THE INTERACTION OF SHOCK WAVES WITH SMALL PERTURBATIONS. I*

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The interaction of oblique shock waves of arbitrary intensity with small perturbations is considered in the linear approximation, within the framework of a two-dimensional stationary problem. The flow behind the shock wave is assumed to be supersonic. The general results are applied to the study of the interaction with weak discontinuities, weak shock waves and with the interface between two media of slightly different acoustical properties.

1. STATEMENT OF THE PROBLEM

In our note of Ref. 1, a complete classification was given of the cases of the interaction of shock waves with weak discontinuities. All these cases can be considered quantitatively by means of perturbation theory. Each weak discontinuity can be represented as a small perturbation superimposed on a smooth distribution of hydrodynamic quantities. If we are interested in the behavior of all quantities in the neighborhood of the intersection of the weak discontinuity with the shock, then the flow in the vicinity of the point of interaction (which is not perturbed by the weak discontinuity) can be considered homogeneous and the shock wave can be regarded as plane. The incident weak discontinuity can be thought of as a small "incoming" perturbation, as a result of the interaction of which with the shock wave, small "outgoing" perturbations are created. The smallness of the perturbation permits us to expand the hydrodynamical equations and the boundary conditions on the shock wave in a series in the perturbations; limiting ourselves to linear terms, we can obtain the complete solution of interest to us.

Thus, let us consider a plane shock wave in homogeneous flow, and, given the incoming perturbation, let us find the outgoing perturbation. We choose the y axis in a direction perpendicular to the shock wave, and let the x axis lie in the plane of the shock wave, with its direction coinciding with the direction of the x component of the velocity; let matter flow into the shock from below.

We denote by p_0, v_0, V_0, s_0, the pressure, velocity, specific volume and entropy of the unperturbed flow before the shock wave (for $y < 0$), and by p, v, V, s, the corresponding quantities after the wave (for $y > 0$); we shall denote all small perturbations of the corresponding quantities by the symbol δ. Expanding the hydrodynamical equations in a series in small values of the perturbation, and limiting ourselves to linear terms, we obtain a system of linear equations with constant coefficients for the flow behind the shock wave:

$$(\mathbf{v}\nabla)\,\delta\mathbf{v} + V\nabla\delta p = 0, \tag{1.1}$$

$$\operatorname{div}\delta\mathbf{v} + (V/c^2)\,\mathbf{v}\nabla\delta p = 0, \tag{1.2}$$

$$(\mathbf{v}\nabla)\,\delta s = 0. \tag{1.3}$$

Similar equations are obtained for the flow in front of the shock if all quantities are given the index 0.

If the flow behind the shock is supersonic, the equations (1.1) — (1.3) are everywhere hyperbolic and have a general solution which is easily found by the method of characteristics. The latter, as is well known, consists of finding linear characteristic combinations of the desired functions which are constant along the determined lines (in our case, straight lines); these characteristics form a one-parameter set. We first note that Eq. (1.3), which has the general solution.

$$\delta s = f(x - y/k_0), \tag{1.4}$$

*The two papers published here belonged to S. P. D'iakov, who perished tragically in 1954. They were prepared for publication by his colleagues from his manuscripts — L. Landau.

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where $k_0 = v_y/v_x$ and f is an arbitrary function of its argument, is already in characteristic form. The solution of (1.4) shows that δs is a characteristic quantity, which is constant along each of the flow lines $y - k_0 x = C$, which form a one-parameter set of characteristics. Graphically speaking, the disturbance δs is "carried along" the line of flow.

To find the three other characteristic quantities,[2] we construct a linear combination of Eqs. (1.1) and (1.2) with coefficients α, β, γ:

$$(\alpha v_x + \gamma)\frac{\partial \delta v_x}{\partial x} + \alpha v_y \frac{\partial \delta v_x}{\partial y} + \beta v_x \frac{\partial \delta v_y}{\partial x} + (\beta v_y + \gamma)\frac{\partial \delta v_y}{\partial y} + \left(\alpha + \frac{v_x\gamma}{c^2}\right) V \frac{\partial \delta p}{\partial x} + \left(\beta + \frac{v_y\gamma}{c^2}\right) V \frac{\partial \delta p}{\partial y} = 0,$$

and require that all the quantities δv_x, δv_y, δp in this equation be differentiated along the same direction i.e., that the ratio of the coefficient for the derivative with respect to x to the coefficient for the derivative with respect to y be the same for all quantities δv_x, δv_y, δp:

$$\frac{\alpha v_y}{\alpha v_x + \gamma} = \frac{\beta v_y + \gamma}{\beta v_x} = \frac{\beta + \gamma v_y/c^2}{\alpha + \gamma v_x/c^2} = k. \tag{1.5}$$

This condition means that the derivative of the combination

$$I = \alpha v_y \delta v_x + (\beta v_y + \gamma)\delta v_y + (\beta + v_y \gamma/c^2) V \delta p$$

along the direction with angular coefficient k is equal to zero, i.e., this combination is characteristic for the set of characteristics $y - kx = C$. In other words, the quantity I is "carried along" the characteristic $y - kx = C$.

We can write the condition in the form of a system of linear homogeneous equations in α, β, γ:

$$(v_y - v_x k)\alpha - k\gamma = 0, \quad (v_y - v_x k)\beta + \gamma = 0, \quad -k\alpha + \beta + c^{-2}(v_y - v_x k)\gamma = 0. \tag{1.6}$$

The system has solutions differing from zero if its determinant vanishes:

$$(v_y - v_x k)\left[(v_y - v_x k)^2 - (1 + k^2)c^2\right] = 0. \tag{1.7}$$

The cubic equation (1.7) has three roots (real, if the motion is supersonic), each of which determines a set of rectilinear characteristics. To each root there corresponds a definite solution of the system (1.6) of α, β, γ, i.e., a definite linear combination I. Let us write out all these characteristic combinations.

1.
$$k_0 = v_y/v_x; \quad \alpha = \beta v_x/v_y; \quad \gamma = 0. \tag{1.8}$$

The characteristic combination is

$$\mathbf{v}\,\delta\mathbf{v} + V\delta p = I_0(x - y/k_0).$$

It is obvious that the flow line $k_0 = v_y/v_x$ is a "twofold characteristic," since two characteristic quantities δs and I_0 are carried along it.

2.
$$k_\pm = \frac{v_x v_y \pm c^2\sqrt{M^2 - 1}}{v_x^2 - c^2}; \quad \alpha = -k\beta; \quad \gamma = -(v_y - v_x k)\beta. \tag{1.9}$$

The characteristic combinations are

$$v_x \delta v_y - v_y \delta v_x \pm \sqrt{M^2 - 1}\, V\delta p = I_\pm(x - y/k),$$

where $M = v/c$. These combinations are perturbations of the acoustic type, which are propagated along the corresponding characteristics.

Assuming that all the characteristic combinations vanish save one, we obtain four different independent solutions of the hydrodynamical equations: each of these solutions contains only a single arbitrary function. It is appropriate to take four equations for the fundamental independent solutions: we can represent an arbitrary solution in the form of their linear combinations. Let us now obtain these fundamental solutions.

1. Setting $I_0 = I_\pm = 0$; $\delta s = \delta s(y - k_0 x)$, we shall have, in correspondence with (1.7) and (1.8), a solution which describes the propagation of the entropy perturbation, for which

$$\delta v = 0; \quad \delta p = 0; \quad \delta s = \delta s(y - k_0 x); \quad \delta V = (\partial V/\partial s)_p \delta s. \tag{1.10}$$

2. Assuming $\delta s = 0$, $I_{\pm} = 0$, we get a solution which corresponds to the propagation of a perturbation of the curve of velocity

$$\delta p = 0; \; v_x \delta v_y - v_y \delta v_x = 0; \; \mathbf{v}\,\delta\,\mathbf{v} = I\,(y - k_0 x). \tag{1.11}$$

Both these solutions, which are connected with the lines of flow, will for convenience be joined in a single "entropy-vortical" solution.

3. Assuming $\delta s = 0$, $I_0 = I_+ = 0$, we get a solution of the acoustic perturbation type:

$$\delta p = \delta p\,(y - k_- x),$$
$$\delta v_x = \frac{-v_x + v_y \sqrt{M^2-1}}{M^2 c^2} V \delta p, \; \delta v_y = -\frac{v_x \sqrt{M^2-1} + v_y}{M^2 c^2} V \delta p. \tag{1.12}$$

4. Finally, for $\delta s = 0$, $I_0 = I_- = 0$, we get an acoustic perturbation with reverse direction

$$\delta p = \delta p\,(y - k_+ x);$$
$$\delta v_x = -\frac{v_y \sqrt{M^2-1} + v_x}{M^2 c^2} V \delta p; \; \delta v_y = \frac{v_x \sqrt{M^2-1} - v_y}{M^2 c^2} V \delta p. \tag{1.13}$$

The position of the characteristics relative to the shock is shown in Fig. 1. The waves (points) and the trajectories (solid lines) are shown in the middle of Fig. 1 for the obstacles A and A′, which are found in front of the shock (A) and behind the shock (A′). These waves and trajectories represent the characteristics of the equations of gas dynamics. Inasmuch as they describe the propagation of a disturbance produced at a point, we can describe the determined directions of propagation — from the points A or A′ — by means of the characteristics. (See Fig. 1, where the directions are shown by arrows.)

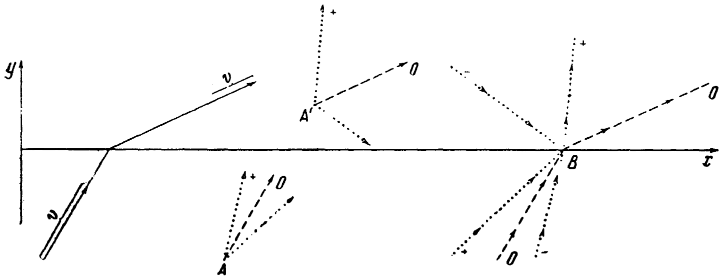

FIG. 1. ——— -shock wave — — — characteristics of the entropy-vortical perturbation, coinciding with the lines of flow, ··· — characteristics of the acoustic perturbation.

The relations between v_y and c for $y < 0$ and $y > 0$, which must hold for the undisturbed shock wave, inevitably lead to the result that for $y < 0$, all three characteristics, originating from an arbitrary point A, abut against the surface of the shock wave; for $y > 0$, one and only one of the characteristics which emanate from A′ abuts the surface of the wave. Keeping the deflections and the directions of propagation of the characteristics, we shall transform them so that the point of intersection coincides with the wave front B; we thus survey all the characteristics which converge on the given point B and emerge from it, and obtain the picture shown on the right side of Fig. 1.

All the drawings are made for a gas with $\gamma = 1.4$, for shock waves in which the pressure increases 10 times and the horizontal velocity exceeds by 1.7 times the sound velocity before the compresion; in this case, we obtain $c/v = 0.87$ (for $y > 0$) in back of the compression.

In order to understand the distribution of the characteristics graphically, let us first represent the unperturbed horizontal surface of the shock and the flow normal to it. As is well known, in such a current, $v_y > c$ for $y < 0$ and $v_y < c$ for $y > 0$; the material velocity relative to the wave is supersonic before the front and subsonic behind the front of the shock. Let us now set up the point of view of an observer moving with velocity v_x with the front: the surface of the shock then remains at rest, and we get for the velocity of the gas the horizontal component v_x both in front of and behind the front. The velocity and trajectory vectors that are shown on the left in Fig. 1. We choose v_x sufficiently large that the motion is supersonic even behind the front, $v = \sqrt{v_x^2 + v_y^2} > c$ for $y > 0$.

An obstacle placed in supersonic flow gives two stationary Mach waves, making equal angles with the trajectory of the particles of the gas which abut the obstacle. The sine of the angle is equal to the ratio of the sound velocity to the flow velocity.

On the side $y < 0$, a disturbance of any of the types enumerated can be incident on the shock wave.

From the side $y > 0$, only an accoustic disturbance of type 3 (sound disturbance "catches up" with the shock wave) can occur. Thus interaction of the shock is possible with five types of disturbances. As the result of the interaction, disturbances are formed of types 1, 2, 4, which emanate from the wave in the direction $y > 0$ along the characteristics k_0 and k_+. For a given type of incident disturbance, the solution of the problem will be a combination of the incident and (in the general case) three emergent disturbances. This linear combination is determined from the boundary conditions on the shock wave, into the derivation of which we shall now go.

Let the equation of the excited shock wave be $y = \eta(x)$, where η is a small quantity; the components of the unit vectors in the directions concerned and normal to the wave (in the approximation of interest to us) are $\mathbf{t}(1, \eta')$ and $\mathbf{n}(-\eta', 1)$, respectively.

Expanding the continuity condition of the tangential component of the velocity

$$(\mathbf{v}_0 + \delta\mathbf{v}_0)\,\mathbf{t} = (\mathbf{v} + \delta\mathbf{v})\,\mathbf{t}$$

in terms of a small disturbance, and limiting ourselves to linear terms, we find

$$\delta v_{0x} - \delta v_x = (v_y - v_{0y})\,\eta'. \tag{1.14}$$

In a similar manner, the condition which connects the velocity jump with the pressure jump

$$(\mathbf{v}_0 + \delta\mathbf{v}_0)\,\mathbf{n} - (\mathbf{v} + \delta\mathbf{v})\,\mathbf{n} = [(p - p_0 + \delta p - \delta p_0)(V_0 - V + \delta V_0 - \delta V)]^{1/2}$$

gives, after expansion,

$$\frac{\delta v_{0y} - \delta v_y}{v_{0y} - v_y} = \frac{1}{2}\left[\frac{\delta p - \delta p_0}{p - p_0} + \frac{\delta V_0 - \delta V}{V_0 - V}\right]. \tag{1.15}$$

The equation for the Hugoniot adiabatics can be written in the general form $V = V(p, p_0, V_0)$, whence

$$\delta V = \frac{\partial V}{\partial p}\delta p + \frac{\partial V}{\partial p_0}\delta p_0 + \frac{\partial V}{\partial V_0}\delta V_0. \tag{1.16}$$

Finally, there remains the condition which determines the matter velocity in front in terms of the jump of the quantities in the discontinuity:

$$[(\mathbf{v}_0 + \delta\mathbf{v}_0)\,\mathbf{n}]^2 = (V_0 + \delta V_0)^2 (p - p_0 + \delta p - \delta p_0)/(V_0 - V + \delta V_0 - \delta V),$$

which, after expansion, yields

$$\frac{2}{v_{0y}}(\delta v_{0y} - v_{0x}\eta') = \frac{2\delta V_0}{V_0} + \frac{\delta p - \delta p_0}{p - p_0} - \frac{\delta V_0 - \delta V}{V_0 - V}. \tag{1.17}$$

The system of boundary conditions (1.14) — (1.16) allows us to determine the solution uniquely.

2. SOLUTIONS OF THE EQUATION

We proceed to find the solution of interest to us, in the case in which the external disturbance takes place from below ($y < 0$). Assuming that all the quantities which describe the onset of the distrubance are given on the shock wave as a function of x, we determine all the quantities on the shock wave (for $y = 0$) which describe the onset of the disturbance, dividing them into parts corresponding to the independent solutions described. Eliminating $\delta V'$ and η' from (1.15) and (1.17), by means of (1.14) and (1.16), we get the following relations:

$$\frac{\delta v_{0y} - \delta v_y}{v_{0y} - v_y} = \frac{1}{2}\left[\frac{\delta p}{p - p_0}\left(1 - j^2\frac{\partial V}{\partial p}\right) - \frac{\delta p_0}{p - p_0}\left(1 + j^2\frac{\partial V}{\partial p_0}\right) + \frac{\delta V_0}{V_0 - V}\left(1 - \frac{\partial V}{\partial V_0}\right)\right],$$

$$\begin{aligned}\frac{\delta v_{0x} - \delta v_x}{v_{0y} - v_y} = &-\frac{\delta v_{0y}}{v_{0x}} + \frac{v_{0y}}{2v_{0x}}\left[\frac{2\delta V_0}{V_0} + \frac{\delta p}{p - p_0}\left(1 + j^2\frac{\partial V}{\partial p}\right)\right.\\ &\left. - \frac{\delta p_0}{p - p_0}\left(1 - j^2\frac{\partial V}{\partial p_0}\right) - \frac{\delta V_0}{V_0 - V}\left(1 - \frac{\partial V}{\partial V_0}\right)\right].\end{aligned} \tag{2.1}$$

Here by $\delta\mathbf{v}$ we mean the linear combination of two quantities, one of which is determined by the entropy vortex $[\delta\mathbf{v}^{(1)}]$ solution, the other, by the departing sound $[\delta\mathbf{v}^{(2)}$: $\delta\mathbf{v} = \delta\mathbf{v}^{(1)} + \delta\mathbf{v}^{(2)}]$. We multiply the first of these equations by v_x, the second by v_y, and subtract one from the other:

$$2\cot\varphi\,(v_y\delta v_x^{(2)} - v_x\delta v_y^{(2)}) - \left[V_0\cot^2\varphi\left(1 - j^2\frac{\partial V}{\partial p}\right) - V\left(1 + j^2\frac{\partial V}{\partial p}\right)\right]\delta p = F(x), \tag{2.2}$$

where

$$F(x) = 2\cot\varphi\,(v_y\delta v_{0x} - v_x\delta v_{0y}) - \left[V_0\cot^2\varphi\left(1 + j^2\frac{\partial V}{\partial p_0}\right) - V\left(1 - j^2\frac{\partial V}{\partial p_0}\right)\right]\delta p_0 + 2j\frac{V}{V_0}(V_0 - V)\,\delta v_{0y}$$
$$+ j^2\left[\left(1 - \frac{\partial V}{\partial V_0}\right)(V + V_0\cot^2\varphi) - 2\frac{V}{V_0}(V_0 - V)\right]\delta V_0. \tag{2.3}$$

In the derivation, we made use of the relations

$$v_x = v_{0x} = jV_0\cot\varphi;\quad v_y = jV;\quad v_{0y} = jV_0.$$

The contribution of the entropy-vertical solution on the left hand side (2.2) is equal to zero identically, by virtue of (1.11).

Furthermore, according to (1.13),

$$v_y\delta v_x^{(2)} - v_x\delta v_y^{(2)} = -\sqrt{M^2 - 1}\,V\delta p.$$

Substituting this in (2.2), we find the expression for δp on the shock:

$$\delta p(x) = -F(x)/D_+, \tag{2.4}$$

$$D_+ = V_0\cot^2\varphi\left(1 - j^2\frac{\partial V}{\partial p}\right) - V\left(1 + j^2\frac{\partial V}{\partial p}\right) + 2\cot\varphi\sqrt{M^2 - 1}\,V. \tag{2.5}$$

Further from, (1.13).

$$\delta v_x^{(2)}(x) = (v_y\sqrt{M^2 - 1} + v_x)\,VF(x)/M^2c^2D_+,\qquad \delta v_y^{(2)}(x) = (v_y - v_x\sqrt{M^2 - 1})\,VF(x)/M^2c^2D_+. \tag{2.6}$$

We now note that $\delta V = \delta V^{(1)} + \delta V^{(2)}$, where $\delta V^{(1)} = (\partial V/\partial s)_p\,\delta s$ is the contribution of the entropy solution and $\delta V^{(2)} = -(V/c)^2\delta p$ that of the sound. Substituting this in (in (1.16), we find:

$$\delta V^{(1)}(x) = \left[\frac{\partial V}{\partial p} + \frac{V^2}{c^2}\right]\delta p(x) + \frac{\partial V}{\partial p_0}\delta p_0(x) + \frac{\partial V}{\partial V_0}\delta V_0(x) = -\frac{\partial V/\partial p + V^2/c^2}{D_+}F(x) + \frac{\partial V}{\partial p_0}\delta p_0(x) + \frac{\partial V}{\partial V_0}\delta V_0(x); \tag{2.7}$$

$$\delta s(x) = (\partial s/\partial V)_p\,\delta V_1(x).$$

We multiply the first equation of (2.1) by v_y, and the second by v_x, and combine:

$$\mathbf{v}(\delta\mathbf{v}^{(1)} + \delta\mathbf{v}^{(2)}) = \mathbf{v}_0\delta\mathbf{v}_0 - \frac{1}{2}\delta p\left[V\left(1 - j^2\frac{\partial V}{\partial p}\right) + V_0\left(1 + j^2\frac{\partial V}{\partial p}\right)\right]$$
$$+ \frac{1}{2}\delta p_0\left[V\left(1 + j^2\frac{\partial V}{\partial p_0}\right) + V_0\left(1 - j^2\frac{\partial V}{\partial p_0}\right)\right] - \frac{1}{2}j^2(V_0 - V)\left(1 + \frac{\partial V}{\partial V_0}\right)\delta V_0.$$

Noting that according to (1.13), $\mathbf{v}\delta\mathbf{v}^{(2)} = -V\delta p$, we finally obtain

$$I_0(x) = \mathbf{v}\,\delta\mathbf{v}^{(1)}(x) = \mathbf{v}_0\,\delta\mathbf{v}_0(x) + \frac{1}{2}\,\frac{(V_0 - V)\left(1 + j^2\frac{\partial V}{\partial p}\right)}{D_+}F(x)$$
$$+ \frac{1}{2}\left[V\left(1 + j^2\frac{\partial V}{\partial p_0}\right) + V_0\left(1 - j^2\frac{\partial V}{\partial p_0}\right)\right]\delta p_0(x) - \frac{1}{2}j^2(V_0 - V)\left(1 + \frac{\partial V}{\partial V_0}\right)\delta V_0(x),$$
$$\delta v_x^{(1)}(x) = (v_x/M^2c^2)\,I_0(x);\quad \delta v_y^{(1)}(x) = (v_y/M^2c^2)\,I_0(x). \tag{2.8}$$

We obtained values of the quantities which describe the excitation reaching the shock wave as a function of x. Now, substituting $x - y/k_0$ for x in δV_1 and I_0, and $x - y/k_+$ for x in δp, and summing the acoustic and the entropy solutions, we find the total solution of the hydrodynamic equations for $y > 0$:

$$\delta p(x, y) = -F(x - y/k_+)/D_+,\quad \delta V(x, y) = \frac{V^2}{c^2D_+}F\left(x - \frac{y}{k_+}\right) - \frac{\partial V/\partial p + V^2/c^2}{D_+}F\left(x - \frac{y}{k_0}\right) + \frac{\partial V}{\partial p_0}\delta p_0\left(x - \frac{y}{k_0}\right) + \frac{\partial V}{\partial V_0}\delta V_0(x),$$
$$\delta v_x(x, y) = \frac{v_y\sqrt{M^2 - 1} + v_x}{M^2c^2D_+}VF\left(x - \frac{y}{k_+}\right) + \frac{v_x}{M^2c^2}I_0\left(x - \frac{y}{k_0}\right)$$
$$\delta v_y(x, y) = \frac{v_x - v_x\sqrt{M^2 - 1}}{M^2c^2D_+}VF\left(x - \frac{y}{k_+}\right) + \frac{v_y}{M^2c^2}I_0\left(x - \frac{y}{k_0}\right). \tag{2.9}$$

Equations (2.9) represent the local linear relation of the departing perturbations with the given incident one. It follows from these equations that the departing perturbations will be of the same type as the incoming. Since, if the arriving perturbation is a weak acoustic shock or the tangential discontinuity is of low intensity, the departing perturbations will also consist of a weak shock wave (or weak rarefaction wave), and a tangential discontinuity of low intensity. If the incident perturbation is a discontinuity of arbitrary hydrodynamical quantities (a weak or a weak tangential discontinuity), the weak and weak tangential discontinuities which are produced as the result of the interaction will also be discontinuities of arbitrary hydrodynamical quantities, etc.

We now determine the form of the perturbation of a shock wave. It is appropiate to characterize it by the curvature K, which is equal to $\eta''(x)$ in our approximation. Substituting the values of δp and δV on the shock in Eq. (1.17), we find

$$K(x)=\eta''(x)=\tan\varphi\left[\frac{\delta v'_{0y}(x)}{v_{0y}}+\frac{1}{2}\frac{\delta p'_0(x)}{p-p_0}\left(1-j^2\frac{\partial V}{\partial p_0}\right)+\frac{2V-V_0\left(1+\frac{\partial V}{\partial V_0}\right)}{2V_0(V_0-V)}\delta V'_0(x)+\frac{1}{2}\frac{F'(x)}{D(p-p_0)}\left(1+j^2\frac{\partial V}{\partial p}\right)\right. . \tag{2.10}$$

We see that the curvature of a shock wave is determined by the first derivatives of hydrodynamical quantities.

3. INTERACTION OF THE SHOCK WAVE WITH A WEAK ACOUSTIC WAVE WHICH ENTERS FROM AN INCOMPRESSIBLE GAS

Let us consider the interaction with a weak acoustic shock wave which can fall along either of two characteristics k_{+0} and k_{-0}; in correspondence with this we must distinguish two cases (Fig. 2a and Fig. 2b). As a result of the interaction there arise, for $y > 0$, a weak shock wave (or a rarefaction wave of low intensity), which is associated with the characteristic k_+, and a tangential discontinuity of low intensity, which is propagated along the characteristic k_0. We compute the intensity of the outgoing discontinuities.

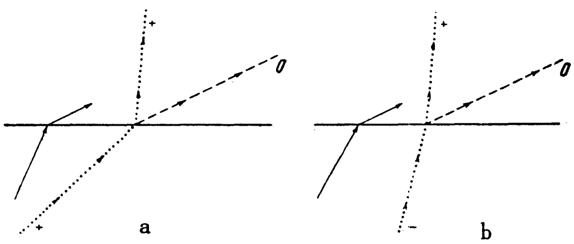

FIG. 2

We choose the origin of the coordinates as the point of intersection of the weak shock wave with the front of the strong shock wave. The incident weak shock is an excitation of the acoustic type:

$$\delta p_0(x,y)=\begin{cases}0 & \text{for } x<y/k_{\pm 0}\\ \delta p_0\equiv\text{const} & \text{for } x>y/k_{\pm 0}.\end{cases} \tag{3.1}$$

The quantity δp_0 can serve as a measure of the intensity of the wave. The quantities δv_0 and δV_0 are connected with δp_0 by Eqs. (1.12) and (1.13). Substituting these quantities in (2.3) and (2.4), we find the following expression for δp:

$$\delta p(x)=a^{(\pm)}_{pp_0}\delta p_0(x), \tag{3.2}$$

where

$$a^{(\pm)}_{pp_0}=\frac{1}{D_+}\left\{\frac{2j^2V_0}{M_0^2c_0^2}\left[\pm\cot\varphi\sqrt{M_0^2-1}\,(V^2+V_0^2\cot^2\varphi)-(V_0-V)(V_0\cot^2\varphi-V)\right]+\left[V_0\cot^2\varphi\left(1+j^2\frac{\partial V}{\partial p_0}\right)-V\left(1-j^2\frac{\partial V}{\partial p_0}\right)\right]\right.$$
$$\left.+\frac{j^2V_0^2}{c_0^2}\left[\left(1-\frac{\partial V}{\partial V_0}\right)(V+V_0\cot^2\varphi)-\frac{2V}{V_0}(V_0-V)\right]\right\}$$

is the "transmission coefficient." If $a_{pp_0} > 0$, the outgoing acoustic perturbation is a weak shock wave; if $a_{pp_0} < 0$, it is a rarefaction wave of low intensity. We turn our attention to the fact that the quantity D can become zero; in this case, the "transmission coefficient" $a^{(\pm)}_{pp_0}$ tends to infinity — a singular "resonance" is observed — i.e., an amplification of the previous shock wave. It is easy to understand what the possibility of such a resonance means physically. For $D_+ = 0$, $a^{(\pm)}_{pp_0} = \infty$, a solution of the equation is possible with finite δp in the absence of a perturbation, which is incident on the shock wave in front; we are dealing with a spontaneous "splitting off" of a weak shock from the shock wave. Thus the condition $D = 0$ stands for such a connection of the parameters of the shock wave for which a Mach reflection of a weak shock wave is possible.[3,4]

Now let us compute the intensity of the tangential discontinuity. In accord with (2.7), we have

$$\delta V^{(1)}(x) = \left(\frac{\partial V}{\partial p} + \frac{V^2}{c^2}\right)\delta p(x) + \left(\frac{\partial V}{\partial p_0} - \frac{V_0^2}{c_0^2}\frac{\partial V}{\partial V_0}\right)\delta p_0(x) = a_{Vp_0}\delta p_0(x),$$
$$a_{Vp_0} = a_{pp_0}(\partial V/\partial p + V^2/c^2) + \partial V/\partial p_0 - (V_0/c_0)^2 \partial V/\partial V_0. \tag{3.3}$$

The jump of the tangential component of the velocity at a tangential discontinuity is determined by the quantity I_0. According to (2.8), we get

$$I(x) = a_{Ip_0}\delta p_0, \quad a_{Ip_0} = -\frac{1}{2}(V_0 - V)\left[\left(1 + j^2\frac{\partial V}{\partial p}\right)a_{pp_0} + \left(1 + j^2\frac{\partial V}{\partial p_0}\right) - \frac{j^2V_0^2}{c_0^2}\left(1 + \frac{\partial V}{\partial V_0}\right)\right]. \tag{3.4}$$

The shock at the origin will obviously have an inclination at a finite angle, proportional to the intensity of the incident weak shock wave. Making use of (2.10), we get the following formula for the angle of inclination χ:

$$\chi \approx a_{\chi p_0}\delta p_0, \tag{3.5}$$

where

$$a_{\chi p_0} = \tan\varphi\left\{\frac{\pm\cot\varphi\sqrt{M_0^2 - 1} - 1}{M_0^2c_0^2}V_0 + \frac{1}{2(p - p_0)}\left[1 - j^2\frac{\partial V}{\partial p_0} - a_p\left(1 + j^2\frac{\partial V}{\partial p}\right) - \frac{j^2V_0}{2c_0^2}\left[2V - V_0\left(1 + \frac{\partial V}{\partial V_0}\right)\right]\right]\right\}.$$

4. INTERACTION OF A WEAK DISCONTINUITY WITH A SHOCK WAVE

Now let us assume that a small perturbation with a weak discontinuity is incident on the shock wave; for definiteness, we shall consider that the weak discontinuity is a discontinuity of the derivatives of the hydrodynamical quantities. We choose the origin of the coordinates as the point of intersection of the weak discontinuity with the shock wave. Expansion of any quantity along the shock wave not far from the origin will have the form

$$f = \begin{cases} f(0) + (A_f - \Delta_f)x & \text{for } x < 0 \\ f(0) + A_f x & \text{for } x > 0, \end{cases} \tag{4.1}$$

where the quantity Δ_f is proportional in the proper fashion to the determined intensity of the incident weak discontinuity. The intensity of the weak discontinuity is appropriately determined as the jump of the derivative of the hydrodynamic quantity along the normal to the discontinuity bearing the characteristic $(\partial f/\partial n)_+ - (\partial f/\partial n)_-$; in this case we have agreed to consider as the positive direction of the normal the direction obtained from the direction of the characteristic turned through an angle of $\pi/2$ in the clockwise direction. Thus, the intensity of a weak discontinuity of the acoustic type will be determined by the jump in the pressure gradient along the normal $n_\pm$ to the acoustic characteristic $\Delta^{(\pm)} = (\partial\delta p/\partial n)_+ - (\partial\delta p/\partial n)_-$, the intensity of the weak tangential discontinuity — by the jumps of the entropy gradient and the quantity I_0:

$$\Delta^{(01)} = \left(\frac{\partial\delta s}{\partial n_0}\right)_+ - \left(\frac{\partial\delta s}{\partial n_0}\right)_-, \quad \Delta^{(02)} = \left(\frac{\partial\delta I_0}{\partial n_0}\right)_+ - \left(\frac{\partial\delta I_0}{\partial n_0}\right)_-$$

along the normal n_0 to the flow line bearing the discontinuity. The connection of quantities of the type Δ_f with Δ, $\Delta^{(1)}$, $\Delta^{(2)}$ is determined from elementary geometrical considerations.

Thus, it is easy to see that

$$\Delta_p = \frac{|k_{(\pm)}|}{\sqrt{1 + k_\pm^2}}\Delta^{(\pm)} = \left|\frac{v_xv_y \pm c^2\sqrt{M^2 - 1}}{c(\pm v_x\sqrt{M^2 - 1} + v_y)}\right|\Delta^{(\pm)}, \quad \Delta_V = \frac{k_0}{\sqrt{1 + k_0^2}}\left(\frac{\partial V}{\partial s}\right)_p\Delta^{(01)} = \frac{v_y}{Mc}\left(\frac{\partial V}{\partial s}\right)_p\Delta^{(01)},$$
$$\Delta_{I_0} = \frac{k_0}{\sqrt{1 + k_0^2}}\Delta^{(02)} = \frac{v_y}{Mc}\Delta^{(02)}. \tag{4.2}$$

After these preliminary remarks, we consider the interaction of a weak discontinuity of the acoustic type with the shock wave. It is obvious that the mutual relations of the quantities Δ_f on the shock wave is the same as for the f themselves; therefore,

$$\Delta_p = a_{pp_0}\Delta_{p_0}, \quad \Delta_V = a_{Vp_0}\Delta_{p_0}, \quad \Delta_{I_0} = a_{Ip_0}\Delta_{p_0}, \tag{4.3}$$

where a_p, a_V, a_I are coefficients determined in Sec. 2.

Now, expressing Δ_{p_0}, Δ_p, Δ_V; Δ_I in terms of the intensity of the incident ($\Delta_0^{(\pm)}$) and the transmitted ($\Delta^{(+)}$, $\Delta^{(01)}$, $\Delta^{(02)}$), weak discontinuities in accord with (4.2), we obtain equations which connect the $\Delta^{(+)}$, $\Delta^{(01)}$, $\Delta^{(02)}$, with $\Delta_0^{(\pm)}$:

$$\Delta^{(+)} = a_{pp_0}^{(\pm)} \frac{c(v_x\sqrt{M^2-1}+v_y)}{v_xv_y+c^2\sqrt{M^2-1}} \left|\frac{v_{0x}v_{0y} \pm c_0^2\sqrt{M_0^2-1}}{c_0(\pm v_{0x}\sqrt{M_0^2-1}+v_{0y})}\right| \Delta_0^{(\pm)}, \quad \Delta^{(01)} = a_{Vp_0}\frac{Mc}{v_y}\left(\frac{\partial s}{\partial V}\right)_p \left|\frac{v_{0x}v_{0y} \pm c_0^2\sqrt{M_0^2-1}}{c_0(\pm v_{0x}\sqrt{M_0^2-1}+v_{0y})}\right| \Delta_0^{(\pm)},$$

$$\Delta^{(02)} = a_{Ip_0}\frac{Mc}{v_y}\left|\frac{v_{0x}v_{0y} \pm c_0^2\sqrt{M_0^2-1}}{c_0(\pm v_{0x}\sqrt{M_0^2-1}+v_{0y})}\right| \Delta_0^{(\pm)}. \tag{4.4}$$

We note that, close to "resonance" ($D_+ = 0$), the intensity of the transmitted discontinuities is strongly increased.

Let us determine the form of the shock wave close to the origin. The relation (2.10) shows that η' is continuous, but the curvature $K \approx \eta''(x)$ has a finite jump Δ_K at the origin, equal to

$$\Delta_K = a_{xp_0}\left|\frac{v_{0x}v_{0y} \pm c_0^2\sqrt{M_0^2-1}}{c_0(\pm v_{0x}\sqrt{M_0^2-1}+v_{0y})}\right| \Delta_0^{(\pm)}. \tag{4.5}$$

5. INTERACTION WITH A SHOCK WAVE OF A TANGENTIAL DISCONTINUITY OF LOW INTENSITY AND OF AN ENTROPY PERTURBATION IN AN INCOMPRESSIBLE GAS

Let a tangential discontinuity of low intensity, which is characterized by two quantities δV_0 and I_0

$$\delta V_0(x, y) = \begin{cases} 0 & \text{for } x < y/k_0^{(0)} \\ \delta V_0 = \text{const} & \text{for } x > y/k_0^{(0)} \end{cases}$$
$$I_0(x, y) = \begin{cases} 0 & \text{for } x < y/k_0^{(0)} \\ I_0 = \text{const} & \text{for } x > y/k_0^{(0)}. \end{cases} \tag{5.1}$$

be incident along the characteristic $k_0^{(0)}$.

As a result of the interaction, there arise for $y > 0$ a tangential discontinuity of low intensity and a weak shock wave (or a rarefaction wave of low intensity) (Fig. 3).

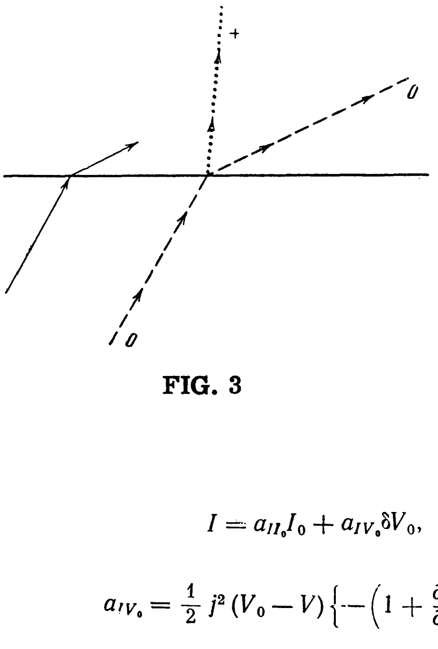

FIG. 3

Elementary calculations, by means of Eqs. (2.3) and (2.4), give the following expression for intensity of the transmitted shock wave (rarefaction wave)

$$\delta p = a_{pI_0}I_0 + a_{pV_0}\delta V_0, \quad a_{pI_0} = \frac{2j^2(V_0-V)}{M_0^2c_0^2D_+}(V_0^2\cot^2\varphi - V)\,a_{pV_0}$$
$$= \frac{j^2}{D_+}\left[\left(1-\frac{\partial V}{\partial V_0}\right)(V+V_0\cot^2\varphi) - 2\frac{V}{V_0}(V_0-V)\right]\delta V_0. \tag{5.2}$$

The intensity of the transmitted tangential discontinuity is determined by means of Eqs. (2.7) and (2.8):

$$\delta V^{(1)} = a_{VI_0}I_0 + a_{VV_0}\delta V_0, \quad a_{VI_0} = \frac{2j^2(V_0-V)}{M_0^2c_0^2D_+}(V_0\cot^2\varphi - V)\left(\frac{\partial V}{\partial p}+\frac{V^2}{c^2}\right)I_0, \tag{5.3}$$
$$a_{VV_0} = \left\{\frac{\partial V}{\partial V_0} - \left(\frac{\partial V}{\partial p}+\frac{V^2}{c^2}\right)\frac{j^2}{D_+}\left[\left(1-\frac{\partial V}{\partial V_0}\right)(V+V_0\cot^2\varphi) - 2\frac{V}{V_0}(V_0-V)\right]\right\}\delta V_0,$$

$$I = a_{II_0}I_0 + a_{IV_0}\delta V_0, \quad a_{II_0} = \left[1 - \frac{1}{2}\frac{j^2(V_0-V)^2}{M_0^2c_0^2D_+}(V_0\cot^2\varphi - V)\left(1+j^2\frac{\partial V}{\partial p}\right)\right], \tag{5.4}$$

$$a_{IV_0} = \frac{1}{2}j^2(V_0-V)\left\{-\left(1+\frac{\partial V}{\partial V_0}\right)+\frac{1}{D_+}\left(1+j^2\frac{\partial V}{\partial p}\right)\left[\left(1-\frac{\partial V}{\partial V_0}\right)(V+V_0\cot^2\varphi) - 2\frac{V}{V_0}(V_0-V)\right]\right\}\delta V_0.$$

The shock wave at the point of intersection will make a break at a finite angle χ, which is determined from (2.10):

$$\chi = a_{\chi I_0} I_0 + a_{\chi V_0}\delta V_0, \qquad a_{\chi I_0} = \tan\varphi\left[1 - \frac{1}{D_+}\left(1 + j^2\frac{\partial V}{\partial p}\right)(V_0\cot^2\varphi - V)\right]\frac{I_0}{M_0^2 c_0^2},$$

$$a_{\chi V_0} = \frac{\tan\varphi}{2(V_0 - V)}\left\{\frac{V}{V_0} - 1 - \frac{\partial V}{\partial V_0} - \frac{1}{D_+}\left(1 + j^2\frac{\partial V}{\partial p}\right)\left[\left(1 - \frac{\partial V}{\partial V_0}\right)(V + V_0\cot^2\varphi) - 2\frac{V}{V_0}(V_0 - V)\right]\right\}. \tag{5.5}$$

We now consider the interaction of a weak tangential discontinuity (the type of discontinuity of derivatives of hydrodynamical quantities) with a shock wave, as a result of which weak and weak-tangential discontinuities are formed for y > 0.

Denoting the intensities of the incident weak tangential discontinuity by $\Delta_0^{(01)}$ and $\Delta_0^{(02)}$, we can easily find the expressions

$$\Delta^{(+)} = \frac{v_{0y}}{M_0 c_0}\frac{c\,(v_x\sqrt{M^2-1} + v_y)}{v_x v_y + c^2\sqrt{M^2-1}}\left[a_{pI_0}\Delta_0^{(02)} + a_{pV_0}\left(\frac{\partial V_0}{\partial s_0}\right)_{p_0}\Delta_0^{(01)}\right],$$

$$\Delta^{(01)} = \frac{V_0}{V}\frac{Mc}{M_0 c_0}\left(\frac{\partial s}{\partial V}\right)_p\left[a_{VI_0}\Delta_0^{(02)} + a_{pV_0}\left(\frac{\partial V_0}{\partial s_0}\right)_{p_0}\Delta_0^{(01)}\right], \quad \Delta^{(02)} = \frac{V_0}{V}\frac{Mc}{M_0 c_0}\left[a_{II_0}\Delta_0^{(02)} + a_{IV_0}\left(\frac{\partial V_0}{\partial s_0}\right)_{p_0}\Delta_0^{(01)}\right]. \tag{5.6}$$

with the aid of (4.2).

The curvature of the shock wave at the origin will have a finite discontinuity Δ_K, equal to

$$\Delta_K = \frac{v_{0y}}{M_0 c_0}\left[a_{\chi V_0}\left(\frac{\partial V_0}{\partial s_0}\right)_{p_0}\Delta_0^{(01)} + a_{\chi I_0}\Delta_0^{(02)}\right]. \tag{5.7}$$

6. REFLECTION FROM A SHOCK OF AN ACOUSTIC WAVE PROPAGATED IN A COMPRESSIBLE GAS AND GENERATING AN ENTROPY-VORTICAL PERTURBATION IN SUCH A CASE

Let us now consider the case in which the incident perturbation comes from the region of flow behind the shock wave ("overtakes" the shock wave). The flow in front of the shock wave in this case is subsonic; in Eqs. (2.1), (2.2), we must set δV_0, δp_0, $\delta \mathbf{v}_0$ equal to zero. The perturbation behind the shock wave will be a combination of the incident acoustic perturbation and two outgoing perturbations — the sonic and the entropy-vortical:

$$\delta p = \delta p^{(0)} + \delta p^{(2)}, \quad \delta V = \delta V^{(0)} + \delta V^{(1)} + \delta V^{(2)}, \quad \delta \mathbf{v} = \delta \mathbf{v}^{(0)} + \delta \mathbf{v}^{(1)} + \delta \mathbf{v}^{(2)}, \tag{6.1}$$

where the quantities $\delta p^{(0)}$, $\delta V^{(0)}$, $\delta \mathbf{v}^{(0)}$ describe the incoming acoustic excitation.

Assuming F = 0 in Eq. (2.2) and substituting $\delta p^{(0)} + \delta p^{(2)}$ for δp, $\delta \mathbf{v}^{(2)} + \delta \mathbf{v}^{(0)}$ for $\delta \mathbf{v}^{(2)}$, we obtain the following ralations:

$$2\cot\varphi\,(v_y\delta v_x^{(0)} - v_x\delta v_y^{(0)}) - \left[V_0\cot^2\varphi\left(1 - j^2\frac{\partial V}{\partial p}\right) - V\left(1 + j^2\frac{\partial V}{\partial p}\right)\right]\delta p^{(0)}$$

$$= -2\cot\varphi\,(v_y\delta v_x^{(2)} - v_x\delta v_y^{(2)}) + \left[V_0\cot^2\varphi\left(1 - j^2\frac{\partial V}{\partial p}\right) - V\left(1 + j^2\frac{\partial V}{\partial p}\right)\right]\delta p^{(2)}. \tag{6.2}$$

Further, for the incoming perturbation, in accord with (1.12), we have

$$v_y\delta v_x^{(0)} - v_x\delta v_y^{(0)} = \sqrt{M^2-1}\,V\delta p^{(0)};$$

for the outgoing acoustic perturbation, in accord with (1.13),

$$v_y\delta v_x^{(2)} - v_x\delta v_y^{(2)} = -\sqrt{M^2-1}\,V\delta p^{(2)};$$

Substituting this in (6.2), we find the quantity $\delta p^{(2)}$ which characterizes the intensity of the outgoing acoustic perturbation

$$\delta p^{(2)} = -(D_-/D_+)\,\delta p^{(0)}, \tag{6.3}$$

where D_- differs from D_+ by the change in sign in the radical $\sqrt{M^2-1}$. The remaining quantities in the outgoing sound perturbation are computed in elementary fashion by means of Eq. (1.13).

We compute the entropy-vortical perturbation. With the aid of (1.15), we find

$$\delta V^{1)} = \frac{4}{D_+} \cot \varphi \sqrt{M^2-1}\, V \left(\frac{\partial V}{\partial p} + \frac{V^2}{c^2}\right) \delta p^{(0)}. \tag{6.4}$$

The quantity I_0, as is easy to see, is equal to

$$I_0 = -\frac{2}{D_+} \cot \varphi \sqrt{M^2-1}\, V (V_0 - V)\left(1 + j^2 \frac{\partial V}{\partial p}\right) \delta p^{(0)}. \tag{6.5}$$

The form of the shock wave is determined directly from (1.17):

$$\eta' = -\frac{2\sqrt{M^2-1}\, V}{j^2 (V_0 - V) D_+}\left(1 + j^2 \frac{\partial V}{\partial p}\right) \delta p^{(0)}. \tag{6.7}$$

Let the incident perturbation be a weak shock wave

$$\delta p^{(0)}(x, y) = \begin{cases} 0 & \text{for } x < y/k_- \\ \delta p^{(0)} = \text{const} & \text{for } x > y/k_-. \end{cases}$$

As a result of the interaction, a reflected weak shock wave (or a rarefaction wave of low intensity) and a tangential discontinuity of low intensity (Fig.4) are formed. In this case, Eq. (6.3) gives the intensity of the reflected shock wave (rarefaction wave) directly. Equations (6.4) and (6.5) determine the intensity of the tangential discontinuity.

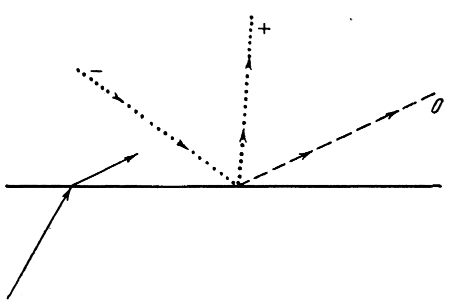

FIG. 4

The shock wave at the origin will undergo a break at a finite angle, the magnitude of which will be determined by the right side of Eq. (6.7).

If the incident excitation is a weak discontinuity (of the type of discontinuity of derivatives of hydrodynamical quantities), then, as a result of the interaction, outgoing weak and weak-tangential discontinuities are formed. Making use of the notations of the previous sections, we find the following relations between the intensities of the outgoing and the incoming weak discontinuities:

$$\Delta = -\frac{D_-}{D_+} \frac{|v_x v_y - c^2\sqrt{M^2-1}|}{v_x v_y + c^2 \sqrt{M^2-1}} \frac{v_x \sqrt{M^2-1} + v_y}{|v_x \sqrt{M^2-1} - v_y|} \Delta^{(0)},$$

$$\Delta_1 = + \frac{4 \cot \varphi \sqrt{M^2-1}\, VM}{D_+} \left(\frac{\partial s}{\partial V}\right)_p \left(\frac{\partial V}{\partial p} + \frac{V^2}{c^2}\right) \left| \frac{v_x v_y - c^2 \sqrt{M^2-1}}{v_y (v_x \sqrt{M^2-1} - v_y)} \right| \Delta^{(0)}, \tag{6.8}$$

$$\Delta_2 = -\frac{2 \cot \varphi \sqrt{M^2-1}\, VM}{D_+} (V_0 - V)\left(1 + j^2 \frac{\partial V}{\partial p}\right) \left| \frac{v_x v_y - c^2 \sqrt{M^2-1}}{v_y (v_x \sqrt{M^2-1} - v_y)} \right| \Delta^{(0)}.$$

At the point of intersection with a weak discontinuity, the curvature of the shock wave has a finite discontinuity, equal to

$$\Delta_K = -\frac{2\sqrt{M^2-1}\, V}{j^2 (V_0 - V) D_+}\left(1 + j^2 \frac{\partial V}{\partial p}\right) \left| \frac{v_x v_y - c^2 \sqrt{M^2-1}}{c (v_x \sqrt{M^2-1} - v_y)} \right| \Delta^{(0)}. \tag{6.9}$$

7. CONCLUSIONS

1. The interaction of a shock wave of arbitrary intensity with small perturbations has been considered in the case in which the motion behind the shock wave is supersonic.

2. The interaction of a shock wave of arbitrary intensity with a weak shock wave and a tangential discontinuity of low intensity has been considered, as a result of which interaction, an outgoing weak shock wave and a tangential discontinuity of low intensity are produced. Equations have been obtained connecting the intensity of the resultant discontinuities with the incident intensity.

3. The interactions of a shock wave with a weak and a weak-tangential discontinuity have been considered. As a result of these, outgoing weak and weak-tangential discontinuities are formed.

4. A singular phenomenon of "resonance" has been discovered, which leads to a significant amplification of the intensity of the incoming discontinuities. The connection is pointed out of this phenomenon with the possibility of splitting of the shock wave.

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INTERACTION OF SHOCK WAVES WITH SMALL PERTURBATIONS II.

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Consideration is given to the general case of the interaction of small perturbations with a shock wave of arbitrary intensity within the framework of a two dimensional stationary problem in the case of subsonic flow behind the shock wave. A solution is given for the interaction of weak and weak-tangential discontinuities with the shock wave. In both cases, the weak-tangential discontinuity formed behind the shock wave possesses a specific logarithmic discontinuity.

1. INTRODUCTION

In the previous paper,[1] the perturbation theory was given for the shock wave with supersonic flow behind it; with the aid of perturbation methods, problems were considered of the interaction of weak singularities with the shock wave. The purpose of the present work is the consideration of analogous interactions in the case of subsonic flow behind the shock wave.

For subsonic flow behind the shock wave, the equations of hydrodynamics are elliptic and the method of characteristics used in the previous paper are not applicable.

Let us consider a plane shock wave in one dimensional flow. As the y axis we choose the perpendicular to the wave in the direction of the normal component of the velocity. Let the x axis lie in the plane of the wave and be directed along the tangent to the component of the velocity.

Inasmuch as the motion is supersonic for $y < 0$, all the incident perturbations are described by solutions obtained in Sec. 1 of the previous paper. For $y > 0$, the system of hydrodynamic equations will possess only one set of real characteristics — flow lines along which the perturbation of the entropy and the curl of the velocity are "transferred." Therefore the entropy-vortical solution is preserved here also. Two sets of characteristics become imaginary. To find the solutions corresponding to the elliptic equation it is expedient to reduce it to Laplace's equation.

*Deceased.