pairs by an electron hitting a nucleus

$$\sigma_{2e} = \int d\sigma(\omega', \omega'')\,\beta(\omega')\,\beta(\omega''), \quad \beta(\omega) \approx \frac{2\alpha}{3\pi}\ln\frac{\omega}{m}. \tag{1}$$

Apart from an undetermined numerical coefficient of the order of unity, we get finally for the total cross-section for the effect of production of two electron-positron pairs by an electron hitting a nucleus in the extreme relativistic approximation (with logarithmic accuracy):

$$\sigma_{2e} \approx (r_0^2 Z^2 \alpha^4/\pi^3)\ln^4(E_0/m). \tag{2}$$

(Here and in what follows E_0 is the energy of the initial particle, electron or photon.) We note that in principle the formula (2) could be obtained in the framework of the Weitzsäcker-Williams method[5,6] by using the effective cross-section for the production of two electron-poitron pairs by a γ-ray quantum in the field of the nucleus: $\gamma + Z \rightarrow 2(e^- + e^+) + Z$. Therefore for the effective cross-section for the formation of two pairs by a photon hitting the nucleus we must have

$$\sigma_{2\gamma} \approx (r_0^2 Z^2 \alpha^3/\pi^2)\ln^2(E_0/m). \tag{3}$$

Comparing Eq. (2) with the Landau-Lifshitz formula[7] for the production of one pair by an electron hitting a nucleus

$$\sigma_{1e} \approx (r_0^2 Z^2 \alpha^2/\pi)\ln^3(E_0/m)^3,$$

and also comparing Eq. (3) with the Bethe-Heitler formula[8] for the production of one pair by a photon colliding with a nucleus

$$\sigma_{1\gamma} \cong r_0^2 Z^2 \alpha \ln(E_0/m),$$

we come to the conclusion that the expansion parameter in terms of the n of pairs produced is

$$\varepsilon = \sigma_{n+1}/\sigma_n \lesssim (\alpha/\pi)^2 \ln(E_0/m).$$

The formulas given here, Eqs. (2), (3), do not include the effect of screening of the nuclear field by the atomic electrons. As is well known, this latter effect to a considerable degree hinders the increase of the cross-sections in the high-energy region. Therefore it is not excluded that effects of greater interest might be those of production of pairs in collisions with electrons, the effective cross-sections for which can be obtained from Eqs. (2) and (3) by simply replacing Z^2 by unity. It must be remarked, however, that according to Ref. 4 the effect of screening on the effects considered above is relatively small.

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RADIATION INSTABILITY IN NUCLEAR MAGNETIC RESONANCE EXPERIMENTS

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In experiments on nuclear magnetic resonance, the substance being investigated is placed in an rf coil which is part of a resonant circuit. Analysis of the oscillating system comprising the specimen with precessing nuclear magnitization and the resonant circuit shows that in certain cases one cannot neglect the change caused in the rf magnetic field within the winding by the coupling of the specimen to the circuit. It has been shown[1-3] that this change in the field leads to increased damping of the free precession of the nuclear magnetization and to broadening of the signals observed under steady-state conditions.

The present note considers the stability of the system consisting of the specimen and the resonant circuit. The properties of the sample are characterized by the macroscopic Bloch equations.[4] It is assumed that the radiofrequency field acting on the sample is due only to the emf induced in the circuit by the sample, i.e., that the circuit is not connected to an external signal generator. The circuit is tuned close to the free-precession frequency of the nuclear moments, and the transmission band of the circuit is assumed large compared with the width of the nuclear magnetic resonance line. Under these assumptions we have

$$\dot{M}_x - \gamma H_0 M_y + M_x/T_2 = 0,\ \dot{M}_y + \gamma H_0 M_x - \gamma H_x M_z + M_y/T_2 = 0,$$
$$\dot{M}_z + \gamma H_x M_y + M_z/T_1 = M_0/T_1,\ H_x = -(4\pi\eta Q/|\gamma|H_0)\dot{M}_x. \tag{1}$$

Here η is a space factor almost equal to unity when the coil is completely filled by the substance being investigated, and Q is the figure of merit of the circuit.

Let us assume that at time $t = 0$, we have $M_x = M_y = H_x = 0$, and $M_z \neq M_0$. Then one has the obvious solution

$$M_x = M_y = H_x = 0,\ M_{z0} = M_0 + A\exp(-t/T_1), \tag{2}$$

where A is a constant. To test for stability, let us consider the set of equations for small deviations from (2), i.e.,

$$\dot{M}_x - \gamma H_0 M_y + M_x/T_2 = 0,\ \dot{M}_y + \gamma H_0 M_x - \gamma H_x M_{z0} + M_y/T_2 = 0,\ \ \dot{M}'_z + M'_z/T_1 = 0,\ H_x = -(4\pi\eta Q/|\gamma|H_0)\dot{M}_x, \tag{3}$$

where $M'_z = M_z - M_{z0}$. In Equation (3) terms of higher order have been dropped. The time dependence of M_x is given by

$$\ddot{M}_x + (4\pi M_{z0}|\gamma|\eta Q + 2/T_2)\dot{M}_x + (\gamma^2 H_0^2 + T_2^{-2})M_x = 0. \tag{4}$$

It can be shown by separating out the oscillating factor in the solution of this equation, that small deviations from $M_x = M_y = H_x = 0$ increase exponentially if

$$-2\pi M_{z0}|\gamma|\eta Q > 1/T_2. \tag{5}$$

Contrary to the statements of Bloembergen and Pound,[2] the state in which the nuclear magnetization is oriented against the field is not nonradiative when condition (5) is fulfilled. Instead of a purely relaxational change in M_z, self-excitation will occur with the nuclear magnetization approaching its equilibrium value more rapidly and part of the magnetic energy being converted into the energy of high-frequency vibrations. Numerical calculations similar to those performed by Bloembergen and Pound[2] show that condition (5) for self-excitation can be fulfilled, for instance, in experiments with water at attainable values of H_z.

The process of self-excitation when $2\pi|M||\gamma|\eta Q \gg 1/T_2$ is described by the equation

$$d\vartheta/dt = -2\pi|M||\gamma|\eta Q\sin\vartheta,\quad \vartheta = \arccos(M_z/|M|).$$

The solution $\tan(\vartheta/2) = \exp(-2\pi|M||\gamma|\eta Qt)$ of this equation has been used to describe radiation damping.[2] When $t < 0$ this same solution can be used to study the process of self-excitation which is accompanied by the appearance of an inductive signal proportional to $\operatorname{sech}(2\pi|M||\gamma|\eta Qt)$ in the circuit.

The system comprising the sample and the resonant circuit can be used, in the self-excitation state and close to it, as a nuclear magnetic signal generator and amplifier. Fluctuations of the nuclear magnetization are low, so that one could expect to obtain noise factors close to unity in this way. Continuously variable apparatus of this type could be designed with continuously replaceable operating substances.

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