

$$\Delta\sigma = \frac{kT\kappa^2}{16\pi} \left( \frac{3}{2} - 2\gamma + \ln \frac{2\epsilon kT}{e^2\kappa} \right) - \frac{kT\kappa^2}{16\pi} \frac{\sum_i (z_i e)^2 n_{0i} \ln(z_i e)}{\sum_i (z_i e)^2 n_{0i}} \quad (6)$$

( $\gamma = 0.577$ , the Euler constant). In practice it is more convenient to use this formula in the following form

$$\Delta\sigma = 39.7 \left( \sum_i z_i P_i \right) \frac{m}{\epsilon} \log[2.28 \cdot 10^{-13} (\epsilon T)^3 / m z_0^4 \sum_i z_i^2 P_i], \quad (7)$$

where  $m$  is the concentration in moles/liter.

In the case of mono-monovalent electrolytes the resultant limiting law becomes the Onsager-Samaras limiting law.<sup>2</sup> The general formula they derived has no meaning, since the correction to the limiting law, contained in this formula, is of the same order as the quantities neglected by these authors.

2. Allowance for the dielectric constant of the external medium. In the case of mono-monovalent electrolytes the potential of the electrostatic field, produced by a fixed ion in its atmosphere, with allowance for the separation surface between the two dielectrics, is determined by the following equation

$$(\Delta - \kappa^2) \varphi = - (4\pi/\epsilon) e \delta(r). \quad (8)$$

The solution of this equation is substantially more complicated than that for the case considered by Onsager and Samaras, owing to the fact that the dielectric constant of the external medium is not assumed to be zero. The adsorption potential (the potential energy of the interaction of the ion with its electrostatic image, with allowance for screening) turns out to be

$$W(z) = \frac{e^2}{2\epsilon} \int_{\kappa}^{\infty} \frac{\lambda\epsilon - \epsilon' \sqrt{\lambda^2 - \kappa^2}}{\lambda\epsilon + \epsilon' \sqrt{\lambda^2 - \kappa^2}} e^{-2\lambda z} d\lambda. \quad (9)$$

In spite of the fact that it is necessary to calculate an integral of a non-elementary function (9), which is furthermore in exponential form, it is possible, using the smallness of the parameter of the theory, to obtain an expression for the change in surface tension  $\Delta\sigma$  in terms of elementary functions

$$\Delta\sigma = \frac{e^2 (\epsilon - \epsilon') n}{2\epsilon (\epsilon + \epsilon')} \left[ \ln \frac{2\sqrt{2} \cdot \epsilon kT (\epsilon + \epsilon')}{\kappa e^2 (\epsilon - \epsilon')} + \frac{3}{2} - 2\gamma - \frac{1}{2} \left( \frac{\epsilon + \epsilon'}{\epsilon - \epsilon'} \right)^2 \ln 2 + \frac{2\epsilon^2}{(\epsilon - \epsilon')^2} \ln \frac{\epsilon + \epsilon'}{\epsilon} \right]. \quad (10)$$

When  $\epsilon' = 0$ , Eq. (10) goes into the limiting Onsager-Samaras law.<sup>2</sup>

<sup>1</sup> P. Debye and F. Hückel, *Phys. Z.* **24**, 1085 (1929).

<sup>2</sup> L. Onsager and N. T. Samaras, *J. Chem. Phys.* **2**, 528 (1934).

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## REFINEMENT OF THE APPROXIMATION FOR SMALL ANGLES IN MULTIPLE-SCATTERING PROBLEMS

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To analyze the scattering of beams of charged particles by metallic foils it is necessary to solve the problem of the spatial and angular distribution of the density produced in the beam as a result of a large number of collisions. At a depth  $\tau$ , the particle density in a monoenergetic ( $E > 10$  Mev) beam  $\psi(\mu, \tau)$

propagating at an angle  $\theta$  relative to the axis  $\tau$  ( $\mu = \cos \theta$ ) is determined from the boundary problem

$$\mu \frac{\partial \psi}{\partial \tau} + \psi(\mu, \tau) - \frac{1}{2} \int_{-1}^{+1} d\mu' \psi(\mu', \tau) \int_0^{2\pi} \frac{d\varphi}{2\pi} \mathcal{F}(\cos \chi) = 0, \quad (1.1)$$

$$\cos \chi = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos \varphi; \quad \psi(\mu, 0) = \delta(\mu - 1), \quad 0 \leq \mu \leq 1, \quad \psi(\mu, h) = 0, \quad -1 \leq \mu \leq 0. \quad (1.2)$$

The multiple-scattering distribution  $\mathcal{F}(\cos \chi)$  is assumed known. Usually the problem is solved in the small-angle approximation, i.e., subject to the following simplifications: (1) the factor  $\mu$  in front of the derivative (1.1) is assumed to be unity, (2) the conditions in (1.2) are replaced by the initial conditions  $\psi(\mu, 0) = \delta(\mu - 1)$  at  $-1 \leq \mu \leq 1$ .

A numerical method proposed by Spencer<sup>1</sup> yields a solution in this approximation with an accuracy as high as desired for a wide class of problems, permitting representation of the function  $\mathcal{F}(\cos \chi)$  by the sum<sup>2</sup>

$$\mathcal{F}(\cos \chi) = \sum_{i=1}^N A_i (1 + x_i - \cos \chi)^{-k_i}, \quad x_i \ll 1, \quad k_i = 1, 2, \dots \quad (2)$$

It is noted in Refs. 3 and 4 that along with the good agreement between solutions in the small-angle approximation and experimental results at small angles, there is considerable discrepancy at large angles (more than 15% at  $\theta > 6^\circ$  for  $E = 15.7$  Mev), which could not be explained by the authors within the framework of this approximation. Investigation of scattering at large angles is very important in connection with the study of the distribution of charge in the nucleus, of the character of the nuclear potential,<sup>3</sup> of phenomena connected with polarization, of differences in the scattering of positive and negative particles, etc.

Many computations were made to refine the small-angular approximation and to determine the region of its validity.<sup>2</sup>

1. Corrections for the small-angle approximation calculated by Spencer<sup>1</sup> were determined for the problem of scattering of 15.7 Mev electrons. In this problem

$$\mathcal{F}(\cos \chi) = x_0 (1 + x_0 - \cos \chi)^{-2}, \quad x_0 = 1.56 \cdot 10^{-6}, \quad h = 118.4.$$

This approximation is close to the Gauss function at small angles  $\theta^2 \leq x_0 h$ , and tends to the function  $4x_0 h / \theta^4$  at larger angles.

We represent  $\psi(\mu, \tau)$  as a sum  $\psi_0 + \psi_1 + \psi_2$  where  $\psi_0$  is the solution to the problem in the small-angle approximation,  $\psi_1$  is the correction in the same approximation, i.e., the solution of the equation

$$\frac{\partial \psi_1}{\partial \tau} + \psi_1 - \frac{1}{2} \int_{-1}^{+1} d\mu' \psi_1(\mu', \tau) \int_0^{2\pi} \frac{d\varphi}{2\pi} \mathcal{F}(\cos \chi) = (1 - \mu) \frac{\partial \psi_0}{\partial \tau} \quad (3)$$

with initial condition  $\psi_1(\mu, 0) = 0$  ( $-1 \leq \mu \leq 1$ ), while the correction  $\psi_2$  is the solution of the inhomogeneous equation corresponding to (1.1) with the right half containing  $(1 - \mu) \partial \psi_1 / \partial \tau$ . The boundary conditions for  $\psi_2(\mu, \tau)$  are:

$$\psi_2(\mu, 0) = 0, \quad 0 \leq \mu \leq 1; \quad \psi_2(\mu, h) = -\psi_0(\mu, h) - \psi_1(\mu, h), \quad -1 \leq \mu \leq 0.$$

The solution of problem (3) in the small-angle region is well represented by the Gauss function (in  $\theta$ ), multiplied by a certain polynomial in  $\theta^2$ ; the asymptotic expression in the large-angle region is  $2x_0 h / \theta^2$ . The absence of a strong anisotropy in the inhomogeneities of the problem for  $\psi_2(\mu, \tau)$  makes it possible

$\cos \theta$	1	0.999	0.99	0.95	0.90
$\psi_1(\mu, h)$	0.1478	0.2349	0.02623	0.004384	0.001985
$2 x_0 h / \theta^2$		0.1940	0.01834	0.003668	0.001814
$\psi_2(\mu, h)$	0.2804	0.2788	0.2651	0.2071	0.1413

to seek a solution in each of the intervals  $(0, 1)$  and  $(-1, 0)$  in the form of an expansion in Legendre polynomials  $P_n(\mu)$  with  $n \leq 3$ . The table gives the values of  $\psi_1(\mu, h)$  for  $\mu \geq 0$ , obtained by numerical integration with Spencer's method, and the calculated results for  $\psi_2(\mu, h)$  for  $\mu \geq 0$ .

2. Several general results were estab-

lished for an arbitrary scattering law, the degree of anisotropy of which is determined by the smallness of the parameter

$$\varepsilon = \frac{1}{2} \int_{-1}^{+1} (1 - \mu) \mathcal{F}(\mu) d\mu,$$

i.e., by the smallness of the rms angle upon single scattering. The small-angle approximation is represented by the sum

$$\delta(\mu - 1) e^{-\tau/\mu} + I_0(\mu, \tau),$$

where  $I_0(\mu, \tau)$  is close to the Gauss function  $\exp(-\theta^2/\bar{\theta}^2)$ ,  $\bar{\theta}^2 \approx h$ , at small angles ( $\theta^2 \lesssim \epsilon h$ ). If  $\mathcal{F}(\cos \chi)$  is taken in the form (2), the asymptotic expression at large angles is

$$2^\eta A_\eta h \theta^{-2\eta}, \quad \eta = \min k_i.$$

The free term in Eq. (3) is a quantity of the order of  $\epsilon$ . The function  $\psi_1 = O(\epsilon h \psi_0)$  at small angles ( $\theta^2 \lesssim h$ ), and  $\psi_1 = O(\theta^2 \psi_0/2)$  at greater angles. Thus, at  $\epsilon h \ll 1$ , as expected, the correction  $\psi_1$  is small in the region  $\theta^2/2 \ll 1$ . On the other hand, the comparatively large value of the correction  $\psi_2(\mu, \tau)$  in this example limits the range of validity of the small-angle approximation to the inequality  $\psi_0 \gg \psi_2$ , i.e.,  $\theta^4 \ll \epsilon h$ . Since in this problem  $\epsilon = x_0 \ln x_0$ , one would expect that the approximation here is good in the region  $\theta \lesssim 1/4_0$  and is acceptable at  $\epsilon h \ll 1$ . To obtain solutions with a sufficient degree of accuracy at large angles it is proposed to use the interpolation method developed in Refs. 2 and 5.

<sup>1</sup>L. V. Spencer, Phys. Rev. **90**, 146 (1953).

<sup>2</sup>T. A. Germogenova, Dissertation, Division of Applied Mathematics, Mathematics Institute, Academy of Sciences, U.S.S.R. (1957).

<sup>3</sup>L. V. Cooper and G. Rainwater, Phys. Rev. **97**, 492 (1955).

<sup>4</sup>L. V. Spencer and J. Blanchard, Phys. Rev. **93**, 114 (1954).

<sup>5</sup>T. A. Germogenova, Dokl. Akad. Nauk SSSR **113**, No. 2 (1957).

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## COMPARISON OF NEUTRON SPECTRA IN THE FISSION OF $U^{233}$ , $U^{235}$ , $Pu^{239}$

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THE spectra of fission neutrons from  $U^{235}$ ,  $U^{233}$ , and  $Pu^{239}$  have been reported in a number of papers.<sup>1-10</sup> Measurements of the fission neutron spectrum from  $U^{235}$  (Refs. 1-8) are in satisfactory agreement with the semi-empirical formula of Watt.<sup>3</sup>

According to the data of Mukhin, Barkov, and Gerasimov,<sup>8</sup> the fission neutron spectra from  $U^{233}$  and  $Pu^{239}$  are the same as the spectrum from  $U^{235}$ , within experimental errors of 10-20%. The data of Nereson<sup>9</sup> and of Grandl and Neuer<sup>10</sup> indicate that the neutrons from  $Pu^{239}$  are somewhat harder than those from  $U^{235}$ .

This note presents a comparison of the neutron spectra from the fission of  $U^{233}$ ,  $U^{235}$ , and  $Pu^{239}$ . Various neutron detectors were used.

The fission neutrons were obtained by irradiating samples of  $U^{233}$ ,  $U^{235}$ , and  $Pu^{239}$  with thermal neutrons from a reactor. In the first series of measurements, the neutrons were detected by using the thresholds of the reactions  $Pr^{141}(n, 2n)Pr^{140}$ ,  $Al^{27}(n, p)Mg^{27}$ ,  $P^{31}(n, p)Si^{31}$ , and  $Au^{197}(n, \gamma)Au^{198}$ . To compare the intensities of the fission neutron sources, we used a fission camera with  $U^{233}$ . The irradiation took place inside a cavity  $20 \times 20 \times 40$  cm in the thermal column of the reactor.