uum due to the distortion of the wave function. This correction is determined by means of the formula

$$\Delta E_L = -\frac{E_{0L}\alpha^2}{3}\left[4\frac{<r^2>}{r_0^2} + \frac{(4\pi)^2}{2}\int_0^\infty \rho(y)\,dy\int_0^y \rho(x)\,x^4dx - \frac{(4\pi)^3}{12}\int_0^\infty \rho(y)\frac{dy}{y^2}\left(\int_0^y \rho(x)\,x^4\,dx\right)^2\right], \tag{8}$$

where E_{0L} is the electromagnetic part of the Lamb shift.

In formula (8) the first term takes into account the change in the normalization $\Delta N/N = -\Delta n/\alpha^2$ and yields -0.006 Mc/sec; the second and third terms take into account the distortion of the wave function. The last part depends essentially on the form of the charge distribution inside the proton; its magnitude, however, does not exceed $\Delta E = -0.012$ Mc/sec (for the charged sphere) and, consequently, lies beyond the range of the present experiment.[6]

Thus taking into account the volume of the proton reduces the Lamb-shift discrepany between theory and experiment from 0.6 to 0.5 Mc/sec. It will be possible to draw further conclusions about the effect of the structure of the elementary particles on the Lamb shift after calculating electromagnetic processes of the fifth order and improving the accuracy of experiment. We are going to apply an analogous method to the calculation of the correction to the hyperfine structure due to the volume of the proton.

In conclusion I consider it my duty to express gratitude to Professor D. D. Ivanenko for constant attention to the work and for a discussion of the results.

[1] E. Salpeter, Phys. Rev. **89**, 92 (1953).
[2] Baranger, Bethe, and Feynman, Phys. Rev. **92**, 482 (1953).
[3] D. D. Ivanenko and N. N. Kolesnikov, Dokl. Akad. Nauk SSSR **91**, 47 (1953).
[4] A. A. Sokolov and D. D. Ivanenko, Квантовая теория поля (Quantum Theory of Fields), Part II, Chapt. III, § 7, Moscow-Leningrad (1952).
[5] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).
[6] Triebwasser, Dayhoff, and Lamb, Phys. Rev. **89**, 98 (1953).

Translated by J. Heberle
168

PARTICLE ANGULAR DISTRIBUTION FUNCTION AT THE CASCADE SHOWER MAXIMUM

I. P. IVANENKO

Moscow State University

Submitted to JETP editor June 25, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 825-827 (September, 1957)

BELEN'KII[1] calculated the distribution function of particles with respect to energy and angle at cascade shower maximum without assuming small deflection angles. The scattering, however, was considered to be mutliple. In the following we shall drop this limitation. We can then write the equation for $P(E, \vartheta)$, integrated over the depth of shower development with boundary conditions corresponding to a single primary electron of energy E_0 incident vertically upon the boundary of the material layer at $t = 0$, in the following form:

$$-\frac{1}{2\pi}\delta(E_0 - E)\,\delta(1-\cos\vartheta)\cos\vartheta = q\frac{N(E_0, E, \vartheta)}{E} + q\frac{\partial N(E_0, E, \vartheta)}{\partial E}$$

$$+ n\int_0^{\chi_{\max}}\int_0^{2\pi} P(E_0, E, \vartheta') - P(E_0, E, \vartheta)]\, f\left(2\sin\frac{\chi}{2}\right)\sin\chi\, d\chi\, d\varphi, \qquad N(E_0, E, \vartheta) = \int_E^\infty P(E_0, E, \vartheta)\, dE. \tag{1}$$

Here $q = 2.29$ and $P(E, \vartheta)$ is the required distribution function of electrons with respect to energy E and angle ϑ. The angle ϑ' is determined by the equation

$$\cos\vartheta' = \cos\vartheta\cos\chi + \sin\vartheta\sin\chi\cos\varphi. \tag{2}$$

where n is the number of scattering nuclei in 1 cm^3 of matter, $f(2\sin(\chi/2))$ is the transverse cross-section for Rutherford scattering, and χ_{max} is an angle depending on the energy and on the scattering medium. We neglect ionization losses and, therefore, our considerations are valid for energies $E > \beta$.

We shall seek the solution of Eq. (1) in the form of a series of Legendre polynomials:

$$N(E_0, E, \vartheta) = \sum_{n=0}^{\infty} \varphi_n(E_0, E)\, P_n(\cos\vartheta)\, \frac{2n+1}{2}. \tag{3}$$

Substituting (3) into (1) and using the addition theorem for Legendre polynomials in transforming the integral term in (1), we obtain

$$-\delta(E_0 - E)/2\pi = q\varphi_n(E_0, E)/E + (q - K_n(E))\,\partial\varphi_n(E_0, E)/\partial E;$$

$$K_n(E) = 2\pi n \int_0^{\chi_{max}} [P_n(\cos\chi) - 1]\, f\left(2\sin\frac{\chi}{2}\right)\sin\chi\, d\chi. \tag{4}$$

Using the explicit expression for the function f,[1] using the expansion of Legendre polynomials in terms of powers of $\sin(\chi/2)$, and taking into account screening and the finite dimensions of the nucleus[1] we obtain, finally, the following expression:

$$K_n(E) = \left(\frac{E_k}{2E}\right)^2 \left(-n(n+1) + \sum_{k=2}^{n} (-1)^k \frac{(n+k)!}{(n-k)!\,(k!)^2\,(2k-2)} \left(\frac{7.33}{2}\, 10^{-3} \frac{mc^2}{E} Z^{1/3}\right)^{2k-2} \frac{1}{L_R} [(181Z^{-1/3})^{4k-4} - 1]\right\}. \tag{5}$$

where $L_R = 2\ln(181\, Z^{-1/3})$, m is the electronic mass and Z the atomic number. The sum in Eq. (5) vanishes for $n < k$.

The solution of Eq. (4), is

$$\varphi_n(E_0, E) = \frac{1}{2\pi}(q - K_n(E_0))^{-1} \exp\left[\int_E^{E_0} \frac{q}{E'}(q - K_n(E'))^{-1} dE'\right]. \tag{6}$$

Expressions (3), (5), and (6) represent the solution of our problem. They determine the energy and angular distribtuion function of electrons.

The explicit expressions for the first few functions φ_n are:

$$\varphi_0(E_0, E) = \frac{E_0}{qE}; \quad \varphi_1(E_0, E) = E_0\left(1 + \frac{2P^2}{E_0^2}\right)^{-1/2}\left(1 + \frac{2P^2}{E^2}\right)^{-1/2}/qE;$$

$$\varphi_2(E_0, E) = \frac{E_0(2 + 6P^2/E^2 - \Delta/E^2)^{1.5P^2/\Delta}(2 + 6P^2/E_0^2 + \Delta/E_0^2)^{1.5P^2/\Delta}}{qE(1 + 6P^2/E_0^2 - 3\alpha P^2/E_0^4)^{3/4}(1 + 6P^2/E^2 - 3\alpha P^2/E^4)^{1/4}(2 + 6P^2/E_0^2 - \Delta/E_0^2)^{1.5P^2/\Delta}} \; \frac{1}{(2 + 6P^2/E^2 + \Delta/E^2)^{1.5P^2/\Delta}}; \tag{7}$$

$$P = \frac{E_k}{2q^{1/2}}; \quad \alpha = \left(\frac{7.33}{2}\, 10^{-3} mc^2 Z^{1/3}\right)\frac{1}{L_R}[(181Z^{-1/3})^4 - 1]; \; \Delta^2 = \frac{3\alpha E_k^2}{q} - 3\left(\frac{E_k^2}{2q}\right)^2.$$

The functions φ_n can be calculated in an analogous way for $n > 2$. Comparison with the values φ_{nL} calculated in the Landau approximation shows that for $n = 0$ and $n = 1$ $\varphi_n = \varphi_{nL}$. For $n \geq 2$ the dependence of φ_n and φ_{nL} on E_0 and E is different. Moreover, for $n \geq 2$, φ_n depends strongly on the atomic number Z.

Using the distribution functions (3) and (7) we can find expressions for $\overline{\cos\vartheta_N}$ and $\overline{\cos^2\vartheta_N}$. In view of the orthogonality of Legendre polynomials, these are determined by the first three terms of series (3). Since $\varphi_n = \varphi_{nL}$ for $n = 0, 1$ the expressions for $\overline{\cos\vartheta_N}$ coincide with $\overline{\cos\vartheta_{NL}}$ obtained in Ref. 1. For the mean square we obtain the following expression:

$$\overline{\cos^2\vartheta_N(E_0, E)} = 1/3 + 2\varphi_2(E_0, E)/3\varphi_0(E_0, E),$$

where φ_0 and $\varphi_2(E_0, E)$ are given by Eq. (7).

For $E_0 \to \infty$ we have

$$\overline{\cos^2\vartheta_N(E)} = \frac{1}{3} + \frac{2}{3}\, \frac{(2 + 6P^2/E^2 - \Delta/E^2)^{1.5P^2/\Delta}}{(1 + 6P^2/E^2 - 3\alpha P^2/E^4)^{1/4}(2 + 6P^2/E^2 + \Delta/E^2)^{1.5P^2/\Delta}}.$$

In lead ($Z = 82$) the ratio $\overline{\cos^2\vartheta_N}/\overline{\cos^2\vartheta_{NL}}$ equals 0.99 and 0.98 for $E = 3\times 10^7$ ev and 1.5×10^7 ev respectively.

---

[1]S. Z. Belen'kii, Лавинные процессы в космических лучах (Cascade Processes in Cosmic Rays), Gostekhizdat, 1948.

Translated by H. Kasha
169

EFFECT OF DAMPING ON POLARIZATION OF DIRAC PARTICLES IN SCATTERING

A. A. SOKOLOV and B. K. KERIMOV

Moscow State University

Submitted to JETP editor June 27, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 827-829 (September, 1957)

THE elastic scattering of both Dirac and spinless particles by a fixed force center was investigated in Refs. 1 to 4 using damping theory. We calculate below, using damping theory, the polarization resulting from elastic scattering of Dirac particles.

The fundamental integral equation of damping theory which determines the scattering amplitude $f'_{s'} \equiv f'_{s'}(\mathbf{k}')$ and is relevant in a discussion of polarization phenomena has the following form (we use the notation of Ref. 3):

$$(f'_{s'} - b'^{+}_{s'} b_s f_s) V_{\mathbf{k}'\mathbf{k}} = \frac{kK}{8\pi^2 c\hbar i} \sum_{s''} \oint d\Omega'' V_{\mathbf{k}'\mathbf{k}''} V_{\mathbf{k}''\mathbf{k}} b'^{+}_{s'} b''_{s''} f''_{s''}, \tag{1}$$

Here $E = c\hbar K$ is the total energy of the particle and $V_{\mathbf{k}'\mathbf{k}''}$ is the Fourier component of the potential $V(\mathbf{r})$.

We shall restrict ourselves to calculating the polarization resulting from elastic scattering of Dirac particles by a delta-function potential $V(\mathbf{r}) = V_0\delta(\mathbf{r})$, $V_{\mathbf{k}'\mathbf{k}''} = V_0$. In that case we have from formulas (5) of Ref. 3:

$$b'^{+}_{s'} b''_{s''} = \sum_{j=1,2} h^{j}_{s's''}, \tag{2}$$

where

$$h^1_{s's''} = \frac{1}{2}\left(1+\frac{k_0}{K}\right)[\cos\theta'_{s'}\cos\theta''_{s''} + e^{i(\varphi''-\varphi')}\sin\theta'_{s'}\sin\theta''_{s''}] \qquad h^2_{s's''} = \frac{1}{2}\left(1-\frac{k_0}{K}\right)s's''[\cos\theta'_{s'}\cos\theta''_{s''} + e^{i(\varphi''-\varphi')}\sin\theta'_{s'}\sin\theta''_{s''}].$$

We seek a solution of integral equation (1) of the form ($s' = 1, -1$):

$$f'_{s'} = \sum_{j=1,2} \varepsilon_j h^j_{s's} f_s. \tag{3}$$

Taking into account the orthogonality condition for $h^j_{s's''}$ [see Eq. (31) of Ref. 3] we obtain for ϵ_1 and ϵ_2 from Eqs. (1)–(3):

$$\varepsilon_{1,2} = \frac{1}{1+i\delta_{1,2}}, \quad \delta_{1,2} = \frac{V_0}{4\pi c\hbar} k(K \pm k_0). \tag{4}$$

From Eqs. (3) and (4) we obtain for the amplitudes $f'_{s'}$ of the first scattering

$$\begin{aligned} f'_1 &= \frac{1}{2}[a_1\varepsilon_1 + a_2\varepsilon_2]\cos\frac{\theta'}{2} f_1 - \frac{1}{2}[a_1\varepsilon_1 - a_2\varepsilon_2] e^{-i\varphi'}\sin\frac{\theta'}{2} f_{-1}, \\ f'_{-1} &= \frac{1}{2}[a_1\varepsilon_1 - a_2\varepsilon_2]\sin\frac{\theta'}{2} f_1 + \frac{1}{2}[a_1\varepsilon_1 + a_2\varepsilon_2] e^{-i\varphi'}\cos\frac{\theta'}{2} f_{-1}, \end{aligned} \tag{5}$$