# TIME REVERSAL AND REACTIONS INVOLVING POLARIZED PARTICLES

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Submitted to JETP editor April 14, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 975-981 (October, 1957)

If the interaction between particles in a reversible process such as a + b $\rightleftarrows$ c + d is invariant under time reversal, one can obtain several relations between the angular distributions, polarization vectors, and polarization tensors of the particles in the direct and inverse reactions (for arbitrary particle spins). These relations may be considered a generalization of the so-called "principle of detailed balance."

$W$E shall consider reactions such as a + b $\rightarrow$ c + d ("direct" reaction D) and its inverse (I) reaction c + d $\rightarrow$ a + b, where a, b, c, and d are "elementary" particles or nuclei with arbitrary spins. We assume that the physical system (consisting of either pair a and b or c and d) is invariant under time reversal.[1,2]

Experiments on such reactions give information on their S matrix. In order to find all the elements of the S matrix we must perform a complete set of experiments, which involves measuring not only the angular distributions of the reaction products, but also their polarization vectors and tensors in experiments with a polarized beam and a polarized target. In elastic reactions of the type a + b $\rightarrow$ a + b, invariance under time reversal makes it possible to decrease the necessary complete set, just as do the conservation laws for angular momentum, etc. It will be shown in the present work (see Sec. 3), for instance, that once having measured the angular distribution for an unpolarized beam and an unpolarized target, we need measure only the polarization vectors of the products of the reaction in the next experiment. In this experiment the initial beam and target are polarized so that only their polarization vectors are nonzero. It is not necessary to measure also the polarization tensors (of rank higher than one), since what such a measurement would tell us about the S matrix will be obtained from other experiments of the complete set. In these other experiments the beam and target are polarized in a more complicated way (with nonzero polarization tensors). In general, we can conclude the following: it is possible to eliminate experiments in which one measures polarization tensors of rank higher than the highest rank polarization tensor describing the spin state of the beam and target.

For a reaction of the type a + b $\rightarrow$ c + d, time reversal invariance leads only to the conclusion that an investigation of the direct reaction can be replaced by an investigation of the inverse reaction.

1. The results of this work will be given in terms of the statistical tensors defined previously.[3] We present here the relation between the components $\rho(q, \nu)$, with $\nu = -q, -q + 1, \ldots, +q$, of a statistical tensor of rank $q$ (with $q = 0, 1, \ldots, 2i$) of a single particle with spin $i$ and the density matrix $(m_1 | \rho | m_2)$ describing the spin state of an ensemble of such particles:

$$\rho(q, \nu) = \sqrt{2i + 1} \sum_{m_1, m_2} (-1)^{i - m_2} (ii m_1 - m_2 | q\nu)(m_1 | \rho | m_2).$$

The symbol $(i_1 i_2 m_1 m_2 | q\nu) = C^{q\nu}_{i_1 m_1 i_2 m_2}$ is a Clebsch-Gordan coefficient, and the rest of the notation is consistent with the previously cited work by the author.[3]

In the a + b $\rightarrow$ c + d reaction we obtain the following expression for the statistical tensor of the particles c and d in terms of the "scattering matrix" R (in the representation in which the spin projection and momentum are diagonal) and the statistical tensor of the beam a and target b (the notation is that used previously[3]):

$$\rho'(\mathbf{p}_c, \alpha'; q_c \tau_c q_d \tau_d) = [(2i_c + 1)(2i_d + 1)]^{1/2} [(2i_a + 1)(2i_b + 1)]^{-1/2}$$

$$\times \sum_{q_a \tau_a q_b \tau_b} (\mathbf{p}_c, \alpha', \alpha'; q_c \tau_c q_d \tau_d | W_D | \mathbf{p}_a, \alpha_1, \alpha_2; q_a \tau_a q_b \tau_b) \rho_{\alpha_1, \alpha_2}(\mathbf{p}_a; q_a \tau_a q_b \tau_b); \tag{1}$$

$$(\mathbf{p}_c, \alpha_1', \alpha_2'; q_c\tau_c q_d\tau_d \mid W_D \mid \mathbf{p}_a, \alpha_1, \alpha_2; q_a\tau_a q_b\tau_b) = \sum_{m_c, m_d, m_c', m_d'} (-1)^{i_c - m_c'} (i_c i_c m_c - m_c' \mid q_c\tau_c)(-1)^{i_d - m_d'} (i_d i_d m_d - m_d' \mid q_d\tau_d) \times \tag{2}$$

$$\sum_{m_a, m_b, m_a', m_b'} (m_c m_d \mathbf{p}_c \alpha_1' \mid R \mid m_a m_b \mathbf{p}_a \alpha_1)(m_c' m_d' \mathbf{p}_c \alpha_2' \mid R \mid m_a' m_b' \mathbf{p}_a \alpha_2)^* (-1)^{-i + m_a'} (i_a i_a m_a - m_a' \mid q_a\tau_a)(-1)^{-i + m_b'} (i_b i_b m_b - m_b' \mid q_b\tau_b).$$

In what follows it is irrelevant whether all the spin indices refer to the same $z$ axis of quantization, or whether $\tau_c$, $\tau_d$, $m_c$, and $m_d$ are defined with respect to an axis of quantization[3] along $\mathbf{p}_c$, and $\tau_a$, $\tau_b$, $m_a$, and $m_b$ are defined with respect to the $\mathbf{p}_a$ direction.

For the inverse reaction we may write two similar equations which we shall call ($1'$) and ($2'$) (the momentum of particles $c$ and $d$ in the rest system is $\mathbf{p}_c$; we are interested in particles $a$ and $b$ emitted in the $\mathbf{p}_a$ direction; the coordinate system is the same). Let us establish the relation between the elements $(m_a m_b \mathbf{p}_a \mid R \mid m_c m_d \mathbf{p}_c)$ in ($2'$) (the $\alpha$ indices are suppressed) and the corresponding elements of (2).

Invariance of this process under time reversal can be written[2]

$$(\Psi, \hat{S}\Phi) = (\Phi_K, \hat{S}\Psi_K) \tag{3}$$

(a similar relation holds for $\hat{R} = \hat{S} = 1$). According to the definition of the wave function of the time-reversed state (see Sec. 3 of Ref. 2) the reversed wave function of $\Psi_{m_a m_b \mathbf{p}}$ describes a state* in which the spin projections and momentum of $a$ and $b$ have the eigenvalues $-m_a$, $-m_b$, and $-\mathbf{p}$,† i.e.,

$$\Psi_{K m_a m_b \mathbf{p}} \approx \Psi_{-m_a, -m_b, -\mathbf{p}}$$

with an accuracy up to a phase factor whose expression is given by Huby[4] and Biedenharn,[5] namely

$$\Psi_{K m_a m_b \mathbf{p}} \equiv \hat{K}\Psi_{m_a m_b \mathbf{p}} = e^{i\beta}(-1)^{m_a + m_b}\Psi_{-m_a, -m_b, -\mathbf{p}}. \tag{4}$$

The phase $\beta$, which may depend on $i_a$ and $i_b$ and even on $p$ will be unimportant in what follows. From the above we find that

$$(m_a m_b \mathbf{p}_a \mid R \mid m_c m_d \mathbf{p}_c)(m_a' m_b' \mathbf{p}_a \mid R \mid m_c' m_d' \mathbf{p}_c)^*$$

in ($2'$) can be replaced by

$$(-1)^{m_a + m_b + m_a' + m_b' + m_c + m_d + m_c' + m_d'} (-m_c, -m_d, -\mathbf{p}_c \mid R \mid -m_a, -m_b, -\mathbf{p}_a)(-m_c' - m_d', -\mathbf{p}_c \mid R \mid -m_a', -m_b', -\mathbf{p}_a)^*,$$

where the phase factor is $(-1)^{\tau_a + \tau_b + \tau_c + \tau_d}$, since $(-1)^{m_a' + m_b' + m_c' + m_d'} = (-1)^{-m_a' - m_b' - m_c' - m_d'} (m_a' + m_b' + m_c' + m_d')$ is an integer and $\tau_a = m_a - m_a'$, etc. [see Eq. (2)].

After this we replace the indices $m$ in ($2'$) by $-m$. This does not change the sums, since the indices $m$ take on the values $-i, -i+1, \ldots, i-1, i$. The factors $(-1)^{\pm i \mp m}(\text{iim} - m \mid q\tau)$ in ($2'$) then become $(-1)^{\pm i \pm m}(\text{ii} - mm \mid q\tau)$, and according to the properties of the Clebsch-Gordan coefficients this is $(-1)^{-q}(-1)^{\mp i \pm m}(\text{iim} - m \mid q - \tau)$. We note that $(-1)^{i-m} = (-1)^{-i+m}$, since $i - m$ is an integer.

We finally find that all the coefficients

$$(\mathbf{p}_a; q_a\tau_a q_b\tau_b \mid W_I \mid \mathbf{p}_c; q_c\tau_c q_d\tau_d)$$

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*The wave function $\Psi_{m_a m_b \mathbf{p}}$ describes the state of the physical system consisting of particles $a$ and $b$ (but not $c$ and $d$) whose momentum in the rest system is $\mathbf{p}$ and whose spin projections are $m_a$ and $m_b$.

†We assume that among the variables $\alpha$ (which represent all the variables other than $\mathbf{p}$, $m_a$, and $m_b$ characterizing the state of the system, e.g., intrinsic particle parity) there are none which change sign under time reversal. In addition, it is assumed for simplicity that the beam and target are not in mixed states with respect to $\alpha$, or that the density matrix $\rho$ in Eq. (1) contains the factor $\delta_{\alpha\alpha_1}\delta_{\alpha\alpha_2'}$ and that after the reaction one measures the probabilities only for possible values of $\alpha$ (if they are measured at all) which can be given by matrix elements of $\rho'$ diagonal in $\alpha$.

of the inverse reaction are known if we know all the coefficients $W_D$ of the direct $a + b \rightarrow c + d$ reaction, and vice versa:

$$(p_a, \alpha; q_a\tau_a q_b\tau_b \mid W_0 \mid p_c, \alpha'; q_c\tau_c q_d\tau_d) = (-1)^{q_a+q_b+q_c+q_d+\tau_a+\tau_b+\tau_c+\tau_d}$$

$$\times (-p_c, \alpha'; q_c, -\tau_c, q_d, -\tau_d \mid W_D \mid -p_a, \alpha; q_a, -\tau_a, q_b, -\tau_b). \tag{5}$$

**2.** The fundamental relation (5) can be obtained in a different way by expressing the invariance under time reversal in terms of the symmetry of the elements of the S (or R) matrix in the representation in which s, $\ell$, J, E, and $\alpha$ are diagonal (see the previous footnote):[2,4]

$$(i_a i_b s l \alpha \mid R^{JE} \mid i_c i_d s' l' \alpha') = (i_c i_d s' l' \alpha' \mid R^{JE} \mid i_a i_b s l \alpha). \tag{3'}$$

Expressions for the statistical tensors of particles c and d in terms of such matrix elements and the statistical tensors of the beam and target have been given previously [Eqs. (2.10) and (2.11) of Ref. 3]; to be specific we shall use henceforth the second of these equations, (2.11). This can be written in the form of (1), where

$$(p_c, \alpha'; q_c\tau_c q_d\tau_d \mid W_D \mid p_a, \alpha; q_a\tau_a q_b\tau_b) = N (4\pi)^{-2} \sum (-1)^{q'+\tau'} (q_c q_d\tau_c\tau_d \mid q'\tau') (2q_c + 1)(2q_d + 1)^{1/2} X (i_c q_c i_c; s_1' q' s_2'; i_d q_d i_d)$$

$$\times G_{\tau'}^* (J_1 l_1' s_1'; J q'; J_2 l_2' s_2') (s_1' l_1' \alpha' \mid R^{J,E(p_a)} \mid s_1 l_1 \alpha) (s_2' l_2' \alpha' \mid R^{J,E(p_a)} \mid s_2 l_2 \alpha)^*$$

$$\times G_\tau (J_1 l_1 s_1; J q; J_2 l_2 s_2) X (i_a q_a i_a; s_1 q s_2; i_b q_b i_b) [(2q_a + 1)(2q_b + 1)]^{1/2} (q_a q_b\tau_a\tau_b \mid q\tau)(-1)^{q+\tau} D_{\tau'\tau}^J (g_c g_a^{-1}). \tag{6}$$

Here $\sum$ denotes summation over $q'$, $s_1'$, $s_2'$, $\ell_1'$, $\ell_2'$, $J_1$, $J_2$, $\ell_1$, $\ell_2$, $s_1$, $s_2$, q, and J. We have written $\tau' = \tau_c + \tau_d$ and $\tau = \tau_a + \tau_b$. The remaining notation is the same as that of the work cited.

We could now write out the expression [let us call it (6')] for the coefficients $W_I$ of the inverse $c + d \rightarrow a + b$ reaction (maintaining the same notation, i.e., using primed indices such as $q'$, $\tau'$, $\ell'$, $s'$ for particles c and d, and unprimed indices for a and b); then (3') can be used to transform (6'), to the extent that this is possible, to the form of (6).

The Clebsch-Gordan coefficients, the X coefficients, and many other factors in the general term of the sum in (6') can simply be permuted so that they occur in the same places as they do in (6). Further,

$$D_{\tau\tau'}^J (g_a g_c^{-1}) = (D^{-1})_{\tau\tau'}^J (g_c g_a^{-1}) = D_{\tau'\tau}^{J*} (g_c g_a^{-1}) = (-1)^{\tau-\tau'} D_{-\tau', -\tau}^J (g_c g_a^{-1}).$$

From Simon's[6] Eq. (B.1) for $G_\tau$ it follows that

$$G_\tau \sim i^{l_1+l_2} \sum_L (l_1 l_2 00 \mid L 0)(q J\tau - \tau \mid L 0) X = i^{l_1+l_2} \sum_L (l_1 l_2 00 \mid L 0)(q J - \tau\tau \mid L 0)(-1)^{L-q-J} X \sim (-1)^{-q-J} G_{-\tau}^*$$

Since $L + \ell_1 + \ell_2$ must be even [the presence of the coefficient $(\ell_1\ell_2 00 \mid \ell_1\ell_2 L0)$], we find that $(-1)^L = (-1)^{\ell_1+\ell_2}$, and $(-1)^{\ell_1+\ell_2} i^{\ell_1+\ell_2} = i^{-\ell_1-\ell_2}$. Thus $G_\tau^* G_{\tau'}$ in (6') can be replaced by $(-1)^{q+q'} G_{-\tau'}^* G_{-\tau}$. Noting also that

$$(q_a q_b\tau_a\tau_b \mid q\tau) = (-1)^{q_a+q_b-q} (q_a q_b - \tau_a - \tau_b \mid q - \tau),$$

we finally obtain a relation which can be written in the form of (5) [or (8); see below].

It should be emphasized that in obtaining (5) by this method we started not only with a formulation of time-reversal invariance different than that of Sec. 1, but also assumed invariance under space rotations. This latter makes is possible to express the fundamental result in a form much more convenient for experiment. This involves rewriting (5) for the case in which the incident beams in the direct and inverse reactions have the same directions (usually chosen as the direction of the z axis) rather than two different directions $p_a$ and $p_c$. This is particularly convenient for a reaction of the type $a + b \rightarrow a + b$.

Let us first note that the element

$$, (p_a; q_a\tau_a q_b\tau_b \mid W_I \mid p_c; q_c\tau_c q_d\tau_d) \tag{7}$$

does not actually depend on $p_a$ and $p_c$ separately, but on their mutual orientation [more exactly, on the rotation parameters $g_c g_a^{-1}$; see Eq. (6) and Sec. 2 of Ref. 3]. Let us rotate the pair of vectors $p_a$ and $p_c$ together so that $p_c$ is directed along the original $p_a$ direction [it is assumed that the whole coordinate system in which the spin state of the beam and target is described is also rotated; so that the spin is expressed in exactly the same way as before the rotation; see Sec. 2 of Ref. 3 and Eq. (6)]. In this

procedure (7) does not change, but the rotated position of $p_a$ and its nonrotated position are symmetric with respect to the nonrotated position of $p_c$. More exactly, if the $z$ axis is chosen along $p_a$, then (7) is equal to $W_I$ for the case in which the products of the inverse reaction are emitted at the angles $\vartheta$, $\varphi + \pi$, if the element $(-p_c \mid W_D \mid -p_a)$ [which can be shown by similar considerations to be equal to $(p_c \mid W_D \mid p_a)$] describes the state in which the products of the direct reaction are emitted at the angles $\vartheta$, $\varphi$.

Thus, after some obvious changes of notation, (5) can be written

$$(q_a\tau_a q_b\tau_b \mid W_I(\vartheta, \varphi + \pi) \mid q_c\tau_c q_d\tau_d) = (-1)^{q_a+q_b+q_c+q_d+\tau_a+\tau_b+\tau_c+\tau_d} (q_c, -\tau_c, q_d, -\tau_d \mid W_D(\vartheta, \varphi) \mid q_a, -\tau_a, q_b, -\tau_b). \qquad (8)$$

3. In this section we shall show how to use (8) to obtain several relations between the experimental observables from the direct and inverse reactions. Generally, speaking, what we shall do is replace the coefficients W of (8) by their expressions in terms of angular distributions, polarization vectors, and other statistical tensors of the direct and inverse reactions, quantities which are given directly by experiment. It is clear that one can obtain as many relations between these quantities as there are coefficients W.

First we shall make one remark (A) and introduce some abbreviated notation (B).

A. Let us go over from the probabilities for processes to their cross sections (adjusted to the conditions of the experiment) and therefore introduce the statistical tensor $\rho'_{rt}$ ($n$, $p$; $q_1\tau_1 q_2\tau_2$) of the reaction products renormalized to the current, which has been defined in Appendix II of Ref. 3 [so that $\rho'_{rt}$($n$, $p$; 0, 0, 0, 0) is equal to the differential cross section per unit solid angle $\sigma(n)$].* It has been shown that the transition from $\rho'$ to $\rho'_{rt}$ is accomplished by replacing $N/(4\pi)^2$ by $h^2/4p_a^2$ in (6) and by $h^2/4p_c^2$ in (6'). Therefore if we replace $\rho'$ by $\rho'_{rt}$ in Eqs. (1) and (1'), then $W_D$ and $W_I$ must be replaced by

$$(h^2/4p_a^2)(4\pi)^2 W_D/N \text{ and } (h^2/4p_c^2)(4\pi)^2 W_I/N.$$

B. Let us denote the statistical tensors of particles $a$ and $b$ in the direct reaction

$$-\rho(p_a; 0, 0, 0, 0), \rho(p_a; 1, -1, 0, 0), \rho(p_a; 1, 0, 0, 0), \rho(p_a; 1, 1, 0, 0),$$

$$\rho(p_a; 0, 0, 1, -1), \rho(p_a; 0, 0, 1, 0), \ldots \rho(p_a; 2, -2, 0, 0), \ldots \rho(p_a; 2i_a, 2i_a, 0, 0),$$

$$\ldots \rho(p_a; 0, 0, 2i_b, 2i_b), \rho(p_a; 1, -1, 1, -1)^\dagger, \ldots \rho(p_a; 2i_a, 2i_a, 2i_b, 2i_b),$$

by $x_0$ (which is equal to unity, as indicated in the preceding footnote *), $x_1$, $x_2$, $x_3$, $\ldots$, $x_m$ [where $m = (2i_a + 1)^2(2i_b + 1)^2 - 1$]. In this way the components of all the statistical tensors are renumbered by a single index. We shall use $x'_0$, $x'_1$, $\ldots$, $x'_n$ and $y'_0$ (= 1), $y'_1$, $\ldots$, $y'_n$ [where $n = (2i_c + 1)^2 \times (2i_d + 1)^2 - 1$] for particles $c$ and $d$ in the direct and inverse reactions, respectively, and $y_0$, $y_1$, $\ldots$, $y_m$ for the statistical tensors of particles $a$ and $b$ in the inverse reaction. The letter $a$ with new indices shall be used to denote the products of $(4\pi h)^2/4N$ [see remark (A)] with both the elements $W_D$ and the equal $W_I$ corresponding to them.

Bearing in mind remark (A), Eq. (1) for the direct reaction and the corresponding equation (1') for the inverse reaction take on the form

$$x'_0 = Fp_a^{-2}(a_{00}x_0 + a_{01}x_1 + a_{02}x_2 + \cdots + a_{0m}x_m),$$

$$x'_1 = Fp_a^{-2}(a_{10}x_0 + a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m),$$

$$\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots$$

$$x'_n = Fp_a^{-2}(a_{n0}x_0 + \cdots + a_{nm}x_m), \qquad (9)$$

in the new notation, where

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*We note that the statistical tensor $\rho(n, p; 0, 0, 0, 0)$ of the beam and target is simply equal to unity (normalization of the density matrix of the initial state).

†All the statistical tensors following on this one are products of pairs of preceding one [divided by $\rho(p; 0, 0, 0, 0)$; see Appendix I in Ref. 3].

$$F = [(2i_c + 1)(2i_d + 1)]^{1/2} [(2i_a + 1)(2i_b + 1)]^{-1/2};$$

$$y_0 = F^{-1} p_c^{-2} (a_{00} y_0' + a_{30} y_1' - a_{20} y_2' + a_{10} y_3' + \cdots), \qquad \text{n terms}$$

$$y_1 = F^{-1} p_c^{-2} (a_{03} y_0' + a_{33} y_1' - a_{23} y_2' + \cdots), \qquad \text{"}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$y_m = F^{-1} p_c^{-2} (a_{0,m-4i_a-4i_b} y_0' + \cdots). \qquad \text{"}$$

(10)

The rectangular matrix $A_I$ of the $a_{ik}$ in (10) is obtained by transposing the matrix $A_D$ of the coefficient $a_{ik}$ in (9) and then performing certain definite permutations and sign changes for some of the elements. For instance, the (1, 2) element (second row, third column) of $A_I$ is

$$(1, -1, 0, 0 \mid W_I(\vartheta, \varphi + \pi) \mid 1, 0, 0, 0) (2\pi\hbar)^2 / N = (-1)(1, 0, 0, 0) \mid W_D(\vartheta, \varphi) \mid 1, 1, 0, 0) (2\pi\hbar)^2 / N = -a_{23}.$$

The method for completely studying the reaction, that is for finding experimentally all the W (or $a_{ik}$)* can be described as follows.

Let us assume we are considering the reaction $a + b \rightarrow c + d$ at some given energy. One must perform m experiments with the beam and target polarized in different ways (see below), measuring all the statistical tensors of the reaction products in each of these experiments (for all angles of emission). Then, for instance, $a_{00}$, $a_{01}$, ..., $a_{0m}$ are found from the set of equations (note that all the $x_0^{(k)}$ are equal to unity; see the footnote * on page 751)

$$(x_0')^{(1)} = F p_a^{-2} (a_{00} x_0^{(1)} + a_{01} x_1^{(1)} + \ldots + a_{0m} x_m^{(1)}),$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$(x_0')^{(m)} = F p_a^{-2} (a_{00} x_0^{(m)} + a_{01} x_1^{(m)} + \ldots + a_{0m} x_m^{(m)}),$$

($11_0$)

if the determinant $\| x_i^{(k)} \|$ does not vanish (this is the condition that the spin states of the beam and target are different in our m experiments).

In principle, the solution of the problem posed in this section can now be given as follows: one must find expressions for the $a_{ik}$ in terms of the statistical tensors of the direct reaction [by making a complete study of the direct reaction and solving a set of equations ($11_i$)] and setting them equal to the expressions for the $a_{ik}$ in terms of the statistical tensors of the inverse reaction (found in the same way). Furthermore, if all the $a_{ik}$ are known, then by inserting them into (10) we immediately obtain m relations between the statistical tensors of the direct and inverse reactions.

4. The well known relation of "semidetailed balance"

$$p_a^2 (2i_a + 1)(2i_b + 1) \sigma_D(\vartheta) = p_c^2 (2i_c + 1)(2i_d + 1) \sigma_0(\vartheta)$$

is a simple consequence of (8). We present still another illustration of the results of Sec. 3 (see also the Introduction).

Suppose we are interested in the angular distribution $\sigma_D'(\vartheta, \varphi)$ of the products of the direct reaction in which the beam a and target b are polarized in an arbitrary way. This is given by the first relation of (9). The $a_{ik}$ can be found by measuring all the statistical tensors $y_1$, $y_2$, ..., $y_m$ for the inverse reaction with completely unpolarized beam and target. If the latter are known, one can find the difference between the desired $\sigma_D'(\vartheta, \varphi)$ and the angular distribution $\sigma_D(\vartheta, \varphi)$ when a and b are unpolarized, namely

$$\sigma_D'(\vartheta, \varphi) - \sigma_D(\vartheta, \varphi) = F^2 p_c^2 p_a^{-2} (y_3 x_1 - y_2 x_2 + y_1 x_3 + y_6 x_4 - \ldots).$$

(12)

In particular, assume that only the target b is polarized so that only its polarization vector (whose cyclic† components $x_1$, $x_2$, and $x_3$ we shall denote by $\rho_{-1}$, $\rho_0$, and $\rho_{+1}$) is nonzero. Then in the inverse

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*If all the W are known, one must first solve the set of linear equations (6) for the products

$$(s_1' l_1' \mid R^{J_1 E} \mid s_1 l_1) (s_2' l_2' \mid R^{J_2 E} \mid s_2 l_2)^*,$$

in order to find the elements of the R matrix, after which the unique values of these products (though not yet of the elements themselves) will be found.

† The relations between the cyclic and the ordinary cartesian components of a vector are

$$\rho_{-1} = (\rho_x + i\rho_y) / \sqrt{2}; \quad \rho_0 = \rho_z; \quad \rho_{+1} = (-\rho_x + i\rho_y) / \sqrt{2}.$$

reaction it is sufficient to measure only the components $\rho'_{-1}$, $\rho'_0$, and $\rho'_{+1}$ of the statistical vector of particles b. Equation (12) then becomes

$$\sigma'_D(\vartheta, \varphi) - \sigma_D(\vartheta, \varphi) = F^2 p_c^2 p_a^{-2} (\rho'_{+1}\rho_{-1} - \rho'_0\rho_0 + \rho'_{-1}\rho_{+1}) = - F^2 p_c^2 p_a^{-2} (\rho'(\vartheta, \varphi + \pi)\rho). \tag{13}$$

Since the beam and target are unpolarized in (10), it can be shown that the statistical vectors of particles b emitted at angles $\vartheta$, $\varphi$ and $\vartheta$, $\varphi + \pi$ are of equal magnitude and oppositely directed (and both perpendicular to the reaction plane). In place of $\rho'$ let us introduce the polarization vector $\mathbf{N}'$ (which for particles of spin $\frac{1}{2}$ is equal to twice the expectation value of the vector spin operator), so that $\rho'(\vartheta, \varphi) = \mathbf{N}'(\vartheta, \varphi) \sigma_I(\vartheta, \varphi)$ (where $\sigma_I(\vartheta, \varphi)$ is the cross section for the inverse reaction with unpolarized c and d). Rewriting (13) (we note that $\rho = \mathbf{N} \cdot 1$) in the form

$$\sigma'_D(\vartheta, \varphi) - \sigma_D(\vartheta, \varphi) = F^2 p_c^2 p_a^{-2} (\mathbf{N}'(\vartheta, \varphi)\rho) \sigma_I(\vartheta, \varphi), \tag{14}$$

we obtain a relation for arbitrary spins which has often been obtained in the literature for various special cases (see, for instance, Lapidus[7] and the literature he refers to).

The author expresses his gratitude to L. I. Lapidus for discussing the present work.

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# ON THE METHOD OF "SECOND QUANTIZATION" IN PHASE SPACE

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Submitted to JETP editor April 24, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 982-990 (October, 1957)

In order to give a statistical description of processes in systems of interacting particles we use as independent variables the numbers of particles in different points of coordinate-momentum phase space, which at every point of phase space are random functions of the time. We give classical and quantum mechanical equations for these functions; from these equations we can, by averaging the random functions over their distributions, obtain a set of equations for the classical and quantum mechanical distribution functions. To illustrate the possible use of this method we obtain expressions for the excitation spectrum and correlation function for systems of particles interacting through central forces in the case where close interactions are unimportant. This method is convenient for considering system of particles and fields, in particular, electromagnetic interactions. It is possible to generalize this method for the relativistic case.

IN the present paper we give a method to introduce collective variables to describe a system of interacting particles which is slightly different from the considerations in the papers of Tomonaga,[1] Bogoliubov and Zubarev[2,3], and Bohm and Pines.[4] We introduce as independent variables the numbers of particles in