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SCATTERING OF ELECTRONS BY PROTONS

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It is shown that under some very general assumptions, scattering of electrons by protons is determined by two real functions $a(q^2)$ and $b(q^2)$ of the invariant $q^2 = (p_1 - p_2)^2$ (p_1 and p_2 are the four-momenta of the electron before and after the collision). Experiments with polarized electrons and protons are considered which permit one in principle to determine the particular form of the functions $a(q^2)$ and $b(q^2)$.

1. In the region of not too high energies, where the proton recoil may be neglected, the scattering of electrons by protons is considered as the scattering of a Dirac particle in an external field. At high energies both particles are relativistic, which leads to a formula of the Møller type for the scattering cross section. Here, we must take account of the anomalous magnetic moment of the proton, which may be done by introducing a suitable term into the Hamiltonian of the interaction of nucleons with the electromagnetic field. But if the nucleon recoil momentum $q \gtrsim \mu c$, where μ is the mass of the π-meson, then the nucleon may not be considered to be a point, and a more detailed analysis is necessary, taking account of the interaction of the nucleon with the meson vacuum. It is clear, from semi-intuitive considerations, that this leads to the appearance of two form factors characterizing the distributions of charge and anomalous magnetic moment of the nucleon. However, a more rigorous investigation of this question is appropriate, particularly in connection with the interpretation Hofstadter's[1] experimental results.

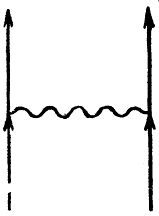

FIG. 1

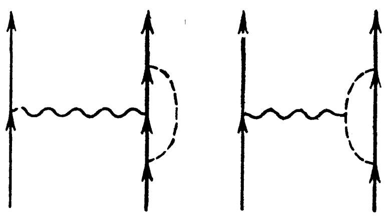

FIG. 2

2. The simplest Feynman diagram corresponding to e–p scattering is shown in Fig. 1 (the heavy line corresponds to the nucleon). The corresponding element of the S-matrix is*

$$S_{i\to f} = -i(2\pi)^4 e^2 \mathfrak{M}_{i\to f}\,\delta(p_1 + P_1 - p_2 - P_2), \tag{1}$$

*We use the system of units in which $\hbar = c = 1$ and $\alpha = e^2/4\pi = 1/137$.

where

$$\mathfrak{M}_{i\to f} = q^{-2}\left(\overline{U}(P_2)\gamma_\mu^{(p)}U(P_1)\right)\left(\overline{u}(p_2)\gamma_\mu^{(e)}u(p_1)\right); \tag{2}$$

the capital letters denote quantities referring to the nucleon, the small letters to the electron, and $q = p_1 - p_2 = P_2 - P_1$.

Limiting ourselves to the first nonvanishing approximation in e, and wishing to take account of all mesonic radiative corrections, we must, along with the diagram of Fig. 1, take into account all diagrams of the types shown in Fig. 2 (the dotted lines refer to π-mesons). Clearly, the totality of all such diagrams together with the diagram of Fig. 1 leads to a matrix element differing from Eq. (2) by the replacement of $\gamma_\mu^{(p)}$ by $\Gamma_\mu^{(p)}(P_2, P_1)$,

$$\mathfrak{M}_{i\to f} = q^{-2}\left(\overline{U}_2\Gamma_\mu^{(p)}(P_2, P_1)U_1\right)\left(\overline{u}_2\gamma_\mu^{(e)}u_1\right). \tag{3}$$

$\Gamma_\mu^{(p)}(P_2, P_1)$ is the vertex operator taking account exactly of the interaction of nucleons with mesons, and to a first approximation, of nucleons with the electromagnetic field.

We will attempt to establish the most general form of the matrix element $(\overline{U}_2\Gamma_\mu^{(p)}(P_2, P_1)U_1)$. The operator Γ_μ is a vector, and must therefore be a sum of expressions containing the vectors $P_{1\mu}$, $P_{2\mu}$ (or, which is the same $q_\mu = P_{2\mu} - P_{1\mu}$) and γ_μ as factors, since there are no other vectors. In addition to the vector factor, each such expression, generally speaking, contains a function of the invariants P_1^2, P_2^2 and $P_1 \cdot P_2$ as a factor, and also the factors $\hat{P}_1$, $\hat{P}_2$ (or $\hat{q}$), arranged in a different order. But for real states of the nucleon, $P_1^2 = P_2^2 = -M^2$, and $P_1 \cdot P_2 = q^2 2/ + M^2$, i.e., the above-mentioned functions depend only on the invariant q^2. Taking the identity $P_{1\mu} = {}^1/_2(\hat{P}_1\gamma_\mu + \gamma_\mu\hat{P}_1)$ and a similar identity for P_2 into account we may limit ourselves to the consideration of only those terms in the expression for Γ_μ which contain the vector factor γ_μ. These terms may still contain some number of factors $\hat{P}_1$ and $\hat{P}_2$ in various orders.

Using the commutation relations, and also the fact that the operator $\hat{P}_2$ in the extreme left position, and $\hat{P}_1$ at the extreme right are equivalent to numerical factors iM, we conclude that effectively [in Eq. (3)] Γ_μ reduces to the sum of four terms, each of which is a product of functions of q^2 and one of the following operators $\hat{P}_1\gamma_\mu$, $\gamma_\mu\hat{P}_2$, $\hat{P}_1\gamma_\mu\hat{P}_2$ and γ_μ. But

$$\hat{P}_1\gamma_\mu = -\hat{q}\gamma_\mu + \hat{P}_2\gamma_\mu \to (-\hat{q} + iM)\gamma_\mu.$$

In the very same way we obtain

$$\gamma_\mu\hat{P}_2 \to \gamma_\mu(\hat{q} + iM), \qquad \hat{P}_1\gamma_\mu\hat{P}_2 = q^2\gamma_\mu - 2q_\mu\hat{q} - \hat{q}\gamma_\mu\hat{P}_1 + \hat{P}_2\gamma_\mu\hat{q} + \hat{P}_2\gamma_\mu\hat{P}_1 \to (-M^2 + q^2)\gamma_\mu - iM(\hat{q}\gamma_\mu - \gamma_\mu\hat{q}),$$

since in view of the equation of continuity

$$(\overline{U}_2 q_\mu \hat{q} U_1) = q_\mu(\overline{U}_2\hat{q}U_1) = 0.$$

Consequently, the operator Γ_μ in Eq. (3) may be represented in the following form:

$$\Gamma_\mu(P_2, P_1) = a(q^2)\gamma_\mu + (i/4M)[b_1(q^2)\gamma_\mu\hat{q} - b_2(q^2)\hat{q}\gamma_\mu]. \tag{4}$$

From the invariance of the theory with respect to charge conjugation, we have[2]

$$\Gamma_\mu(P_2, P_1) = -C\Gamma_\mu^T(-P_1, -P_2)C^{-1}.$$

Hence, it is not difficult to find that $b_1(q^2) = b_2(q^2)$. Taking into account further that $\hat{q}\gamma_\mu = -\gamma_\mu\hat{q} + 2q_\mu$ and that terms containing the factor q_μ do not contribute to the matrix element (3) [since in view of the equation of continuity for the electron flux $q_\mu(\overline{u}_2\gamma_\mu^{(e)}u_1) = (\overline{u}_2\hat{q}u_1) = 0$], we finally obtain* for Γ_μ

$$\Gamma_\mu(P_2, P_1) = a(q^2)\gamma_\mu + (ib(q^2)/2M)\gamma_\mu\hat{q}. \tag{5}$$

The quantities $a(q^2)$ and $b(q^2)$ may be considered as the form factors of the electric charge and the anomalous magnetic moment of the proton. For $q \ll \mu$ the quantity $a(q^2)$ is practically equal to unity,

*After completing the present work, we learned that Salzman[3] arrived at the same result using the gauge invariance of the theory. The authors thank Dr. G. Salzman for sending them a prepublication copy of his work, "Nucleon Structure in Statistical Theory," which contains a refrerence to the above mentioned article.

and $b(q^2)$ to the anomalous magnetic moment of the proton, expressed in nuclear magnetons. Eq. (5) refers as well to the neutron (of course, with other functions $a(q^2)$ and $b(q^2)$: for the neutron, as $q^2 \to 0$, $a(q^2)$ approaches zero, and $b(q^2)$ approaches the anomalous magnetic moment of the neutron).

3. We will prove that the quantities $a(q^2)$ and $b(q^2)$ are real. For this purpose, we will consider the scattering of a proton in an external, infinitely weak electromagnetic field δA_μ. In this case, the S-matrix may be written in the form

$$S = S_0 + S_1, \tag{6}$$

where S_0 is the scattering matrix in the absence of an external field, and S_1 is linear in $\delta A\mu$. From the unitarity of S_0 and S_1, we have

$$S_1 = -S_0 S_1^+ S_0. \tag{7}$$

For the matrix element between states in which there is only one proton with momentum $\mathbf{P}_1$ or $\mathbf{P}_2$, respectively we obtain from,

$$(1\mathbf{P}_2 | S_1 | 1\mathbf{P}_1) = -\sum_{m,\,n} (1\mathbf{P}_2 | S_0 | m)(m | S_1^+ | n)(n | S_0 | 1\mathbf{P}_1), \tag{8}$$

where m and n are sets occupation numbers, characterizing the specified states. Clearly, the element $(n|S_0|1\mathbf{P}_1)$ differs from zero only if the state n is a state with only one proton* with momentum $\mathbf{P}_1$

$$(1\mathbf{P}_1 | S_0 | 1\mathbf{P}_1) = 1. \tag{9}$$

Therefore,

$$(1\mathbf{P}_2 | S_1 | 1\mathbf{P}_1) = -(1\mathbf{P}_2 | S_1^+ | 1\mathbf{P}_1) = -(1\mathbf{P}_1 | S_1 | 1\mathbf{P}_2)^*, \tag{10}$$

On the other hand, the same matrix element may be calculated directly. To the first approximation in e, and without taking account of mesonic-radiative corrections, we have

$$(1\mathbf{P}_2 | S_1 | 1\mathbf{P}_1) = -e(\overline{U}_2 \gamma_\mu U_1)\,\delta A_\mu(q), \quad \delta A_\mu(q) = \int \delta A_\mu(x)\, e^{-iqx} dx. \tag{11}$$

Taking account of all mesonic-radiative corrections reduces to the replacement of γ_μ by $\Gamma_\mu(P_2, P_1)$, U_2 by $Z_2^{1/2} U_2$ and U_1 by $Z_2^{1/2} U_1$ in Eq. (11).† Equation (10) now gives

$$(\overline{U}_2 \Gamma_\mu(P_2, P_1) U_1)\,\delta A_\mu(P_2 - P_1) = -\{(\overline{U}_1 \Gamma_\mu(P_1, P_2) U_2)\,\delta A_\mu(P_1 - P_2)\}^*. \tag{12}$$

Taking into account the relations

$$\delta A_\mu(q) = \{\delta A_\mu(-q)\}^* \quad \text{for} \quad \mu = 1, 2, 3, \quad \delta A_4(q) = -\{\delta A_4(-q)\}^* \tag{13}$$

and the transversality of $\delta A\mu$

$$q_\mu\, \delta A_\mu(q) = 0, \tag{14}$$

we obtain for the transverse part of $\Gamma_\mu(P_2, P_1)$, which is the only significant part,

$$(\overline{U}_2 \Gamma_\mu(P_2, P_1) U_1) = -(\overline{U}_1 \Gamma_\mu(P_1, P_2) U_2)^* \quad \text{for} \quad \mu = 1,2,3 \quad (\overline{U}_2 \Gamma_4(P_2, P_1) U_1) = (\overline{U}_1 \Gamma_4(P_1, P_2) U_2)^*. \tag{15}$$

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*Eq. (9) is valid if mass renormalization is performed.

†We do not make the transition from $\delta A_\mu(q)$ to $\delta A'_\mu(q)$ since we are considering the electromagnetic interaction only to the first approximation, and the interaction of the electromagnetic field with π-mesons does not contribute to $\delta A'_\mu(q)$ because of the even charge parity of π^0. If neutral mesons, interacting strongly with nucleons and decaying into an odd number of photons, existed, then instead of $\Gamma_\mu(P_2, P_1)$ in Eq. (3), we would have the operator $V_\mu(P_2, P_1)$ which differs from Γ_μ by a contribution from graphs of the type shown in Fig. 3 (page 591; the double dotted line refers to a meson of odd charge parity). In this connection, Eq. (5) would hold for the operator V_μ, and all the equations obtained below would remain unchanged.

Now substituting Eq. (5) for Γ_μ, we finally obtain

$$a^*(q^2) = a(q^2), \quad b^*(q^2) = b(q^2). \tag{16}$$

That the functions $a(q^2)$ and $b(q^2)$ are real may also be proved without introducing an external field, by consideraing directly the scattering of electrons by protons to the first novanishing approximation in e. For this purpose, we must expand the S-matrix in a series of powers of e, and limiting ourselves to three terms of the series, $S = S_0 + S_1 + S_2$ (S_0, S_1 and S_2 are the terms of zero, first and second order in e, respectively), we must use the unitarity of S and S_0 on the one hand, and Eq. (3) on the other.

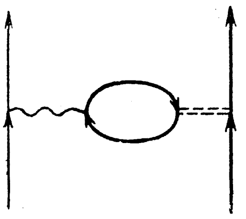

FIG. 3.

4. In the present state of theory, the form factors $a(q^2)$ and $b(q^2)$ cannot be calculated. Their determination from experiments on the scattering of electrons by protons could yield important information about the structure of the nucleon. In principle, knowing the elastic scattering cross section $d\sigma_0$ as a function of the scattering angle ϑ and the electron energy ϵ_1, each of the form factors a and b may be found separately, since one and the same value of q^2 is obtained for various ϑ and ϵ_1, and in the expression for $d\sigma_0$, the varaibles ϑ and ϵ_1 enter not only in the combination in which they are contained in q^2 [see Eq. (28)]. But the unique determination of the quantities a and b is made difficult by the unavoidable errors in the experimental values of $d\sigma_0$.

The problem would be simplified considerably, given an additional relation between a and b, which could possibly be obtained from experiments with polarized particles. However, as will be shown below, the simplest experiments of this kind (for example, investigations of the azimuthal asymmetry arising in the scattering of a polarized beam on an unpolarized target, or the polarization of particles obtained by the scattering of an unpolarized beam on an unpolarized target) cannot accomplish our purpose, since the corresponding polarization effects are proportional to the quantity $\mathrm{Im}(ab^*)$, which vanishes in view of conditions (16).

Before going on to the consideration of more complicated polarization experiments, we will introduce some equations, with the aid of which the state of polarization of a beam of relativistic particles with spin 1/2 may be described.[4] The corresponding density matrix for particles with mass m and momentum p is

$$\rho = {}^1\!/_2 (1 + i\gamma_5 \hat{\zeta})\, \eta^{(+)}(p), \tag{17}$$

where $\eta^{(+)}(p) = (1/2\epsilon)(m - i\hat{p})\gamma_4$ is a projection operator onto states of positive energy $\epsilon = +\sqrt{m^2 + p^2}$. The four-vector $\zeta = (\boldsymbol{\zeta}, \zeta_4)$ describes the polarization of the beam. In a coordinate system in which $p = 0$, the fourth component of the vector ζ vanishes: $\zeta = (\boldsymbol{\zeta}^0, 0)$; in an arbitrary coordinate system,

$$\boldsymbol{\zeta} = \boldsymbol{\zeta}_t^0 + \frac{\epsilon}{m}\boldsymbol{\zeta}_l^0, \quad \zeta_4 = \frac{i}{m}\mathbf{p}\boldsymbol{\zeta}^0. \tag{18}$$

$\boldsymbol{\zeta}_t^0$ and $\boldsymbol{\zeta}_l^0$ are the transverse and longitudinal components of the vector $\boldsymbol{\zeta}_0$ (we note that $\zeta \cdot p = \sum_{\mu=1}^{4} \zeta_\mu p_\mu = 0$). The three-vector $\boldsymbol{\zeta}$ is determined through the known density matrix ρ by the relation $\boldsymbol{\zeta} = (\epsilon/m)\,\mathrm{Sp}\,(i\gamma\gamma_4\gamma_5\rho)$. In the future, we will denote the polarization vector of electrons by $\boldsymbol{\zeta}$, and of protons by Z.

Information about the quantities a and b may be obtained from the following polarization experiments.

(a) Scattering of polarized electrons on a polarized proton target. It will be shown below that the corresponding cross section is

$$d\sigma = d\sigma_0 (1 + M_{ik} \zeta_{1i}^0 Z_{1k}^0), \tag{19}$$

where $d\sigma_0$ is the scattering cross section in the absence of polarization, and the coefficients M_{ik} contain the ratio b/a in a known form. We note that terms of the type $K_i \zeta_{1i}^0$ and $L_k Z_{1k}^0$ are absent from (19), i.e., as already noted, polarization of the target alone, or of the incident beam alone, has no effect on the scattering.

(b) The resulting polarization $\mathbf{Z}_2$ of the recoil protons during the scattering of a polarized beam of electrons ($\boldsymbol{\zeta}_1$) on an unpolarized target ($\mathbf{Z}_1 = 0$). In this case,

$$Z_{2i} = \alpha_{ik} \zeta_{1k}. \tag{20}$$

Similarly, the components of the polarization vector of the scattered electrons ζ_2 will be

$$\zeta_{2i} = \beta_{ik}\zeta_{1k}. \tag{21}$$

The coefficients α_{ik} and β_{ik} also contain the ratio b/a.

(c) Similar effects arise in an unpolarized beam of electrons is scattered on a polarized proton target (now the vectors $\mathbf{Z}_2$ and $\boldsymbol{\zeta}_2$ will be linear vector-functions of the vector $\mathbf{Z}_1$).

(d) Still more complicated experiments associated with the measurement of the polarization of the scattered electrons or the recoil protons, arising when a polarized beam falls on a polarized target.

We shall consider only the simplest cases (a) and (b), prove relations (19) and (20), and calculate the quantities M_{ik} and α_{ik}.

In the case of the scattering of a polarized beam of electrons on a polarized proton target, the scattering cross section is

$$d\sigma = \frac{e^4}{(2\pi)^2}\frac{p_2\varepsilon_2 d\Omega}{v_1\,|\,\partial W/\partial\varepsilon_2\,|}\,\mathrm{Sp}\,[\mathfrak{M}\rho_1\mathfrak{M}^+\eta_m^{(+)}(p_2)\,\eta_M^{(+)}(P_2)]. \tag{22}$$

$\mathfrak{M}$ is the transition matrix element, ρ_1 is the density matrix of the initial state and is the direct product of the corresponding electron and proton matrices,

$$\rho_1 = {}^1/_2\,(1+\gamma_5\hat{\zeta}_1)\,\eta_m^{(+)}(p_1) \times {}^1/_2\,(1+i\gamma_5\hat{Z}_1)\,\eta_M^{(+)}(P_1), \tag{23}$$

and $\eta_m^{(+)}$ and $\eta_M^{(+)}$ are projection operators for the electron and the proton.

Performing the calculations in the laboratory system of coordinates ($\mathbf{P}_1 = 0$), we have

$$d\sigma = \left(\frac{e^2}{4\pi}\right)^2\frac{p_2 d\Omega}{p_1E_2\,|\,\partial W/\partial\varepsilon_2\,|}\,[P_{\mu\nu}+R_{\mu\nu}(\zeta_1)]\,[Q_{\mu\nu}+S_{\mu\nu}(Z_1)]; \tag{24}$$

where

$$P_{\mu\nu} = P_{\nu\mu} = \frac{1}{4}\mathrm{Sp}\,[\gamma_\mu(m-i\hat{p}_1)\gamma_\nu(m-i\hat{p}_2)],\quad Q_{\mu\nu} = \frac{1}{4}\mathrm{Sp}\left[\gamma_\mu\left(a+\frac{ib}{2M}\hat{q}\right)(1+\gamma_4)\left(a^*+\frac{ib^*}{2M}\hat{q}\right)\gamma_\nu(M-i\hat{P}_2)\right),$$
$$R_{\mu\nu} = -R_{\nu\mu} = \frac{1}{4}\mathrm{Sp}\,[\gamma_\mu i\gamma_5\hat{\zeta}_1(m-i\hat{p}_1)\gamma_\nu(m-i\hat{p}_2)],\; S_{\mu\nu} = \frac{1}{4}\mathrm{Sp}\left[\gamma_\mu\left(a+\frac{ib}{2M}\hat{q}\right)i\gamma_5\hat{Z}_1(1+\gamma_4)\left(a^*+\frac{ib^*}{2M}\hat{q}\right)\gamma_\nu(M-i\hat{P}_2)\right]. \tag{25}$$

$Q_{\mu\nu}$ and $S_{\mu\nu}$ may be represented in the form of a sum of symmetric and antisymmetric parts with respect to the indices μ and ν. But we may convince ourselves by direct calculations that the antisymmetric part of $Q_{\mu\nu}$ and the symmetric part of $S_{\mu\nu}$ are proportional to ab* − a* b, i.e., are equal to zero in view of the reality of a and b. Consequently,

$$Q_{\mu\nu} = Q_{\nu\mu},\quad S_{\mu\nu} = -S_{\nu\mu},$$

and (24) takes the form

$$d\sigma = d\sigma_0\,[1+R_{\mu\nu}(\zeta_1)\,S_{\mu\nu}(Z_1)\,/\,P_{\varkappa\lambda}Q_{\varkappa\lambda}],\quad d\sigma_0 = (e^2/4\pi)^2\,(\varepsilon_2^2 d\Omega\,/\,\varepsilon_1^2 Mq^4)\,P_{\mu\nu}Q_{\mu\nu}. \tag{26}$$

After some tedious calculations in which we everywhere neglect the mass of the electron m compared with M and ϵ_1 and assume that $\vartheta \gg m/\epsilon_1$, we obtain*

$$\frac{d\sigma}{\sigma\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0[1+(\zeta_1^0\mathbf{k})\,(\mathbf{Z}_1^0, M_{11}\mathbf{k}+M_{13}\mathbf{l})], \tag{27}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{e^2}{4\pi}\right)^2\frac{a^2\cos^2\frac{\vartheta}{2}\left[1+\eta\mu^2+2\eta(1+\mu)^2\tan^2\frac{\vartheta}{2}\right]}{4M^2\xi^2\left(1+2\xi\sin^2\frac{\vartheta}{2}\right)\sin^4\frac{\vartheta}{2}}, \tag{28}$$

$$M_{11} = \rho\tan\frac{\vartheta}{2}\left[\eta\left(1-\mu+\frac{1}{\xi}\right)-\xi\right],\quad M_{13} = \rho\eta\left(\frac{1}{\xi}-\mu\right), \tag{29}$$

$$\rho = \frac{2(1+\mu)\tan(\vartheta/2)}{1+\eta\mu^2+2\eta(1+\mu)^2\tan^2(\vartheta/2)}; \tag{30}$$

*Equation 28 was obtained by Rosenbluth[5], who attempted to find an explicit form of the form factors a(q^2) and b(q^2), considering several of the simplest Feynman diagrams of the type shown in Fig. 2.

where ϑ is the scattering angle of the electron in the laboratory system, $\xi = \epsilon_1/M$ is the electron energy in the same system, expressed in units of M,

$$\eta = \frac{q^2}{4M^2} = \xi^2 \sin^2\frac{\vartheta}{2} \Big/ \left(1 + 2\xi \sin^2\frac{\vartheta}{2}\right), \tag{31}$$

$$\mu(\eta) = b(q^2)/a(q^2), \tag{32}$$

$\mathbf{k} = \mathbf{p}_1/|\mathbf{p}_1|$ is a unit vector in the direction of the incident beam of electrons $\boldsymbol{l} = [\mathbf{k} \times \mathbf{n}]$, and $\mathbf{n} = [\mathbf{p}_1 \times \mathbf{p}_2]/|[\mathbf{p}_1 \times \mathbf{p}_2]|$ is a unit vector normal to the plane of scattering. We used (18) in the derivation of (27). Terms containing $\zeta_1^0 \boldsymbol{l}$, $\zeta_1^0 \mathbf{n}$ and $\mathbf{Z}_1^0 \mathbf{n}$ were of the order m/M, and were neglected.

Taking the axes 1, 2, 3 along the vectors $\mathbf{k}$, $\mathbf{n}$, $\boldsymbol{l}$ we see that (27) reduces to (19), and (29) gives those coefficients M_{ik} which differ significantly from zero. To give some idea of the orders of magnitude which may be expected, the coefficients M_{11} and M_{13} are shown in Fig. 4 as a function of energy ξ at $\vartheta = \pi/2$ for a point proton ($\mu = 1.79$).

Let us now consider case (b). The polarization of recoil protons, resulting from the scattering of a polarized beam of electrons on an unpolarized target is

$$\mathbf{Z}_2 = \frac{(E_2/M)\mathrm{Sp}[i\gamma\gamma_4\gamma_5\eta_M^{(+)}(P_2)\mathfrak{M}^1/_2(1+i\gamma_5\hat{\zeta}_1)\eta_m^{(+)}(p_1)^1/_2\eta_M^{(+)}(P_1)\,\mathfrak{M}^+\eta_m^{(+)}(p_2)\eta_M^{(+)}(P_2)]}{\mathrm{Sp}\,[\mathfrak{M}\,^1/_2\,(1 + i\gamma_5\zeta_1)\eta_m^{(+)}\,(p_1)\,^1/_2\,\eta_M^{(+)}\,(P_1)\mathfrak{M}^+\eta_m^{(+)}\,(p_2)\,\eta_M^{(+)}\,(P_2)]}. \tag{33}$$

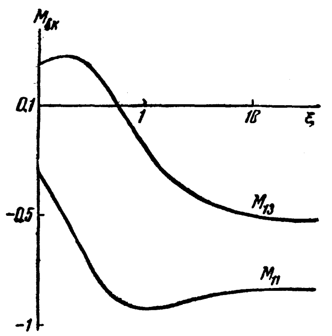

FIG. 4

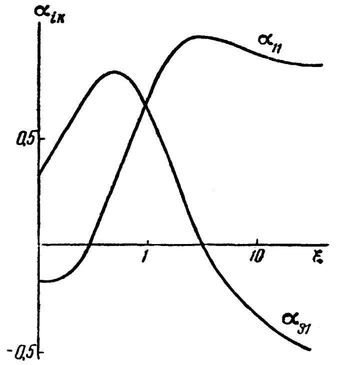

FIG. 5

Letting

$$\mathbf{B}_{\mu\nu} = -\mathbf{B}_{\nu\mu} = \frac{1}{8}\,\mathrm{Sp}\Big[i\gamma_5\gamma\,(M - i\hat{P}_2) \times \gamma_\mu\left(a + \frac{ib}{2M}\hat{q}\right)(1+\gamma_4) \times \left(a + \frac{ib}{2M}\hat{q}\right)\gamma_\nu\,(M - i\hat{P}_2)\Big] \tag{34}$$

we obtain

$$\mathbf{Z}_2 = \mathbf{B}_{\mu\nu}R_{\mu\nu}(\zeta_1)/MP_{\varkappa\lambda}Q_{\varkappa\lambda}, \tag{35}$$

where $P_{\mu\nu}$, $Q_{\mu\nu}$ and $R_{\mu\nu}$ are determined by (25). We note that the antisymmetry of $B_{\mu\nu}$ results from the reality of a and b.

Afer some tedious calculations, we find to within quantities of the order m/M, m/ϵ_1

$$\mathbf{Z}_2^0 = (\zeta_1^0\mathbf{k})(\alpha_{11}\mathbf{k} + \alpha_{31}\mathbf{l}),\ \ \alpha_{11} = \frac{\eta\rho\,\mathrm{tg}\,(\vartheta/2)}{\eta+1}\left\{-\eta\left[\mu\left(3+\frac{2}{\xi}\right) + 1 + \frac{1}{\xi}\right] + \mu(2\xi+1) + \xi + 1 - \frac{1}{\xi}\cot^2\frac{\vartheta}{2}\right\},$$

$$\alpha_{31} = \frac{\eta\rho}{\eta+1}\left\{-\eta\left[\mu\left(1+\frac{2}{\xi}\right) + \frac{1}{\xi}\right] + \mu + 2 + \frac{1}{\xi}\right\} \tag{36}$$

[compare with Eq. (20)]. The coefficients α_{11} and α_{31} are shown in Fig. 5 as functions of ξ at $\vartheta = \pi/2$ for a point proton.

We note in conclusion that since the quantities a and b depend on q^2, an unlimited number of independent experiments exist, through which the two quantities a and b may be determined for a definite value of q^2. Even if we limit ourselves to electrons of definite energy ϵ_1, the number of independent experiments involving polarization is greater than two. Therefore, it is possible to compare the results obtained for the quantities a and b from various experiments. If such a comparison leads to contradictions at high electron energies, it will be a significant indication of the inapplicability of the existing theory at sufficiently great electron energies, or, which is the same thing, at sufficiently small distances.

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THEORY OF PARAMAGNETIC RESONANCE OF F CENTERS IN IONIC CRYSTALS

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A theory is developed for the hyperfine interaction of a localized electron with nuclear magnetic moments, displaced by a certain distance from the center of symmetry of the electron wave function. The hyperfine structure of the electron energy levels has been derived. The calculation is performed with the aid of "smoothed" and detailed F-center wave functions. The result enables one to develop a theory of paramagnetic absorption of radio-frequency waves by F-centers. As examples, the shape and width of the absorption lines in KCl and NaCl crystals has been obtained.

1. INTRODUCTION

THE application of the methods of radio-spectroscopy to the investigation of localized electronic centers in dielectrics and semiconductors has led to a series of important new results.[1-5] In particular, it must be pointed out, that these experiments made it possible for the first time to learn something quantitative about the distribution of the electronic ψ-function in a crystal;[4] to observe the very small effect of the g-shift[1] (spin-electron resonance); to determine the effective mass tensor[5] (cyclotron resonance), etc.

The spin-resonance absorption of radio-frequency waves by F-centers in alkali halide crystals has been subjected to a particularly thorough experimental investigation. It was shown that the half-width of the absorption line in these crystals was several tens of oersteds. The intensity curves have a nearly gaussian shape. Attempts to explain such a large half-width of the absorption line by means of the interaction of the magnetic moments of the electrons in the various F-centers were not successful, since, for the F-center concentrations attained in the experiments ($10^{17} - 10^{18}$ cm^{-3}), this interaction leads to a narrow absorption line, whose half-width is of the order of a few hundredths of an oersted. Kip, Kittel, Levy, and Portis[4] proposed that the reason for the widening of the absorption line was the interaction of the electron in the F-center with the magnetic moments of the nuclei of the metal ions which surround the missing halogen ion.

As is well known, two models are currently accepted in the theory of F-centers, i.e., the "continuum model" and the "molecular orbital."[7,8] Application of the orbital model, according to which the wave function of the electron has the form of a linear combination of the wave functions of the atoms surrounding the vacancy, led to the correct order of magnitude for the half-width of the absorption line. In contrast to this, estimates of the half-width based on the continuum model resulted in a disagreement between theory and experiment of from three to four orders of magnitude (in Ref. 4, the disagreement was somewhat less because the authors of that work used the somewhat incorrect model of Simpson,[9] instead of the results of Refs. 6 and 7). Hence, it was concluded that the continuum model was not valid. As we shall