find that the total intensity is $I = (1.88 \pm 0.05) \times 10^{-4}\,cm^{-2}\,sec^{-1}\,sterad^{-1}$, which is 2.4% of the number of particles which will penetrate 15 cm Pb.

The maximum of the proton spectrum at sea level is at 0.5 — 0.6 Bev/c. The maximum in the proton spectrum at 3250 m. above sea level is at about the same place.[15] The shapes of the two spectra are the same within experimental error, which is to be expected since for the momenta considered both are the spectra of secondary particles. The shape of the spectrum and the position of the maximum agree well with the theoretical spectrum for secondary protons as computed by Rossi.[16]

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Translated by R. Krotkov
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CYCLOTRON RESONANCE IN PLASMA

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Submitted to JETP editor May 21, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 544-546 (August, 1957)

THE physical mechanism of cyclotron resonance in metals, for which a theory was formulated in Refs. 1 and 2 leads to the belief that a similar effect should occur in a compensated electron-ion plasma. Let us recall that cyclotron resonance in metals occurs only in a direct magnetic field H parallel to the metal surface* when an electron is repeatedly returned under the influence of the magnetic field at the necessary phase to a skin layer whose thickness δ is small compared with the radius of the electron orbit r in the magnetic field. Naturally, a sufficiently large number of revolutions must be performed in the length of the free path ℓ before resonance can be observed, i.e., $\ell \gg 2\pi r$. Otherwise, resonance is generally absent. Cyclotron resonance occurs when the external field frequency ω is a multiple of the Larmor frequency $\Omega = eH/mc$.

Let us consider the conditions under which resonance in plasma is possible (we neglect ion motion). It is necessary for resonance that "anomalous skin effect" occur:[1,2]

*Cyclotron resonance was recently observed in tin and bismuth.[3]

$$\delta \ll r \ll l/2\pi. \tag{1}$$

The inequality to the right is satisfied very well in plasma because of the large free path which, as is known,[4,5] is determined only by the "distant" interactions ($l \sim (kT)^2/Ne^4L$, L is the Coulomb logarithm, N the density, e the charge on the electron, T the temperature of the electron gas, and k the Boltzmann constant), by collisions with ions and neutral molecules, or by the characteristic dimensions of the system in the case of bounded plasma.

A sharply pronounced skin effect [the left inequality in (1)] is possible in a plasma because of the negative dielectric constant of the plasma, when the displacement current can be neglected in comparison with the conduction current. Estimates show (see Ref. 2 for example) that the inequality $\delta \ll r$ is equivalent to

$$\bar{v}/c \gg \Omega\,(m/Ne^2\omega\tau)^{1/2} \sim (\omega/\omega_0)\cdot(2\pi/\omega\tau)^{1/2}. \tag{2}$$

Here $\bar{v} \sim (kT/m)^{1/2}$ is the average thermal velocity, m the mass, τ the electron free path time, and $\omega_0 = (4\pi Ne^2/m)^{1/2}$ the plasma frequency. We assume that $\omega \approx \Omega$. Thus, for example, inequalities (1), (2) will be satisfied for $T \sim 10^7$ °K, $N \sim 10^{14}\,\text{cm}^{-3}$ and $\omega \sim 10^{10}$/sec ($\lambda \sim 20$ cm) if $l \gg 1$ cm. If conditions (1) and (2) are satisfied, the entire theory of cyclotron resonance developed in Refs. 1 and 2 for metals can be extended to include plasma. The only difference is that a Maxwellian rather than Fermi equilibrium holds in the plasma: $f_0 = A\exp(-mv^2/kT)$.

Let us cite the final result of a computation of the high-frequency surface impedance Z near resonance for a half-space occupied by plasma:

$$Z = R + iX = Z_0\,[1 - \exp(-2\pi i\omega/\Omega - 2\pi/\Omega\tau)]^{1/3} \approx Z_0\,(2\pi)^{1/3}\,[1/\Omega\tau + \pi(\omega - n\Omega)^2/\Omega^2 + i(\omega - n\Omega)/\Omega]^{1/3};$$
$$Z_0 = (\sqrt{3}\,\pi\omega^2/c^4B)^{1/3}(1 + i\sqrt{3});\quad 1/B = (m/Ne^2)(9\pi kT/8m)^{1/2}. \tag{3}$$

The surface impedance is $Z = (4\pi/c)E(0)/H(0)$, where $E(0)$ and $H(0)$ are the alternating electric and magnetic field intensities on the plasma boundaries. Formula (3) is derived under the assumption that the constant magnetic field is strictly parallel to the interface surface,* that the electrons are scattered from the plasma boundaries in an arbitrary manner (but not specularly!), and that the electromagnetic wave is incident perpendicularly on the plasma surface.

At resonance ($\omega \approx n\Omega$, $n \ll \omega\tau/2\pi$), R and X have a minimum, since the external field whose energy the electrons absorb is of fixed intensity. The relative depth of resonance for R/R_0, X/X_0, X/R and the respective resonance fractions are

$$R_{\text{res}}/R_0 = (2\pi n/\omega\tau)^{2/3};\quad \Omega_{\text{res}} = (\omega/n)(1 - 1/\sqrt{2\pi n\omega\tau});\quad X_{\text{res}}/X_0 = 2\cdot 3^{-1/2}(\pi n/\omega\tau)^{1/3};$$
$$\Omega_{\text{res}} = (\omega/n)(1 + 1/\omega\tau);\quad (X/R)_{\text{res}} = {}^3/_2\,(\omega\tau/\pi n)^{1/3};\quad \Omega_{\text{res}} = (\omega/n)(1 - 1/\sqrt{\pi n\omega\tau}). \tag{4}$$

The small shift in the resonant frequency relative to ω/n is due to the different effective depth of variation in the amplitude and phase of the electric field (see Refs. 1 and 2 for details). The order of magnitude of the relative width of the resonance curves is

$$\Delta H/H \sim |\omega - n\Omega_{\text{res}}|/\omega. \tag{5}$$

Apparently, cyclotron resonance in plasma can be detected by selective absorption of an electromagnetic wave in the resonant region. A "plasma plate" is transparent in a weak magnetic field with $\omega_0 < \omega$ and in a strong magnetic field when (2) is violated, but absorbs strongly in the resonance region.

Moreover, cyclotron resonance permits relaxation time in a plasma to be measured more exactly than usual.

In conclusion, I thank I. M. Lifshitz and Ia. B. Fainberg for discussing the results.

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*The admissible angular deviation of the magnetic field relative to the surface must not exceed $(\sqrt{2\pi}\,c\sqrt{v}\,\omega\omega_0\tau^2)^{1/3}$. Otherwise there is no substantial dependence on H.

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Translated by M. D. Friedman
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ON THE RANGE OF MANY-BODY FORCES BETWEEN A Λ^0-HYPERON AND NUCLEONS

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Submitted to JETP editor May 3, 1957

J. Exptl. Theoret. Phys. 33, 546-547 (August, 1957)

THE forces between a Λ^0-hyperon and one nucleon [($\Lambda^0 - N$) forces], admissible on grounds of isotopic invariance, have a range several times shorter than the usual nuclear forces.[1,2] Thus, the forces suggested by the experiments on pair production and due to virtual processes with K-meson exchange

$$\Lambda^0 + N \rightarrow N' + \bar{K} + N \rightarrow N' + \Lambda^{0\prime}, \tag{1}$$

have a range $1/m_K$, i.e., a range about three times shorter than the radius of nucleon-nucleon forces, which is of order $1/m_\pi$. However, isotopic invariance allows an interaction in which the Λ^0-hyperon exchanges with the nucleon two (in general, an even number of) π-mesons:

$$\Lambda^0 + N \rightarrow \Lambda^{0\prime} + \pi + \pi + N \rightarrow \Lambda^{0\prime} + N'. \tag{2}$$

One can also conceive of similar 2π-meson forces involving Σ-hyperons:[3]

$$\Lambda^0 + N \rightarrow \Sigma + \pi + N \rightarrow \Sigma + N' \rightarrow \Lambda^{0\prime} + \pi + N' \rightarrow \Lambda^{0\prime} + N'. \tag{3}$$

The forces due to (2) and (3) have a range $1/2m_\pi$.

The existence of Λ^0-hyperfragments indicates that the ($\Lambda^0 - N$) forces are sufficiently strong to lead to binding of a Λ^0-hyperon to nucleons. In this connection it is interesting to determine whether there exist ($\Lambda^0 - N$) forces with the usual range $1/m_\pi$.

Such forces indeed are possible if we consider the interaction of the Λ^0-hyperon with two nucleons instead of one. Thus, the 2π-meson forces between a Λ^0 particle and two nucleons, realizably by virtual processes of the form

$$\Lambda^0 + N_1 + N_2 \rightarrow \Lambda^{0\prime} + \pi + \pi + N_1 + N_2 \rightarrow \Lambda^{0\prime} + N_1' + N_2', \tag{4}$$

have a range $1/m_\pi$, since the Λ^0-hyperon exchanges with each of the nucleons only one π-meson. This conclusion regarding the range of the forces is purely qualitative and follows from the uncertainty relation for the energy and time, and actually involves no approximations.

As an illustration it is easy to calculate the potential of the ($\Lambda^0 - N$) forces of the type (4) in the static approximation. This potential contains the product of the exponentials $\exp(-m_\pi r_{1\Lambda}) \exp(-m_\pi r_{2\Lambda})$, where r_i is the distance between the Λ-hyperon and the i-th nucleon. Thus many-body forces of range $1/m_\pi$ between a Λ^0-hyperon and nucleons are possible, in contrast to two-body ($\Lambda^0 - N$) forces, whose range is less. One can therefore assume that the suggested many-body forces may play an important role in the interaction of the Λ^0-hyperon with nucleons.

This hypothesis could be checked by measuring the cross section for the scattering of a slow Λ^0 particle by a proton and by some simple nucleus such as deuterium or helium. If the present hypothesis is true the cross sections should be considerably larger for scattering by nuclei than for scattering by protons.

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Translated by M. Danos
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ROTATIONAL BANDS OF EVEN-EVEN AXIALLY SYMMETRIC NUCLEI

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Submitted to JETP editor May 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 547-549 (August, 1957)

It has been shown by Davydov and Filippov[1] that by using the Hamiltonian obtained by Ford[2] in averaging the interaction between the external nucleons and the nuclear core, one can write the equation for the collective motion of an axially symmetric even-even nucleus with total angular momentum $\hbar J$ in the form

$$d^2U_\nu / d\zeta^2 - 2\zeta dU_\nu / d\zeta + 2\nu U_\nu = 0, \tag{1}$$

where U_ν satisfies the boundary condition

$$U_\nu(-\delta\xi) = 0,\quad U_\nu(\zeta)e^{-\zeta^2/2} \to 0, \qquad \text{for } \zeta \to \infty. \tag{2}$$

The eigenvalue ν of Eq. (1) is not in general an integer, and determines the energy $\epsilon_\nu(J)$ of the collective nuclear motion by the equation

$$\varepsilon_\nu(J)/\hbar\omega_0 = (\nu + 1/2)\sqrt{1 + J(J+1)/\delta^4\xi^4} + J(J+1)/6\delta^2\xi^2 + 1/2\,\delta^2(\xi - 1)^2,$$
$$\xi^3(\xi - 1) = J(J+1)/3\delta^4. \tag{3}$$

Thus the energy of nuclear collective motion for each value of J = 0, 2, 4, . . . is determined uniquely by just the two parameters ω_0 and δ, which are related to parameters of Bohr and Mottelson's[3] generalized nuclear model by the expressions $\omega_0 = \sqrt{C/B}$, $\delta = \beta(BC/\hbar^2)^{1/4}$.

Davydov and Filippov[1] investigated the solution of Eq. (1) for the case $\delta \leq 1$. In this note we present the results of a solution of this set of equations for the case $\delta > 1$.

The figure gives a graph of $\epsilon_\nu(J)/\hbar\omega_0$ vs. δ; the numbers on the curves give the values of J. It is seen from the figure that when $\delta > 2.5$, the energy spectrum of collective excitations of even-even nuclei breaks up into a set of rotational-vibrational bands. In Table 1 we present a comparison of the theoretical excitation energies of the first and second rotational band for certain nuclei with the experimental data. We also give the values of $\hbar\omega_0$ and δ which have been used to calculate the theoretical excitation energy.

TABLE 1

Nucleus and literature reference	J	Energy level (kev)		$\hbar\omega_0$ (kev)	δ
		Theory	Experiment		
W^{182} [4,5]	2	100.09	100.09		
	4	320.3	329.36		
	6	641.6	677.6		
	0	1101	—	1101	3.48
	2	1222	1222		
	4	1481	1488.6		
Th^{232} [6]	2	50	50		
	4	163	165		
	6	332	—		
	0	710	—	710	3.93
	2	770	770		
	4	901	—		
U^{234} [5]	2	43	43		
	4	141	142		
	6	290	295		
	0	803	803	803	4.48
	2	855	—		
	4	966	—		
Pu^{238} [5]	2	44.2	44.2		
	4	147.7	146		
	6	304.8	303		
	0	935	935	935	4.73
	2	986	986		
	4	1100	1073		

In Table 2 we give the δ dependence of the energy ratios of the first and second (1 and 2) rotational-state sublevels in the first and second (I and II) bands of the rotational states of the nucleus.

If the energy of collective oscillations is approximated in the form

$$E_J = n\hbar\omega_0 + AJ(J+1) - BJ^2(J+1)^2, \qquad A = \hbar^2/2I, \quad B = a(\hbar\omega_0)^{-2}(\hbar/I)^3, \tag{4}$$