at $\eta = (\alpha\beta)^{1/2}/J_k$, that is when $|\gamma|\,H_1 = (T_1T_2)^{-1/2}J_k$, and then decrease. The maximum values of the amplitudes are independent of the small parameters. Thus, for instance, the maximum amplitude of the central component of the dispersion signal (the term proportional to sin τ) is $2M_0J_1(T_2/T_1)^{1/2}$. It is significant that the conditions for the optimum signal strength are attained within the region of applicability of Eq. (4). The condition that Eq. (4) be applicable corresponds to choosing a frequency high enough for the nuclear magnetic resonance signal to have a resolved modulation structure.

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MULTIPLE PAIR PRODUCTION IN QUANTUM ELECTRODYNAMICS

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THE results of recent studies* of the production of electron-positron pairs in photographic emulsions by cosmic rays apparently confirm the reality of the previously observed[1,2] cases of production of two electron-positron pairs in a single act.

It has been pointed out repeatedly that effects of multiple pair production can be used as a criterion for the applicability of the present quantum theory of electromagnetic interactions, just in the range of phenomena in which the correspondence principle can no longer be used. Since up to this time there have not been in the literature any statements on the theory of multiple production of electron-positron pairs, it seemed worth while to make an estimate of the values of the effective cross-sections in question. In the present note we present approximate expressions for the effective cross-sections for production of two electron-positron pairs in the collision of an electron with an atomic nucleus and in the collision of a photon with a nucleus. These processes correspond to the 5th and 6th approximations of the perturbation theory in quantum electrodynamics, so that the direct computation of the values of these cross-sections involves extremely great labor.

The estimates of the cross-sections given below were obtained by the use of a considerably simpler method, the usefulness of which in the high-energy region that concerns us here was considered in a paper by Oppenheimer.[3] The effective cross-section for the production of two pairs in an electron-nucleus collision, $e^- + Z \rightarrow 2(e^- + e^+) + e^- + Z$, can thus be expressed in terms of the effective cross-section for the emission of two Bremsstrahlung γ-ray quanta of frequencies ω', ω'' (ω', $\omega'' \gg m$) and the asymptotic values of the pair-conversion coefficients for γ-ray quanta, $\beta(\omega)$.

Using the formula of Gupta[4] for the Bremsstrahlung radiation of two γ-ray quanta

$$d\sigma \approx \frac{28}{3\pi} r_0^2 \alpha^2 Z^2 \frac{d\omega'}{\omega'} \frac{d\omega''}{\omega''},$$

where $\alpha = 1/137$ and $r_0 = e^2/m$, we have for the total cross-section for the process of formation of two

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pairs by an electron hitting a nucleus

$$\sigma_{2e} = \int d\sigma(\omega', \omega'')\,\beta(\omega')\,\beta(\omega''), \quad \beta(\omega) \approx \frac{2\alpha}{3\pi} \ln \frac{\omega}{m}\,. \tag{1}$$

Apart from an undetermined numerical coefficient of the order of unity, we get finally for the total cross-section for the effect of production of two electron-positron pairs by an electron hitting a nucleus in the extreme relativistic approximation (with logarithmic accuracy):

$$\sigma_{2e} \approx (r_0^2 Z^2 \alpha^4/\pi^3) \ln^4 (E_0/m). \tag{2}$$

(Here and in what follows E_0 is the energy of the initial particle, electron or photon.) We note that in principle the formula (2) could be obtained in the framework of the Weitzsäcker-Williams method[5,6] by using the effective cross-section for the production of two electron-poitron pairs by a γ-ray quantum in the field of the nucleus: $\gamma + Z \rightarrow 2(e^- + e^+) + Z$. Therefore for the effective cross-section for the formation of two pairs by a photon hitting the nucleus we must have

$$\sigma_{2\gamma} \approx (r_0^2 Z^2 \alpha^3/\pi^2) \ln^2 (E_0/m). \tag{3}$$

Comparing Eq. (2) with the Landau-Lifshitz formula[7] for the production of one pair by an electron hitting a nucleus

$$\sigma_{1e} \approx (r_0^2 Z^2 \alpha^2/\pi) \ln^3 (E_0/m)^3,$$

and also comparing Eq. (3) with the Bethe-Heitler formula[8] for the production of one pair by a photon colliding with a nucleus

$$\sigma_{1\gamma} \cong r_0^2 Z^2 \alpha \ln (E_0/m),$$

we come to the conclusion that the expansion parameter in terms of the n of pairs produced is

$$\varepsilon = \sigma_{n+1}/\sigma_n \lesssim (\alpha/\pi)^2 \ln (E_0/m).$$

The formulas given here, Eqs. (2), (3), do not include the effect of screening of the nuclear field by the atomic electrons. As is well known, this latter effect to a considerable degree hinders the increase of the cross-sections in the high-energy region. Therefore it is not excluded that effects of greater interest might be those of production of pairs in collisions with electrons, the effective cross-sections for which can be obtained from Eqs. (2) and (3) by simply replacing Z^2 by unity. It must be remarked, however, that according to Ref. 4 the effect of screening on the effects considered above is relatively small.

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RADIATION INSTABILITY IN NUCLEAR MAGNETIC RESONANCE EXPERIMENTS

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In experiments on nuclear magnetic resonance, the substance being investigated is placed in an rf coil which is part of a resonant circuit. Analysis of the oscillating system comprising the specimen with precessing nuclear magnitization and the resonant circuit shows that in certain cases one cannot neglect the