has changed substantially (the maximum has disappeared). The existence of the maximum in the curve $\rho(T)$ and its location as a function of temperature is determined by the form of the temperature dependence of $\Delta R/\Delta H$ and R; a change in the temperature dependence of $\Delta R/\Delta H$ and R shifts the maximum towards lower temperatures, or obliterates the maximum, or emphasizes it. It therefore follows that the increase in ρ with decreasing temperatures, first observed in Ref. 1, is not of great significance. The important factor in the phenomenon under discussion is the existence of a "residual" galvanomagnetic effect at 0° K due to paramagnetism.

Besides the 42% Ni, 58% Fe alloy we have investigated pure nickel, iron-nickel alloys containing 50% and 78% Ni, copper-nickel alloys containing 20% and 25% Cu, and a 23% Mn, 77% Ni alloy (in a hardened, non-ordered state). With the exception of Ni and permalloy (78% Ni, 22% Fe) in which ρ is very small at low temperatures, we have obtained for all the alloys curves very similar to those in Fig. 2. Therefore, contrary to Smith, one may conclude that the phenomenon in question is not peculiar to the 42% Ni, 58% Fe alloy; it also occurs in certain other ferromagnetic alloys. We give below values of ρ measured by us at liquid helium temperature in unannealed Ni-Fe and Cu-Ni alloys and in a hardened Mn-Ni alloy.

Alloy	$\rho \times 10^8$
42% Ni, 58% Fe	31.6
50% Ni, 50% Fe	15.6
20% Cu, 80% Ni	25.5
25% Cu, 75% Ni	11.6
23% Mn, 77% Ni	23.6

The existence of the "residual" galvanomagnetic effect due to paramagnetism at 0° K in other ferromagnetic alloys beside the 42% Ni, 58% Fe alloy raises doubts as to the validity of the explanation given in Ref. 1. We believe that the existence of this effect is connected with the influence of structure imperfections on the exchange interaction.

We take this opportunity to express appreciation to A. J. Shalnikov for his advice and interest in this work.

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Translated by A. Bincer
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GREEN'S FUNCTION FOR DIFFUSION OF RADIATION

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Submitted to JETP editor June 20, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 817-819 (September, 1957)

BIBERMAN[1] suggested a theory for the diffusion of resonance radiation, which takes account of the possibility of changing the photon frequency in each reradiation event. It was assumed that the mean free time of a photon is small compared to the duration of the excited state. The integral equation obtained was solved numerically for various stationary problems. Later a similar equation was obtained by Holstein.[2] In solving the nonstationary problem, Holstein was interested only in the first eigenfunction of the equation, and obtained it using the Rayleigh-Ritz method.

In the present note it is shown that by maintaining Biberman's assumptions and treating the diffusion of radiation in an infinite medium, one can obtain an analytic expression for the Green's function $f(\mathbf{r}, t)$ of this problem.

In this case $f(\mathbf{r}, t)dV$ is the probability that at time t there is an excited atom in the neighborhood of $\mathbf{r}$, if at the initial time ($t = 0$) there existed only one excited atom and it was located at $\mathbf{r} = 0$. For a homogeneous medium $f(\mathbf{r}, t) = f(r, t)$, where r is the magnitude of $\mathbf{r}$. The desired Green's function must satisfy the equation

$$\frac{\partial f(\mathbf{r}, t)}{\partial t} = \frac{1}{4\pi\tau}\iint \varepsilon_\nu k_\nu f(\mathbf{r}_1, t)\frac{e^{-k_\nu|\mathbf{r}-\mathbf{r}_1|}}{|\mathbf{r}-\mathbf{r}_1|^2}\,d\nu\,dV_1 - \left(\frac{1}{\tau}+\sigma\right)f(\mathbf{r}, t) \tag{1}$$

which has been investigated by Biberman and Holstein[1,2] for the initial condition $f(\mathbf{r}, t) = \delta(\mathbf{r})$ at $t = 0$. The notation in Eq. (1) is the following: τ is the mean lifetime of the excited atom, σ is the probability of a damping collision, ν is the photon frequency, and ϵ_ν and k_ν characterize the radiation and absorption line shapes, respectively.

The use of an elegant transformation suggested by Ambartsumian[3] brings Eq. (1) into the form

$$\frac{\partial A(r, t)}{\partial t} = \frac{1}{2\tau}\int_{-\infty}^{\infty}\int_0^{\infty}\varepsilon_\nu k_\nu \,\mathrm{Ei}\,(k_\nu|r-r_1|)\,A(r_1, t)\,d\nu\,dr_1 - \left(\frac{1}{\tau}+\sigma\right)A(r, t) \tag{2}$$

with the initial condition

$$A(r, t) = \delta(r)/2\pi \quad \text{for } t = 0,$$

where

$$A(r, t) = \int_r^{\infty} r_1 f(r_1, t)\,dr_1,\; f(r, t) = -\frac{1}{r}\frac{\partial A(r, t)}{\partial r}. \tag{3}$$

Here Ei is the exponential integral function.

Equation (2) can be solved by Fourier transformation. We finally obtain, using (3),

$$f(r, t) = -\frac{e^{-t(1/\tau+\sigma)}}{(2\pi)^2 r}\frac{\partial}{\partial r}\left\{\int_{-\infty}^{\infty} e^{-ipr}\left[\exp\left(\frac{t}{\tau p}\int_0^{\infty}\varepsilon_\nu k_\nu \tan^{-1}\frac{p}{k_\nu}\,d\nu\right)-1\right]dp + 2\pi\delta(r)\right\}.$$

Let $\epsilon_\nu = \delta(\nu)$, and $k_\nu = k$ be a constant, corresponding to diffusion without a frequency change. In this case

$$\int_0^{\infty}\varepsilon_\nu k_\nu \tan^{-1}\frac{p}{k_\nu}\,d\nu = k\tan^{-1}\frac{p}{k}.$$

Further calculation of Eq. (4) must be performed numerically, although by using known methods it is not difficult to obtain an asymptotic expression for $f(r, t)$ in the form

$$f(r, t) \cong [4\pi Dt]^{-3/2} e^{-\sigma t} e^{-r^2/4Dt},\quad D = 1/3\tau k^2. \tag{5}$$

Thus in the present case of diffusion of radiation, the first approximation is similar to diffusion of particles. This similarity has already been noted by Compton.[4]

As a second example, let us consider the spectroscopically important problem of the dispersion shape of a spectral line

$$k_\nu = \frac{k_0}{1+(\nu-\nu_0)^2/\gamma^2};\quad \varepsilon_\nu = k_\nu \Big/ \int_0^{\infty} k_\nu\,d\nu,$$

where k_0 is the absorption coefficient for the frequency ν_0 corresponding to the center of the spectral line, and γ is the line half-width.

It can be shown that

$$\frac{1}{p}\int_0^{\infty}\varepsilon_\nu k_\nu \tan^{-1}\frac{p}{k_\nu}\,d\nu = 1 - \frac{1}{p}\left\{\frac{\sqrt{p}}{2}[\varphi_1(k_0;p)+\varphi_2(k_0;p)] + k_0\tan^{-1}\frac{\varphi_1(k_0;p)-\sqrt{p}}{\varphi_2(k_0;p)-\sqrt{p}}\right\};\quad (p \geqslant 0),$$

where

$$\varphi_1(k_0;\ p) = (\sqrt{k_0^2 + p^2} + k_0)^{1/2};\quad \varphi_2(k_0;\ p) = (\sqrt{k_0^2 + p^2} - k_0)^{1/2}.$$

The asymptotic expression analogous to (5) now becomes

$$f(r,\ t) \cong -\frac{e^{-\sigma t}}{\pi^2 r}\frac{\partial}{\partial r}\frac{\vartheta}{r^{3/2}}\int_0^\infty \exp\left(-2\frac{\vartheta}{\sqrt{r}}x\right)\sin x^2 dx$$
$$= -\frac{e^{-\sigma t}}{2\pi r}\frac{\partial}{\partial r}\left\{\frac{1}{\vartheta}\sqrt{\frac{r}{2\pi}}\frac{\partial}{\partial r}\left(\cos\frac{\vartheta^2}{r} - \sin\frac{\vartheta^2}{r}\right) + \frac{\partial}{\partial r}\left[C^2\left(\frac{\vartheta}{\sqrt{r}}\right) + S^2\left(\frac{\vartheta}{\sqrt{r}}\right)\right]\right\}, \tag{6}$$

where

$$\vartheta = \frac{t}{3\tau\sqrt{2k_0}},\quad S(x) = \frac{2}{\sqrt{2\pi}}\int_0^x \sin t^2 dt,\ C(x) = \frac{2}{\sqrt{2\pi}}\int_0^x \cos t^2 dt$$

is the Fresnel integral. It follows from (6) that for large optical path lengths

$$f(r,\ t) \approx (4\pi)^{-3/2}(t/\tau\sqrt{k_0 r})\,e^{-\sigma t}/r^3. \tag{7}$$

Comparing (5) and (7) we see that the function for diffusion with redistribution of the photon frequencies decreases much slower than the other. A similar result can be obtained also for the Doppler shape of a spectral line. This is related to the slow decrease of the kernel of the integro-differential equation (1), as has been pointed out by Biberman.[1]

We note further that Ambartsumian's transformation makes it possible to obtain an analytic expression for the Green's function in the problem of diffusion of radiation if one accounts for the motion of the atoms.

In conclusion I express my gratitude to L. M. Biberman for his direction in performing the present work.

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Translated by E. J. Saletan
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EFFECT OF PROTON SIZE ON THE POSITION OF ELECTRONIC LEVELS IN HYDROGEN AND DEUTERIUM

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Submitted to JETP editor June 20, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 819-821 (September, 1957)

RECENT experiments[1] on the scattering of electrons on protons show that the rms radius of the electric charge distribution in the proton is $(0.77 \pm 0.10) \times 10^{-13}$ cm, whereas the rms radius of the neutron is possibly smaller.[2,3] This leads to a reduction of the binding energy of the electron in atoms, i.e., to a correction to the Lamb shift. In the calculation of a similar effect, one may confine oneself to the investigation of the nonrelativistic problem, taking into account additionally the distortions of the electronic wave functions, since the corrections due to these are not large in the cases — of interest to us — of hydrogen and deuterium[4,5] for which experimental data[6] exist.