LAGRANGIAN FOR THE INTERACTION OF K-MESONS AND HYPERONS

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1. In a preceding note,[1] the hypothesis about the conservation of only the combination of parities IC in all interactions was introduced, and it was shown that in certain cases conservation of IC leads to conservation of spatial parity I and invariance with respect to the operation of charge conjugation C, and in other cases, I is not conserved. In the present note the consequences following from the hypothesis of conservation of only the combination of parities in all interactions are considered, with application to the interaction of K-mesons and hyperons.

2. To explain the experimental data, Schwinger[2] proposed the existence of a strong K-π interaction. He succeeded in constructing the Lagrangian of that interaction, considering that the K-mesons do not have a definite parity. The hypothesis about the conservation of only the combination of parities makes it possible to write the Lagrangian of a strong K-π interaction in the following form:

$$L_{K\pi} = ig_{K\pi}\{K^*K_0\Phi - KK_0^*\Phi^*\}, \tag{1}$$

where K, K_0, and Φ are the operators of the K and π-meson fields, respectively. This Lagrangian is not invariant with respect to rotations in isotropic spin space. Thus, the local Lagrangian of a direct K-π interaction, invariant with respect to the operation IC, cannot be written in an isotopic-invariant form. We note that a non-local interaction Lagrangian (for example, the Lagrangian where positive and negative frequency parts of the wave functions are separated) can be written in a charge-invariant form.

3. Salam[4] found a Lagrangian for strong interaction of nucleons and hyperons with K- and π-mesons, invariant with respect to the operations I, C, T. We construct an addition L′ to this Lagrangian, invariant relative to the operation IC, but in which I and C are not separately conserved. Keeping, in the main, the notation of Ref. 3, and assuming the K-meson to be scalar, we obtain L′ in the following form

$$\begin{aligned} L' = {} & ig_1'(\overline{\psi}_p\psi_n\Phi - \overline{\psi}_n\psi_p\Phi^*) + ig_2'(\overline{\Lambda}\Phi^\lambda\Sigma^\lambda - \overline{\Sigma}^\lambda\Phi^\lambda\Lambda) + (g_3'/2i)\,\mathrm{Sp}\,(\tau^\lambda\tau^{\lambda'}\tau^{\lambda''})\,\overline{\Sigma}^\lambda\Phi^{\lambda'}\Sigma^{\lambda''} + ig_4'(\overline{\Xi}^-\Xi^0\Phi^* - \overline{\Xi}^0\Xi^-\Phi) \\ & + ig_5'(\overline{N}(i\gamma_5)\Lambda\theta - \overline{\Lambda}\theta^*(i\gamma_5)N) + ig_6'(\overline{N}(i\gamma_5)\tau^\lambda\Sigma^\lambda\theta - \theta^*\overline{\Xi}^\lambda(i\gamma_5)\tau^\lambda N) \\ & + g_7'(\overline{\Xi}(i\gamma_5)\tau^2\theta^*\Lambda + \overline{\Lambda}\theta(i\gamma_5)\tau^2\Xi) + g_8'(\overline{\Xi}(i\gamma_5)\tau^2\tau^\lambda\Sigma^\lambda\theta^* + \theta\tau^\lambda\overline{\Sigma}^\lambda\tau^2(i\gamma_5)\Xi). \end{aligned} \tag{2}$$

If the K-meson has a negative internal parity (pseudoscalar), then in the terms g_5', g_6', g_7', and g_8', the matrix $i\gamma_5$ should be replaced by 1.

From Eq. (2) it can be seen that the renormalized interactions in which I and C are not separately conserved, can be written in charge-invariant form, if they contain only vertices in which the fermion changes only one of its basic characteristics — mass, charge, or strangeness. This relates to the interactions of K-mesons with baryons and π-mesons with Λ- and Σ-hyperons. Those terms in the interaction Lagrangian in which the initial and final fermions differ only by momentum cannot be described in an isotopic-invariant form. This relates to the interaction of nucleons and cascade hyperons with π-mesons.

We note that interactions invariant with respect to IC conserve the z-component of isotopic spin J_z and the relation connecting J_z, the number of nucleons n and the strangeness s with the electrical charge, i.e., $Q/e = J_z + (n + s)/2$.

4. The hypothesis of conservation of only the combination of parities leads, firstly, to a simple explanation of K-meson decays into both two and three mesons, introducing neither a particle with non-definite parity[2] nor a parity doublet[3] and, secondly, to the possibility of building a direct K-π interaction. However, a new difficulty arises: it is impossible to describe all renormalized and local interaction Lagrangians invariant with respect to IC in a charge-invariant form.

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INTERACTION OF FERMIONS AND THE $K_{\mu 3}$-DECAY

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THE weak interaction of four fermions plays, apparently, an important role in the decays of K-mesons into leptons ($K_{\mu 2}$, $K_{\mu 3}$, K_{e3}). An important problem is the clarification of the variants of this interaction in K_{e3}- and $K_{\mu 3}$-decays. With this in mind, the energy spectrum of the electrons and μ-mesons[1-3] and the angular correlation between the π-meson and electron (μ-meson)[4] were studied.

In the usual investigation of the energy spectrum of μ-mesons[1-3] (electrons) it is necessary, as a result of lack of information about strong interactions, to make definite assumptions about the dependence of these interactions on the particle momenta (it is easy to see[4] that in this problem there is only one independent momentum, for example, p_π). However, Okun'[5] proposed to study the spectrum of electrons from K_{e3}-decay, fixing the energy of the π-mesons. In this investigation definite conclusions can be drawn about the presence of this or that variant and about the conservation of parity with respect to reflection in time in the K_{e3}-decay process, independently of knowledge of strong interactions.

In this note we consider the analogous case of $K_{\mu 3}$-decay. Although here, as compared to K_{e3}, the analysis is made more difficult because it is impossible to neglect the mass of the μ-meson, some conclusions can be drawn all the same. This is of some interest also in connection with the long-standing question as to whether the μ-meson and electron are interchangeable in weak interactions of four fermions.

According to Pais and Treiman[4] the matrix element of $K_{\mu 3}$-decay of a K-meson at rest has, in the general case, the following form (without introduction of derivative lepton functions):

$$R = \left[\left(f_S - g_V \frac{m_\mu}{M}\right) \bar{\psi}_\mu \psi_\nu + f_V \bar{\psi}_\mu \gamma_4 \psi_\nu + \frac{f_T}{M} \bar{\psi}_\mu \gamma_4 \gamma P_\pi \psi_\nu\right] (2M^{3/2} E_\pi^{1/2})^{-1}. \quad (1)$$

Here $f_{S,V,T}$ and g_V are dimensionless scalar functions of the π-meson energy, expressing the contributions of strong interactions in intermediate states (at the present time they are unknown); M is the mass of the K-meson, $P_\pi = \sqrt{E_\pi^2 - m_\pi^2}$; $\hbar = c = 1$ everywhere.

Calculation of the probability of emission of a μ-meson with energy E_μ and a π-meson of energy E_π gives

$$\begin{aligned} w(E_\pi, \varepsilon) = \frac{(M-E_\pi)^3}{32\pi^3 M^3} \Big\{ & |f_{S'}|^2 (1 - \varepsilon_0^2 + \mu_0^2) + |f_V|^2 (\varkappa^2 - (1-\varepsilon)^2) + |f_T|^2 \left(1 - \frac{E_\pi}{M}\right)^2 [\mu_0^2 (\varkappa^2 + 2(1-\varepsilon)) \\ & + (1-\varepsilon_0^2)(1-\varepsilon)^2] + 4\,\mathrm{Re}(f_S f_V^+) \mu_0 (1-\varepsilon) + 2\,\mathrm{Im}(f_S f_T^+) \left(1 - \frac{E_\pi}{M}\right) [\mu_0^2 + (1-\varepsilon_0^2)(1-\varepsilon)] \\ & + 2\,\mathrm{Im}(f_V f_T^+) \mu_0 \left(1 - \frac{E_\pi}{M}\right)(1 + \varkappa^2 - \varepsilon) \Big\}; \quad \varepsilon_0 = P_\pi/(M - E_\pi); \quad \varepsilon = 2E_\mu/(M - E_\pi); \\ & f_{S'} = f_S - g_V \frac{m_\mu}{M}; \quad \varkappa^2 = \varepsilon_0^2 + \mu_0^2; \quad \mu_0 = m_\mu/(M - E_\pi); \quad 0 \leqslant \varepsilon_0/\mu_0 \leqslant 2. \end{aligned} \quad (2)$$

Following Okun', we consider the spectrum of μ-mesons for a fixed energy of the π-meson. Then we will have