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THE OPTICAL MODEL OF THE NUCLEUS AND THE SHELL MODEL

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We investigate the bound states of a neutron in a potential whose parameters are obtained from the optical model of the nucleus, plus a spin-orbit interaction. Agreement with experimental data is obtained for the ordering of single particle levels for nuclei consisting of a closed shell plus one nucleon.

It is known that the shell model is successful only for nuclei consisting of a closed shell or of a closed shell plus one particle. For nuclei which have one particle outside of a closed shell, the excited states can be divided into two groups. Among the levels belonging to the first group are those which have a high probability of excitation in (n, γ), (p, γ), (d, p), and (d, n) reactions. Such levels are usually called single particle levels, since it is assumed that they occur when the odd nucleon occupies a level in the field of the core. Other levels are excited weakly in the reactions cited; this may be regarded as a consequence of a change of state of the core in the formation of such levels. This notion is confirmed in various cases by the values of the spin and parity of the levels (for example, for O^{17}). The latest experiments of Groshev and co-workers[1] on (n, γ) reactions and the experiments of several authors on (d, p) reactions have shown that in many other odd nuclei one group of levels is excited very strongly compared to the other levels. The data on (d, p) reactions, which refer to states with arbitrary angular momentum, are especially instructive. These experiments show that single particle levels occur quite commonly, i.e., the model using an average core field has a wide range of validity. One may raise the question whether the scheme of single-particle levels could be obtained from a potential well model. In doing this it would be desirable for the parameters of the potential well to be determined from other experiments.

There is a direct connection between the problem of single particle levels and that of the broad neutron resonances which have been explained on the basis of the optical model. The first model of Weisskopf, Feshbach, and Porter,[2] which used a square well, was crude and did not explain the large value of the reaction cross section at low energies. Later, the present author[3] improved this model by considering a nucleus with a diffuse edge. The model enables one to explain the fundamental regularities in neutron cross sections, angular distributions and polarizations for scattering by medium and heavy nuclei (excluding the case of highly deformed nuclei).

Computations of cross sections done on the "Strela" computer enabled us to obtain the most reasonable set of parameters for describing the nuclear potential.

1. CHOICE OF POTENTIAL

The potential is complex. The assumption was made that a potential with the same parameters, but omitting the imaginary part (so that there are no incoming waves), should describe the stationary single particle levels. The parameters of this potential turned out to be quite close to those used in the calculations of Ross et al.,[4] although they differed from their values in various respects. The real part of the optical potential consists of two terms:

1) the average potential, which is the same for all ℓ and j, and is equal to

$$V_1 = V_0 \Big/ \left[1 + \exp\left(\frac{r - R_0}{a}\right)\right];$$

2) the spin-orbit interaction, which is chosen in the form

$$V_2 = \frac{\varkappa}{r} \frac{dV_1}{dr} (\mathbf{l}\cdot\sigma).$$

The constant in the spin-orbit interaction was taken from the experiments of Levintov,[5] whom the author wishes to thank. This value, $\kappa = 3.3 \times 10^{-27}$ cm^2 is somewhat lower than that of Ross et al. A calculation of the polarization of 400 kev neutrons shows that this value of κ is in agreement with the experiments of Adair. The quantity α which characterizes the diffuseness of the boundary was taken equal to $1/k_0$, where

$$k_0 = \sqrt{2mV_0}/h$$

This value of α gives reasonable values for the neutron strength function. The values of k_0 and R_0 were selected by comparing calculated and experimental curves of total cross sections and angular distributions. The final parameter values chosen were: $V_0 = 50$ Mev, $R_0 = 1.23 \times 10^{-13}$ $A^{1/3}$ cm.

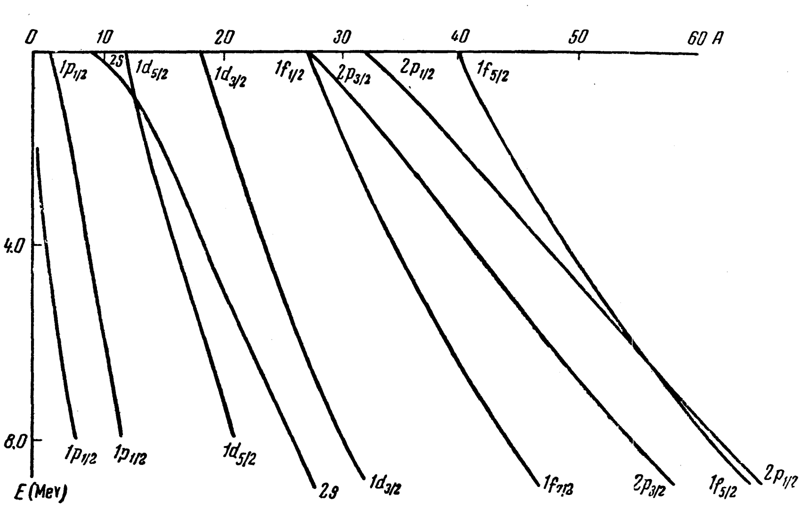
With this potential, we calculated the dependence of level positions on R_0.

2. CALCULATION OF BOUND STATES

The calculation of bound states was done by hand computation. The solution of the Schrödinger equation was represented in the form

$$\psi = x^{-1/2} K_{l+1/2}(x)\, v_l(x),$$

where $x = \kappa r$ and $\kappa = \sqrt{-2mE}/h$, and E is the level energy. At infinity, $v_l(x) \to 1$. We tried to find a value of R_0 for which $v_l(x)$, for a preassigned value of E, goes to zero at $x = 0$. Then the function $\psi(x)$ satisfies the correct boundary conditions at $x = 0$ and $x = \infty$, and the corresponding value of E is the level energy for given R_0, l, and j. Level positions were calculated for energies from zero up to 8 — 9 Mev. There is no sense in computing lower levels since they are already filled and cannot appear as one-particle levels. Ross et al. computed precisely those levels whose binding energy is greater than or equal to the binding energy of the first occupied level. Thus our work fills the gap between the continuous spectrum and the first capture level. At present, the calculations have been done for 1s, $1p_{3/2}$, $1p_{1/2}$, 2s, $1d_{5/2}$, $1d_{3/2}$, $1f_{7/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$, i.e., for nuclei with $A < 50$. For heavier nuclei the level positions are of less interest. The calculation is interesting once more only in the region of lead, for which we plan to do it later. The dependence of level energy on nuclear radius is shown in the figure.



Dependence of Nuclear Energy Levels on Atomic Weight

The most important feature shown in the figure is the change in the order of levels with changing nuclear radius, i.e., the crossing of terms. When the nuclear radius is small or, what amounts to the same thing, when the term has a high excitation, the ordering is essentially different from the ordering of the ground states. Thus at high excitations the 2s level has lower energy than the $1d_{5/2}$, while the $1f_{5/2}$ lies above the $2p_{1/2}$, $2p_{3/2}$ levels. As we shift to the ground states, the $1d_{5/2}$ drops below the 2s level, while the $1f_{5/2}$ goes below the $2p_{1/2}$ level and, for large A, below the $2p_{3/2}$ level. This crossing of terms is caused by the centrifugal potential which, in nuclei of small radius, causes an appreciable increase in the energy of levels with large l, and pushes them into the continuous spectrum.

With increasing radius, the influence of the centrifugal potential diminishes, and levels with large l drop more rapidly than those with small l.

3. COMPARISON WITH EXPERIMENTAL DATA

We should like first to compare the level scheme we have obtained with the experimental data for nuclei consisting of a closed shell plus one particle. These are the nuclei O^{17} and Ca^{41}, and also those nuclei which correspond to a closed subshell plus one particle; C^{13}, Si^{29}, Si^{31}, and S^{33}. For the value of R_0 which we have used, the O^{16} nucleus has a radius of 3.11×10^{-13}. With this R_0, the order of the levels is the following: 1s, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, 2s, $1d_{3/2}$. The 1s, $1p_{3/2}$, and $1p_{1/2}$ are filled, and the ninth neutron, in agreement with experiment, is in the $1d_{5/2}$ state, whose energy, according to the theoretical curve, is 4.1 Mev in good agreement with experiment. The first excited state is 2s. The experimental value of the excitation energy is 0.5 Mev less than calculated, but better quantitative agreement should not be expected. The $1d_{3/2}$ state lies at about 1 Mev up in the continuous spectrum, in complete agreement with experiment. Other low-lying states must be assumed to be empty.

Now we consider C^{13}. For this nucleus, the binding energy of the $1p_{1/2}$ ground state is in poor agreement with experiment (the computed value is 8 Mev). Such a deviation may be related to the inexact nature of the j-j approximation. If the coupling of orbital moment and spin is intermediate, then C^{12} does not represent a closed shell and the $1p_{1/2}$ level is not a rigorous single-particle level for C^{13}. On the other hand, the excited states have the opposite parity, and for them the field of the C^{12} nucleus may be considered to be some average field. It is therefore not surprising that better agreement with the theory is obtained for the excited 2s and $1d_{5/2}$ states. The energy of the excited state is about 1 Mev higher than the experimental value, but the order of the levels is the same as in experiment, and the splitting is ~ 0.5 Mev instead of the experimental value of 0.7 Mev.

For nuclei with $A > 17$ the binding energy of the $1d_{5/2}$ state should not be compared with the theory, since there are now many particles in the $1d_{5/2}$ state, and their interaction with the last odd particle cannot be described by means of a self-consistent field. The single-particle 2s and $1d_{3/2}$ states must be assigned in each particular case. This can be done, for example, from the probability of excitation in deuteron stripping. If we assume that the ground state of Si^{29} is a single-particle 2s state, then the computed binding energy is 9.3 Mev, while experiment gives 8.5 Mev, so that the agreement is satisfactory. For S^{33} the ground state is $1d_{3/2}$, and the binding energy is also in satisfactory agreement with experiment. For nuclei with $N < 20$, the $1f_{7/2}$, $2p_{3/2}$, and $2p_{1/2}$ states should be excited. Since the parity of these states is opposite to that of the partially filled shells ($1d_{5/2}$, 2s, $1d_{3/2}$), the single-particle model can here give a satisfactory description even for incomplete shells, i.e., for any odd nucleus. Actually, the situation is made essentially more complicated by the large deformations of the nuclei in the region $20 < A < 30$. Experiments on the (n, γ) reaction indicate the presence of a bound $2p_{3/2}$ level for Mg^{25} (with a binding energy of 4 Mev). However, according to the calculation, for Mg^{25} the $1f_{7/2}$ and $2p_{3/2}$ states should still lie in the continuous spectrum. These states begin to be bound only at Si^{29}. Also the excitation energy of the $3/2^-$ state increases as we go to Si^{29}, which is in qualitative disagreement with the theory for spherical nuclei. For Si^{29} the $1f_{7/2}$ and $2p_{3/2}$ states have $E < 0$, but the excitation energies of these states are considerably higher than the experimental values. On the other hand, $\Delta E = E(f_{7/2}) - E(p_{3/2})$ is close to the experimental value. The agreement with experiment improves for S^{33}, although the excitation is again too high.

The level scheme for Ca^{41} is most interesting. It has been pointed out that for this nucleus, excited single-particle $2p_{3/2}$ and $2p_{1/2}$ states are observed, while the $1f_{5/2}$ state is not. According to the computation, the binding energy of the $1f_{5/2}$ state is close to zero, and one is unable to observe this state by, for example, deuteron stripping. The order of the other states and their energy differences are given more or less satisfactorily, but all the states have binding energies somewhat lower than the experimental values.

On the whole we may say that the optical model gives the correct order of excited single-particle states of light nuclei, but cannot give the position of individual levels with sufficient accuracy. This may be partially due to the fact that the $A^{1/3}$ law is not exact. A deviation of 1% from this law would be sufficient to shift a level by 0.5 Mev. In addition, the shape of the potential near the nuclear boundary is approximate. However, in some cases one may hope to go a little farther and calculate the probability of radiative transitions, starting from the singe-particle model.

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SCATTERING OF ELECTROMAGNETIC WAVES IN A PLASMA

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Combination scattering by plasma density oscillations may occur when electromagnetic waves are propagated in a plasma. The intensity of combination scattering of electromagnetic waves in a plasma is determined in the absence and in the presence of a constant uniform magnetic field.

I. It is well known that there can exist in a plasma weakly damped electromagnetic oscillations which are associated with oscillations of plasma density whose frequency (without taking dispersion into account) is given by[1] $\Omega = \sqrt{4\pi n_0 e^2/m}$. The existence of these oscillations leads to a periodic variation of the dielectric constant in the plasma. Because of this, combination scattering of electromagnetic waves propagated in the plasma becomes possible, i.e., if a wave of frequency $\omega_0 > \Omega$ is propagated in the plasma then at the same time waves with frequencies $\omega = \omega_0 \pm n\Omega$, where n is an integer, will also be propagated. The object of this paper is to determine the intensity of these waves.

Let us first determine the dielectric constant of the plasma. We start with the equation

$$\mathrm{curl}\,\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{1}{c}\frac{d\mathbf{E}}{dt} + \frac{4\pi}{c}\mathbf{j}_{\mathrm{free}}, \tag{1}$$

where $\mathbf{j}_{\mathrm{free}}$ is the current density associated with the motion of the plasma electrons and is equal to $\mathbf{j}_{\mathrm{free}} = e n \mathbf{v}$, n is the electron density, $\mathbf{v}$ is the electron velocity which is related to the electric field by the equation $\dot{\mathbf{v}} = e\mathbf{E}/m$. From these equations it follows that

$$\mathbf{v}(\mathbf{r}, t) = \frac{e}{m}\int^{t} \mathbf{E}(\mathbf{r}, t')\,dt',$$

$$\mathbf{j}_{\mathrm{free}}(\mathbf{r}, t) = \frac{e^2}{m} n(\mathbf{r}, t)\int^{t} \mathbf{E}(\mathbf{r}, t')\,dt'.$$

Substituting this expression for $\mathbf{j}_{\mathrm{free}}$ into Eq. (1), we find

$$\frac{\partial \mathbf{D}}{\partial t} = \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi e^2}{m} n(\mathbf{r}, t)\int^{t} \mathbf{E}(\mathbf{r}, t')\,dt'.$$

We now introduce the dielectric constant operator by $\mathbf{D} = \hat{\epsilon}\mathbf{E}$. Then

$$\hat{\varepsilon}(\mathbf{r}, t)\,\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + \frac{4\pi e^2}{m}\int^{t} dt' n(\mathbf{r}, t')\int^{t'} dt''\mathbf{E}(\mathbf{r}, t''). \tag{2}$$