_3 = -i(p_1\partial/\partial p_2 - p_2\partial/\partial p_1), \tag{18}$$
$$N_1 = ip\partial/\partial p_1 \mp \Sigma p_2 p_3/(p_1^2+p_2^2), \quad N_2 = ip\partial/\partial p_2 \pm \Sigma p_1 p_3/(p_1^2+p_2^2), \quad N_3 = ip\partial/\partial p_3.$$

With respect to the classification of the unitary representations, the results found here coincide with those of Wigner.[3] We give a table of all the irreducible representations of class P_0:

Representation	Unitarity. Dimensionality in the spin variable.	Fundamental Invariants $p_\mu^2=0$, $\Gamma_\sigma^2=\alpha$	Additional invariants.
$P_{\pm 0}^{\pm\Sigma}$	Unitary, one-dimensional	$\alpha=0$	$S_H=\pm 1$, $S_{\Gamma_0}=\pm 1$, $\Sigma=0,1,2,\ldots$
$P'^{\pm\Sigma}_{\pm 0}$	" "	$\alpha=0$	$S_H=\pm 1$, $S_{\Gamma_0}=\pm 1$, $\Sigma={}^1/_2,{}^3/_2,\ldots$
$P^c_{\pm 0}$	Unitary, infinite-dimensional	$\alpha=c>0$	$S_H=\pm 1$
$P'^c_{\pm 0}$	" "	$\alpha=c>0$	$S_H=\pm 1$
$P^{-c}_{\pm 0}$	Non-unitary, infinite-dimensional	$\alpha=-c<0$	$S_H=\pm 1$
$P'^{-c}_{\pm 0}$	" "	$\alpha=-c<0$	$S_H=\pm 1$
$P^\alpha_{\pm 0}$	" "	$\alpha=$ complex	$S_H=\pm 1$
$P'^\alpha_{\pm 0}$	" "	$\alpha=$ complex	$S_H=\pm 1$

2. THE CLASS O_0

Unlike the three and four-dimensional rotation groups, the inhomogeneous Lorentz group is not simple, but contains a normal divisor (invariant subgroup), the subgroup of translations.

We shall denote by b, b′, ... those elements of G which are in the subgroup of translations, and by a, a′, ... the remaining elements, which represent rotations about different points. By definition, a subgroup is a normal divisor if its right and left residue classes with respect to any of the elements of the group coincide, i.e., if the sets of elements ba, b′a, b″a ... and ab, ab′, ab″ ... coincide. In other words, for any a, b, there exists an element b′ such that ba = ab′, or

$$a^{-1}ba = b'. \tag{19}$$

We can verify directly that (19) is satisfied for the group G:

$$x_\mu = a_{\mu\nu}x'_\nu = a_{\mu\nu}(x''_\nu + b_\nu) = x'''_\mu + a_{\mu\nu}b_\nu \equiv x'''_\mu + b'_\mu.$$

The classes corresponding to elements a_1, a_2, will be denoted by A_1, A_2. Geometrically, a class consists of rotations through the same angle about different points. Each of the elements of the group G is either in one (and only one) of the residue classes, or is in the translation subgroup B. If the product of the elements a_2, a_1, taken from classes A_2, A_1, belongs to the class A_3,

$$a_2a_1 = a_3, \tag{20}$$

then the product of any other pair of elements a'_2, a'_1, taken from these respective classes also will belong to A_3,

$$a'_2a'_1 = a'_3. \tag{21}$$

We can therefore introduce an operation of class multiplication, defining the product of classes as the set of all possible elements of the form a_2a_1. All elements of this type are in class A_3, so that we may write

$$A_2A_1 = A_3. \quad (22)$$

The translation subgroup B plays the role of the identity in the class multiplication,

$$AB = BA = A.$$

Together with B, the set of residue classes A forms a group with respect to the multiplicative operation which we have introduced. This group, which we denote by L, is the factor group of G with respect to the invariant subgroup B. More detailed information concerning factor groups is given, for example, in Pontriagin's[4] book, where, in particular, proofs of all the above assertions are given.

The factor group L is isomorphic to the homogeneous Lorentz group. Now let us assume that we have found some representation of L, i.e., we have associated with each of the elements A_1, A_2, ... a matrix U_1, U_2, ... so that for each relation of type (22) we have

$$U_2U_1 = U_3. \quad (23)$$

It is not difficult to see that the matrices U will at the same time also give a representation of the group G, if we associate the matrix U_1 with all the elements a_1', a_1'', a_1''', ... in the class A_1, etc., and if the elements b, b′, ... are associated with the unit matrix. Thus to each irreducible representation of the homogeneous Lorentz group there corresponds an irreducible representation of the inhomogeneous group, in which all the translations are associated with the unit operator, so that for these representations $p_\mu = 0$, and all of them belong to the class O_0. The invariants for the class O_0 are the invariants of the homogeneous Lorentz group:

$$F = {}^1/_2 M_{\mu\nu}^2 = \mathbf{M}^2 - \mathbf{N}^2, \quad (24)$$

$$W = (1/4i)\,\varepsilon_{\mu\nu\lambda\sigma} M_{\mu\nu} M_{\lambda\sigma} = \mathbf{MN}. \quad (25)$$

All the representations of the homogeneous Lorentz group were found by Gel'fand and Naimark,[5] and we shall not review them in detail. Each representation is defined by a pair of numbers (k_0, c), where k_0 is non-negative and integral or half-integral, and c is an arbitrary complex number. The representation is unitary and infinite-dimensional if c is pure imaginary (fundamental series), or if c is real and lies between zero and one, $1 > c \geq 0$ with $k_0 = 0$ (the complementary series). The invariants F, W are expressed in terms of k_0, c by

$$F = k_0^2 + c^2 - 1, \quad W = ik_0c. \quad (26)$$

We give the operators **M**, **N** for the finite-dimensional non-unitary representations, which are the most interesting for physics, and which are obtained for $c = k_0 + n$, $n = -(2k_0 - 1), \ldots -1, 0, 1, 2, \ldots$ The symmetric undotted spinor of rank $2k_0$ transforms according to the representations $S_{k_0}^{k_0+1}$ $(c = k_0 + 1)$, for which $M_{\mu\nu}$ is

$$\mathbf{M} = \mathbf{S}, \quad \mathbf{N} = i\mathbf{S} \quad (27)$$

In (27) **S** is the operator for the three-dimensional angular momentum of magnitude k_0,

$$[S_i, S_j] = i\varepsilon_{ijk} S_k, \quad \mathbf{S}^2\Omega_{k_0} = k_0(k_0+1)\,\Omega_{k_0}. \quad (28)$$

The dotted spinor of rank $2k_0$ transforms according to the representation $S_{k_0}^{-k_0-1}$ $(c = -k_0 - 1)$, for which $M_{\mu\nu}$ is

$$\mathbf{M} = \mathbf{S}, \quad \mathbf{N} = -i\mathbf{S}. \quad (29)$$

All other finite-dimensional irreducible representations are found by taking direct products $S_{k_0'}^{1-k_0'} \times S_{k_0''}^{k_0''+1}$ with all possible k_0', k_0''. For such irreducible representations the operator $M_{\mu\nu}$ has the form

$$\mathbf{M} = \mathbf{S}_1 + \mathbf{S}_2, \quad \mathbf{N} = i\mathbf{S}_1 - i\mathbf{S}_2, \quad (30)$$

where

$$\mathbf{S}_1^2\Omega = k_0'(k_0'+1)\,\Omega, \quad \mathbf{S}_2^2\Omega = k_0''(k_0''+1)\,\Omega. \quad (31)$$

A straightforward computation of the invariants of the representation (30) will show the validity of the relations

$$S_{k_0'}^{1-k_0'} \times S_{k_0''}^{k_0''+1} = S_{k_0'-k_0''}^{k_0'+k_0''+1} \ (k_0' > k_0''); \quad S_{k_0'}^{1-k_0'} \times S_{k_0''}^{k_0''+1} = S_{k_0'+k_0''+1}^{k_0''-k_0'} \ (k_0'' \geqslant k_0'), \tag{32}$$

which show that we have obtained all the finite-dimensional irreducible representations of class O_0.

3. THE NON-COMPLETELY-REDUCIBLE REPRESENTATIONS OF THE INHOMOGENEOUS LORENTZ GROUP

The reducible representations which we have treated up to now were not only reducible, but also fully reducible. In mathematics, there occur, in addition, representations which are not fully reducible.* A representation is said to be non-completely-reducible if all its matrices can be brought simultaneously to the form

$$U = \begin{pmatrix} U^{(1)} & 0 \\ U^{(21)} & U^{(2)} \end{pmatrix}, \tag{33}$$

where $U^{(21)}$ is not identically zero. For the product of two matrices U_1, U_2, of type (33) we find

$$U_1 U_2 = U_3 = \begin{pmatrix} U_1^{(1)} & 0 \\ U_1^{(21)} & U_1^{(2)} \end{pmatrix} \begin{pmatrix} U_2^{(1)} & 0 \\ U_2^{(21)} & U_2^{(2)} \end{pmatrix} = \begin{pmatrix} U_1^{(1)} U_2^{(1)} & 0 \\ U_1^{(21)} U_2^{(1)} + U_1^{(2)} U_2^{(21)} & U_1^{(2)} U_2^{(2)} \end{pmatrix}, \tag{34}$$

from which we see that the matrices $U^{(1)}$ and $U^{(2)}$ also give representations of the group. The representation $U^{(1)}$ is either fully reducible, or non-completely-reducible, or irreducible. In the first case we can expand $U^{(1)}$ in irreducible representations, while in the second case we can use the procedure just described to separate out representations of lower dimension from $U^{(1)}$. Continuing this process until we obtain irreducible representations, we find that any non-completely-reducible representation is built up from irreducible representations in the sense that all its matrices can be brought to the form

$$U = \begin{pmatrix} U^{(1)} & 0 & 0 & \ldots \\ U^{(21)} & U^{(2)} & 0 & \ldots \\ U^{(31)} & U^{(32)} & U^{(3)} & \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}, \tag{35}$$

where $U^{(1)}$, $U^{(2)}$, ... constitute irreducible representations, and $U^{(21)}$, $U^{(31)}$, ... are not all zero.

Non-completely-reducible representations have various features different from those of irreducible representations. In particular, whereas a matrix which commutes with all the matrices of an irreducible representation must be the unit matrix, for a matrix commuting with all the matrices of a non-completely-reducible representation we can only say that its eigenvalues are equal (cf. Ref. 6, p. 39). Thus the non-completely-reducible representations cannot be classified by means of the invariants defined in Ref. 1. All the non-completely-reducible representations are non-unitary.

One might get the impression that non-completely-reducible representations are of purely mathematical interest and have nothing to do with physics. This is not the case. The simplest example of a physical quantity transforming according to a non-completely-reducible representation is the non-relativistic 4-dimensional radius vector $(\mathbf{x}, t)$. This vector transforms according to the non-completely-reducible representation of the Galilei group:

$$t = t', \quad \mathbf{x} = \mathbf{x}' + \mathbf{v}t'. \tag{36}$$

In fact, (36) can be written as

$$x_\alpha = V_{\alpha\beta} x'_\beta, \tag{37}$$

where α, $\beta = 0, 1, 2, 3$; $x_0 = t$; $x_\alpha = (t, x_1, x_2, x_3)$,

*Everywhere except in the present section we have, in order to simplify the terminology, used the term "reducible" for "fully reducible" representations, since both terms are equivalent for all questions in which we can disregard the existence of non-completely-reducible representations.

$$V_{\alpha\beta}=\begin{pmatrix}1&0&0&0\\ v_1&1&0&0\\ v_2&0&1&0\\ v_3&0&0&1\end{pmatrix}. \qquad (38)$$

The matrices $V_{\alpha\beta}$ are obviously not fully reducible.

Going on to the inhomogeneous Lorentz group,

$$x_\mu = a_{\mu\nu}x'_\nu + b_\mu, \qquad (39)$$

we rewrite (39) in homogeneous coordinates $y_1, \ldots y_5$,

$$x_\mu = y_\mu / y_5. \qquad (40)$$

We get

$$y_s = A_{ss'}y'_{s'}, \qquad (41)$$

where $s, s' = 1, 2, 3, 4, 5$,

$$A_{ss'}=\begin{pmatrix}a_{\mu\nu}b_\mu\\ 0\quad 1\end{pmatrix}. \qquad (42)$$

The five-rowed matrices $A_{ss'}$ form a non-completely-reducible representation of the group G. (It is interesting that the representation (42) is not only finite-dimensional, but also faithful, i.e., different elements of the group correspond to different matrices.) Thus the relativistic radius vector, expressed in homogeneous coordinates, transforms according to a non-completely-reducible representation of the group G.

By considering direct products of the representation (42) with itself, we can get new non-completely-reducible representations of the group G.

The representation (42) is fully reducible with respect to rotations, while the matrices for translations have diagonal elements equal to unity. We may pose the problem of finding all possible representations of this type, for which the matrices of rotations and translations have the respective forms

$$\begin{pmatrix}U^{(1)}&0\\ 0&U^{(2)}\end{pmatrix}, \qquad \begin{pmatrix}1&U^{(12)}\\ 0&1\end{pmatrix}. \qquad (43)$$

For such representations, the infinitesimal operators $M_{\mu\nu}$, p_λ have the form

$$M_{\mu\nu}=\begin{pmatrix}M^{(1)}_{\mu\nu}&0\\ 0&M^{(2)}_{\mu\nu}\end{pmatrix}, \qquad p_\lambda=\begin{pmatrix}0&L_\lambda\\ 0&0\end{pmatrix}, \qquad (44)$$

while the commutation relations reduce to

$$[M^{(1)}_{\mu\nu}M^{(1)}_{\lambda\sigma}] = i(\delta_{\mu\sigma}M^{(1)}_{\lambda\nu}+\delta_{\mu\lambda}M^{(1)}_{\nu\sigma}+\delta_{\nu\sigma}M^{(1)}_{\mu\lambda}+\delta_{\lambda\nu}M^{(1)}_{\sigma\mu}), \qquad [M^{(2)}_{\mu\nu}M^{(2)}_{\lambda\sigma}] = i(\delta_{\mu\sigma}M^{(2)}_{\lambda\nu}+\delta_{\mu\lambda}M^{(2)}_{\nu\sigma}+\delta_{\nu\sigma}M^{(2)}_{\mu\lambda}+\delta_{\lambda\nu}M^{(2)}_{\sigma\mu}), \qquad (45)$$

$$M^{(1)}_{\mu\nu}L_\lambda - L_\lambda M^{(2)}_{\mu\nu} = i(L_\nu\delta_{\mu\lambda} - L_\mu\delta_{\nu\lambda}). \qquad (46)$$

In general the matrices $M^{(1)}_{\mu\nu}$, $M^{(2)}_{\mu\nu}$ have different dimensions, so that the matrices L_μ can be rectangular. According to (44),

$$p_\mu p_\nu \equiv 0, \qquad (47)$$

and no requirements are imposed on the commutation relations between the matrices L_μ. For the special case of $M^{(1)}_{\mu\nu} = M^{(2)}_{\mu\nu}$, the matrices L_μ are square, and the relation (46) becomes

$$[M^{(1)}_{\mu\nu}, L_\lambda] = i(L_\nu\delta_{\mu\lambda} - L_\mu\delta_{\nu\lambda}). \qquad (48)$$

On the other hand, we know that any relativistic equation for an elementary particle can be written in the form:[7]

$$\left(\frac{1}{i}L_\mu\frac{\partial}{\partial x_\mu} + \varkappa_0\right)\psi = 0, \qquad (49)$$

where the L_μ are matrices satisfying (48). Thus the problem of finding the possible equations for elementary particles reduces to the problem of finding the possible non-completely-reducible representations of type (44) with $M^{(1)}_{\mu\nu} = M^{(2)}_{\mu\nu}$.

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EQUATIONS FOR THE GREEN'S FUNCTIONS OF A SYSTEM OF FUNDAMENTAL PARTICLES

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Green's functions for fundamental particles are introduced. On the basis of Lagrangians describing all strong interactions of mesons and baryons, closed systems of equations for the Green's functions are obtained in variational derivatives with respect to the external currents, in both three-dimensional and four-dimensional isotopic spin space.

THE scheme of Gell-Mann[1] for the description of heavy mesons and hyperons is well confirmed by the experimental data. On the basis of this scheme d'Espagnat and Prentki,[2,3] Salam,[4] Matthews and Salam,[5] and others have constructed Lagrangians that describe all the strong interactions of the fundamental particles.

The purpose of the present work is to obtain a complete system of equations for the Green's functions of the fundamental particles.

1. THE INTERACTION LAGRANGIAN. THE GREEN'S FUNCTIONS

Let us consider all the strong interactions of mesons and baryons. Let the spinors $\Lambda(x)$, $\Sigma(x)$, $\Xi(x)$, and $N(x)$ describe Λ, Σ, and Ξ hyperons and nucleons, and let the pseudoscalar $\varphi(x)$ and the scalar $k_1(x)$ describe π and K mesons, respectively. Furthermore we assume that in the isotopic spin space Λ is a scalar, $\varphi = \begin{pmatrix}\varphi_1\\ \varphi_2\\ \varphi_3\end{pmatrix}$ and $\Sigma = \begin{pmatrix}\Sigma_1\\ \Sigma_2\\ \Sigma_3\end{pmatrix}$ are pseudovectors, $N = \begin{pmatrix}p\\ n\end{pmatrix}$ and $k_1 = \begin{pmatrix}k^+\\ k^0\end{pmatrix}$ are