Recently Sachs[4] has suggested a semi-empirical model, describing a direct decay into light nuclei under the action of low-energy nucleons, and examined as an example the Be^9(n, 2n) Be^8 reaction. Using the value of $\bar{\sigma}$ (see Refs. 2 and 3) for the spectrum of a Ra + Be source, he obtained a curve for the effective cross section of the Be^9(n, 2n) Be^8 reaction which differed sharply from Mamasakhlisov's curve and from our experimental results (see figure). A set of constants, characterizing the size of the interaction, derived from our data would yield R = 4.44 and a = —1.16, and using these the dependence of the cross section would change and become similar to Mamasakhlisov's curve. In this way, the dependence of the cross section of the Be^9(n, 2n) reaction obtained experimentally agrees, within the limits of error, with the dependence of the cross section given by Ref. 1 and by Sachs' formula with corrected coefficients in the energy interval from 4 to 10 Mev. It is not possible, on the basis of the obtained experimental results, to make a choice between the assumptions made in these papers on the reaction mechanism, but it is possible that it will allow in the future the working over of the available experimental material.

As can be seen from the drawing, the theoretically determined dependence of the cross section from the quoted works differs from the experimental dependence for energies greater than 10 Mev. There is no reason to assume the existence of any appreciable errors for high energies. Because of this one may think that for high energies another reaction mechanism begins to operate to an appreciable extent. Indeed the following types of reactions are possible:

$$(1)\ Be^9(n, n')\, Be^{9*} \rightarrow n'' + Be^8(2\alpha); \quad (2)\ Be^9(n, 2n)\, Be^8(2\alpha); \quad (3)\ Be^9(n, 2n2\alpha).$$

Calculation of the excitation energy for Be^8 at $E_n \lesssim 10$ Mev has indicated the presence of known levels of this nucleus, which is evidence of the formation of Be^8 in the course of the Be^9(n, 2n) reaction, that is of the prevalence of reactions (1) and (2) for this primary neutron energy.

For energies greater than 10 Mev, the third type of reaction apparently plays a part to an appreciable extent: the simultaneous decay of the compound nucleus into two neutrons and two α-particles, which follows from the comparison of the α-particle energy distribution, obtained from experiment, with the distribution computed on the principle given in the work of Ref. 5. At the same time, in the center-of-mass system, an anisotropy is observed with a preferential forward emission of the neutrons and α-particles. This provides a basis for assuming also the existence of a direct interaction of the incident neutron with one of the subgroups of the Be^9 nucleus.

The effective cross section, obtained by us for the Be^9(n, α) He^6 reaction (see figure) for corresponding values of the neutron energy, agrees within the limits of error with the results obtained in other works.[6]

Besides several disintegration events were found, which can be related to the Be^9(n, t) Li^7 reaction, and for which the obtained cross sections are exhibited on the same drawing.

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MODULATION EFFECTS IN NUCLEAR MAGNETIC RESONANCE

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MODULATION effects in nuclear magnetic resonance were experimentally observed and explained in a previous work by the author[1] in terms of the dynamics of nuclear magnetization. The existence of such effects has been noted also in optical and microwave spectroscopy.[2] In nuclear magnetic resonance, the relaxation processes in the most important substances (diamagnetic liquids) take place very slowly, and the instantaneous values of the nuclear magnetization M are observed in the form of large changes of the signal waveform even for very slow variation of the external field. In order to choose the experimental conditions and to interpret the results, in this case, it is necessary to use the theory in the form of the solution of the Bloch equations[3] or other more general ones. The solutions for weak rf fields[4-6] and for low modulation[7] are entirely valid, but their use restricts the choice of experimental conditions. In particular, they do not allow one to use conditions in which the greatest signal strength may be attained. Mathematically, this corresponds to the fact that the solutions contain small parameters in the form of factors. The present note attempts to show that for large relaxation times[3] T_1 and T_2, one may obtain strong signals in a form which is easily interpreted.

Let us consider the Bloch equations for a sinusoidally modulated field $H_z = H_0 + H_m \cos \Omega t$, namely

$$dF/d\tau + [\beta + i(\delta + \varkappa\cos\tau)]F + \eta M_z = 0, \quad dM_z/d\tau + \alpha M_z - \eta v = \alpha M_0;$$
$$\tau = \Omega t, \quad F = v + iu, \quad \alpha = 1/\Omega T_1, \quad \beta = 1/\Omega T_0, \quad \delta = (|\gamma| H_0 - \omega)/\Omega, \quad \eta = |\gamma| H_1/\Omega, \quad \varkappa = |\gamma| H_m/\Omega, \tag{1}$$

where $2H_1$ is the amplitude and $\omega/2\pi$ is the frequency of the rf magnetic field, and γ is the gyromagnetic ratio of the nuclei. We shall attempt to find a solution by treating the parameters α, β, η, which determine the components of the modulation structure, as quantities of the same order of smallness.[1-5] We shall consider the quantity $\delta + k$ a small parameter of the same order, where k is an integer. This means that we shall treat the neighborhood of one of the points in which the solution has a resonant character. We shall consider the modulation amplitude arbitrary, since from the point of view of the signal strength the optimum condition corresponds to[5] $\kappa \sim 1$. Let us introduce the arbitrary parameter ϵ as a factor into those terms of Eq. (1) containing the small parameters. We then have

$$dF/d\tau + i(\varkappa\cos\tau - k)F + \varepsilon[\beta + i(\delta + k)]F + \varepsilon\eta M_z = 0, \qquad dM_z/d\tau = \varepsilon\alpha M_z - \varepsilon\eta v = \varepsilon\alpha M_0 \tag{2}$$

and wish to find a steady-state periodic solution in the form of a power series in ϵ, that is $F = F_0 + \epsilon F_1 + \ldots$, and $M_z = M_{z0} + \epsilon M_{z1} + \ldots$

We obtain sets of differential equations for the coefficients in these series, the first two of each set being

$$dF_0/d\tau + i(\varkappa\cos\tau - k)F_0 = 0, \quad dM_{z0}/d\tau = 0; \tag{3a}$$

$$dF_1/d\tau + i(\varkappa\cos\tau - k)F_1 = -[\beta + i(\delta + k)]F_0 - \eta M_{z0}, \quad dM_{z1}/d\tau = \eta v_0 + \alpha(M_0 - M_{z0}). \tag{3b}$$

Then setting $\epsilon = 1$, we obtain a solution of (1) in the form of the series $F = F_0 + F_1 + \ldots$, and $M_z = M_{z0} + M_{z1} \ldots$, where the index on the term gives its order of smallness.

It is sufficient for our purposes to find the zeroth approximation, i.e.,

$$F_0 = A\exp\{i(k\tau - \varkappa\sin\tau)\}, \quad M_{z0} = \text{const}, \tag{4}$$

where A and M_{z0} are constants which are determined by the condition that time dependent terms are absent. For Eqs. (3b) we have

$$A = -M_0\frac{J_k\eta[\beta - i(\delta + k)]}{\beta^2 + (\delta + k)^2 + (\beta/\alpha)J_k^2\eta^2}, \quad M_{z0} = M_0\frac{\beta^2 + (\delta + k)^2}{\beta^2 + (\delta + k)^2 + (\beta/\alpha)J_k^2\eta^2}.$$

Here J_k is the k-th order Bessel function of the argument κ. Comparison with experiment is more convenient in terms of the Fourier series

$$F_0 = -M_0\eta\sum_{l=-\infty}^{\infty} J_k J_l \frac{e^{i(k-l)\tau}[\beta - i(\delta + k)]}{\beta^2 + (\delta + k)^2 + (\beta/\alpha)J_k^2\eta^2}.$$

The solution obtained has an extremely simple geometric meaning. The vector M precesses about the direction of H_z. The time variation of M_z and the vertex angle of the cone of precession are both small, the fundamental effect being the variation of the phase of precession as a result of the variation of H_z. For weak rf fields, Eq. (4) agrees with the previous solution.[5] The general structure of the solution hardly changes in the interval $1 \gg \eta \gtrsim \beta$. Similarly as in the case of "slow transmission"[3] the effect of saturation is the weakening and broadening of the signal components. The displacement of the side components towards the central one[1,8] is an effect of order η and does not occur in the zeroth approximation. The amplitudes of the dispersion components (corresponding to terms containing the factor $\delta + k$) increase monotonically with the rf field, and the amplitudes of the absorption components attain their maximum value at $\eta = (\alpha\beta)^{1/2}/J_k$, that is when $|\gamma| H_1 = (T_1 T_2)^{-1/2} J_k$, and then decrease. The maximum values of the amplitudes are independent of the small parameters. Thus, for instance, the maximum amplitude of the central component of the dispersion signal (the term proportional to sin τ) is $2M_0J_1 (T_2/T_1)^{1/2}$. It is significant that the conditions for the optimum signal strength are attained within the region of applicability of Eq. (4). The condition that Eq. (4) be applicable corresponds to choosing a frequency high enough for the nuclear magnetic resonance signal to have a resolved modulation structure.

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MULTIPLE PAIR PRODUCTION IN QUANTUM ELECTRODYNAMICS

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THE results of recent studies* of the production of electron-positron pairs in photographic emulsions by cosmic rays apparently confirm the reality of the previously observed[1,2] cases of production of two electron-positron pairs in a single act.

It has been pointed out repeatedly that effects of multiple pair production can be used as a criterion for the applicability of the present quantum theory of electromagnetic interactions, just in the range of phenomena in which the correspondence principle can no longer be used. Since up to this time there have not been in the literature any statements on the theory of multiple production of electron-positron pairs, it seemed worth while to make an estimate of the values of the effective cross-sections in question. In the present note we present approximate expressions for the effective cross-sections for production of two electron-positron pairs in the collision of an electron with an atomic nucleus and in the collision of a photon with a nucleus. These processes correspond to the 5th and 6th approximations of the perturbation theory in quantum electrodynamics, so that the direct computation of the values of these cross-sections involves extremely great labor.

The estimates of the cross-sections given below were obtained by the use of a considerably simpler method, the usefulness of which in the high-energy region that concerns us here was considered in a paper by Oppenheimer.[3] The effective cross-section for the production of two pairs in an electron-nucleus collision, $e^- + Z \rightarrow 2(e^- + e^+) + e^- + Z$, can thus be expressed in terms of the effective cross-section for the emission of two Bremsstrahlung γ-ray quanta of frequencies ω', ω'' (ω', $\omega'' \gg$ m) and the asymptotic values of the pair-conversion coefficients for γ-ray quanta, $\beta(\omega)$.

Using the formula of Gupta[4] for the Bremsstrahlung radiation of two γ-ray quanta

$$d\sigma \approx \frac{28}{3\pi} r_0^2 \alpha^2 Z^2 \frac{d\omega'}{\omega'} \frac{d\omega''}{\omega''},$$

where $\alpha = 1/137$ and $r_0 = e^2/m$, we have for the total cross-section for the process of formation of two

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