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Translated by A. Bincer
154

DISPERSION RELATIONS FOR S AND P WAVES FOR MESON PHOTOPRODUCTION IN FIRST ORDER OF 1/m

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Submitted to JETP editor June 14, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 801-803 (September, 1957)

THE matrix element of the R-matrix for the transition corresponding to the photoproduction of a meson on a nucleon can be written in the form[1]

$$\langle\pi\,|\,R\,|\,\gamma\rangle=\frac{i\,(2\pi)^4}{\sqrt{4k^0q^0}}\,\delta\,(p_1+q-p-k)\sum_{i=1}^{4}\left(\delta_{\rho 3}A_i^{(1)}+\tau_\rho A_i^{(2)}+\frac{1}{2}\,[\tau_\rho,\,\tau_3]\,A_i^{(3)}\right)\eta_i. \tag{1}$$

Here k, q, p, and p_1 are the momenta of the photon, meson, and nucleon in the initial and final state respectively; η_i are the spin operators which in the center of mass system have the form (e — polarization vector of the photon)

$$\eta_1=i\,(\sigma\mathbf{e});\quad \eta_2=k^{-1}q^{-1}\,(\sigma\mathbf{q})\,(\sigma[\mathbf{k}\times\mathbf{e}]);\;\eta_3=ik^{-1}q^{-1}\,(\sigma\mathbf{k})\,(\mathbf{qe});\quad \eta_4=iq^{-2}\,(\sigma\mathbf{q})\,(\mathbf{qe})$$

and $A_i^{(\lambda)}=A_i^{(\lambda)}(W,x)$, where W is the total energy of the system, $x=\cos\theta$ (θ — scattering angle). The quantities $A_i^{(\lambda)}$ as functions of W obey the dispersion relations[1]

$$\operatorname{Re}A_i^{(\lambda)}(W,x)=\overset{0}{A}{}_i^{(\lambda)}(W,x)+\frac{1}{\pi}\,\mathrm{P}\int_{m+\mu}^{\infty}dW'\sum_{i'=1}^{4}f_{ii'}^{(\lambda)}(W,\,W',\,x)\operatorname{Im}A_{i'}^{(\lambda)}(W',x'), \tag{2}$$

where $\overset{0}{A}{}_i^{(\lambda)}$ and $f_{ii'}^{(\lambda)}$ are known functions, and x', the cosine of the primed angle, is connected with x, W, and W′ by the relation

$$k\,(\omega-qx)=k'\,(\omega'-q'x').$$

It follows from this expression that in the c.m.s., i.e., where $\mathbf{p}+\mathbf{p}_1=0$, the unobservable energy range corresponds to the unobservable range of the primed angles, i.e., the range where $-\infty<x'<-1$. One therefore has to know the analytical properties of $A_i^{(\lambda)}$ as a function of x.

By means of a phase-shift analysis one can obtain expressions for the A_i in terms of Legendre polynomials; for example

$$A_1=\sum_{l=0}^{\infty}\{[lM_{l+}+E_{l+}]P'_{l+1}(x)+[(l+1)\,M_{l-}+E_{l-}]P'_{l-1}(x)\}\ \text{etc.}$$

(ℓ — angular momentum of the meson; the subscript ± refers to the total angular momentum $\ell\pm\frac{1}{2}$ respectively; $M_{\ell\pm}$ corresponds to magnetic and $E_{\ell\pm}$ to electric multipoles). Let us assume that these infinite series can be terminated [for this to be true the integrals in (2) have to be sufficiently strongly convergent]. Then the $A_i^{(\lambda)}$ will be analytic functions of x. Further one can in (2) eliminate the angles and write down dispersion relations for $M_{\ell\pm}^{(\lambda)}$, $E_{\ell\pm}^{(\lambda)}$. We shall give these limiting ourselves to S and P waves and including recoil corrections up to the order 1/m. Introducing new variables $\epsilon=W-m$; $\epsilon'=W'-m$ we have

$$\mathrm{Re}\,E_{0+}^{(\lambda)}(\varepsilon)=\overset{0}{M}{}_{03}^{(\lambda)}+\frac{\varepsilon}{\pi}\,\mathrm{P}\int_{\mu}^{\infty}d\varepsilon'\left\{\left(\frac{1}{\varepsilon'-\varepsilon}\pm\frac{1}{\varepsilon'+\varepsilon}\right)\frac{1}{\varepsilon'}\,\mathrm{Im}\,E_{0+}^{(\lambda)}(\varepsilon')-\left[1\pm1-\frac{1}{2m}\left[\varepsilon'+\varepsilon\pm\left(\varepsilon'-\varepsilon+\frac{2\,(3\varepsilon^{2}+2q^{2})}{3\,(\varepsilon'+\varepsilon)}\right.\right.\right.\right.$$

$$\left.\left.\left.-\frac{2\varepsilon q^{2}}{3\,(\varepsilon'+\varepsilon)^{2}}\right)\right]\right]\frac{1}{\varepsilon' q'}\,\mathrm{Im}\,[M_{1-}^{(\lambda)}(\varepsilon')-M_{1+}^{(\lambda)}(\varepsilon')]+3\left[\varepsilon'+2\varepsilon\pm(\varepsilon'-2\varepsilon)-\frac{1}{2m}\left[\varepsilon'(\varepsilon'+\varepsilon)+2\,(\varepsilon^{2}+q^{2})\right.\right.$$

$$\left.\left.\pm\left(\varepsilon'(\varepsilon'-\varepsilon)-\frac{2\varepsilon\,(\varepsilon^{2}+q^{2})}{\varepsilon'+\varepsilon}+\frac{2\varepsilon' q^{2}}{3\,(\varepsilon'+\varepsilon)^{2}}\right)\right]\right]\frac{1}{\varepsilon'^{2}q'}\,\mathrm{Im}\,E_{1+}^{(\lambda)}(\varepsilon')\Bigg\}.$$

$$\mathrm{Re}\,[M_{1-}^{(\lambda)}(\varepsilon)-M_{1+}^{(\lambda)}(\varepsilon)]-\overset{0}{M}{}_{11}^{(\lambda)}+\frac{\varepsilon q}{\pi}\,\mathrm{P}\int_{\mu}^{\infty}d\varepsilon'\left\{\mp\frac{2\varepsilon'+\varepsilon}{2m\,(\varepsilon'+\varepsilon)^{2}}\frac{1}{\varepsilon'}\,\mathrm{Im}\,E_{0+}^{(\lambda)}(\varepsilon')\right.$$

$$\left.+\left[\frac{1}{\varepsilon'-\varepsilon}\mp\frac{1}{\varepsilon'+\varepsilon}\left(1-\frac{\varepsilon'}{m}\right)\right]\frac{1}{\varepsilon' q'}\,\mathrm{Im}\,[M_{1-}^{(\lambda)}(\varepsilon')-M_{1+}^{(\lambda)}(\varepsilon')]+\left[1\mp1-\frac{\varepsilon\pm(2\varepsilon'-3\varepsilon)}{2m}\right]\frac{1}{\varepsilon'^{2}q'}\,\mathrm{Im}\,E_{1+}^{(\lambda)}(\varepsilon')\right\}.$$

$$\mathrm{Re}\,[2\,M_{1+}^{(\lambda)}(\varepsilon)+M_{1-}^{(\lambda)}(\varepsilon)]=\overset{0}{M}{}_{1,2}^{(\lambda)}+\frac{\varepsilon q}{\pi}\,\mathrm{P}\int_{\mu}^{\infty}d\varepsilon'\left[\frac{1}{\varepsilon'-\varepsilon}\pm\frac{1}{\varepsilon'+\varepsilon}\left(1+\frac{\varepsilon}{m}\right)\right]\frac{1}{\varepsilon' q'}\,\mathrm{Im}\,[2M_{1+}^{(\lambda)}(\varepsilon')+M_{1-}^{(\lambda)}(\varepsilon')].$$

$$\mathrm{Re}\,E_{1+}^{(\lambda)}(\varepsilon)=\overset{0}{M}{}_{13}^{(\lambda)}+\frac{\varepsilon^{2}q}{\pi}\,\mathrm{P}\int_{\mu}^{\infty}d\varepsilon'\left\{\mp\frac{q}{6m\,(\varepsilon'+\varepsilon)^{2}}\frac{1}{\varepsilon'}\,\mathrm{Im}\,E_{0+}^{(\lambda)}(\varepsilon')+\left[\frac{1}{\varepsilon'-\varepsilon}\pm\frac{1}{\varepsilon'+\varepsilon}+\frac{1}{2m}\left(1\mp\frac{\varepsilon'-\varepsilon}{\varepsilon'+\varepsilon}\right)\right]\frac{1}{\varepsilon'^{2}q'}\,\mathrm{Im}\,E_{1+}^{(\lambda)}(\varepsilon')\right\}.$$

We take here the upper sign for $\lambda = 1, 2$ and the lower sign for $\lambda = 3$. The terms $\overset{0}{M}{}_{\ell i}^{(\lambda)} = \overset{0}{M}{}_{\ell ie}^{(\lambda)} + \overset{0}{M}{}_{\ell i\mu}^{(\lambda)}$ are given within the present accuracy by

$$\overset{0}{M}{}_{03e}^{(\lambda)}=-\frac{ef}{2\mu}\left[(1\mp1)\,F_{s}-\frac{\omega}{2m}(1\pm1)\pm\frac{q^{2}}{3\,m\omega}\right];\quad \overset{0}{M}{}_{11e}^{(\lambda)}=-\frac{ef}{2\mu}\left[(1\mp1)\,F_{M}-\frac{q}{2m}(1\pm1)\right],$$

$$\overset{0}{M}{}_{12e}^{(\lambda)}=\frac{ef}{2\mu}\cdot\frac{q\,(1\mp1)}{2m};\quad \overset{0}{M}{}_{13e}^{(\lambda)}=-\frac{1}{3}\cdot\frac{ef}{2\mu}\cdot(1\pm1)\,F_{Q}$$

($\lambda = 1, 2$ — upper sign, $\lambda = 3$ — lower sign), and

$$F_{s}=1-\frac{\omega}{2k}F;\quad F_{M}=\frac{4\omega}{3q}F;\quad F_{Q}=\frac{q}{k}\left[1-\frac{3\omega}{4q^{2}}(2\omega-k)\,F\right]\quad F=1+\frac{\mu^{2}}{2\omega q}\ln\frac{\omega-q}{\omega+q}$$

$$\overset{0}{M}{}_{03\mu}^{(\lambda)}=\frac{f\,(\mu_{p}'\mp\mu_{n})}{\mu}\left(\omega-\frac{3\,\omega^{2}-2q^{2}}{6m}\right);\quad \overset{0}{M}{}_{03\mu}^{(3)}=\frac{f\,(\mu_{p}'-\mu_{n})}{\mu}\frac{q^{2}}{6m},\quad \overset{0}{M}{}_{11\mu}^{(\lambda)}=\frac{f\,(\mu_{p}'\mp\mu_{n})}{\mu}q;\quad \overset{0}{M}{}_{11\mu}^{(3)}=0$$

$$\overset{0}{M}{}_{12\mu}^{(\lambda)}=-\frac{f\,(\mu_{p}'\mp\mu_{n})}{\mu}\,q\,\frac{\omega}{2m};\quad \overset{0}{M}{}_{12\mu}^{(3)}=\frac{f\,(\mu_{p}'-\mu_{n})}{\mu}\,q\left(1+\frac{\omega}{2m}\right)$$

($\lambda = 1$ — upper sign, $\lambda = 2$ — lower sign).

$$\overset{0}{M}{}_{13\mu}^{(\lambda)}=0\ (\lambda=1,2,3).$$

The equations for the P-wave are in essence similar to the corresponding expressions given by Chew and Low.[2] They differ in the meaning of the integration variable, in certain interchanges of E and M terms, and in the addition of some correction terms of the order $1/m$ [which are not important in the region of the (33) resonance]. From the equation for the S-wave we have the expression for $E_{0+}^{(3)}$ which is important in comparison with experiments:

$$-2\sqrt{2}\,E_{0+}^{(3)}=E_{0+}(\gamma p\to n\pi^{+})+E_{0+}(\gamma n\to p\pi^{-}),$$

in which the integral converges sufficiently rapidly for large ϵ'.

I take the opportunity to express my gratitude to A. A. Logunov for his interest in the present work.

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Translated by M. Danos