ON K_{e3} DECAY

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Submitted to JETP editor May 7, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 525-526 (August, 1957)

The electron and π meson energy distributions from K_{e3} decay are calculated. Measurement of these distributions will make it possible to establish the type of decay interaction and to determine the strong interaction form factors $g(E_\pi)$ in these decays.

1. The investigation of K_{e3} decays ($K^\pm \to e^\pm + \nu + \pi^0$ and $K^0 \to e^\pm + \nu + \pi^\pm$) is very important for the clarification of the character of weak electron interactions. In the general case, any matrix element for the K_{e3} decay of a K meson at rest, which does not contain products of lepton functions, is of the form

$$\mathfrak{M} = \{g_S \bar{\psi}_e \psi_\nu + g_V \bar{\psi}_e \gamma_4 \psi_\nu + i g_T \bar{\psi}_e \gamma_4 \gamma \psi_\nu \mathbf{k}_\pi M^{-1}\} (2M^{3/2} E_\pi^{1/2})^{-1}. \quad (1)$$

Here $g_{S,V,T}$ are functions of the π meson energy E_π corresponding to the scalar (S), vector (V), and tensor (T) interactions. The dependence of g on E_π cannot be calculated, since there exists no theory for strong interaction between the K and π mesons. The matrix element $\mathfrak{M}$ is normalized to make the $g(E_\pi)$ functions dimensionless and constant in first-order perturbation theory with respect to strong interaction between the K and π mesons. Further, M is the mass of the K meson, $k_\pi = E_\pi^2 - m_\pi^2$, and $\hbar = c = 1$. (For a detailed discussion of the form of $\mathfrak{M}$, see the works of Furuichi et al.[1] and Pais and Treiman.[2])

2. With the aid of (1) one easily obtains an expression for the probability of emitting an electron with energy E_e and a π meson with energy E_π, namely

$$W(E_\pi, E_e)\, dE_\pi dE_e = \{ |g_S|^2 [(M - E_\pi)^2 - k_\pi^2] + |g_V|^2 [k_\pi^2 - (M - E_\pi, - 2E_e)^2] \quad (2)$$

$$+ |g_T|^2 [(M - E_\pi)^2 - k_\pi^2] [M - E_\pi - 2E_e]^2 M^{-2} + i(g_S g_T^* + g_S^* g_T) [(M - E_\pi)^2 - k_\pi^2] [M - E_\pi - 2E_e] M^{-1}\} (32\pi^3 M^3)^{-1} dE_\pi dE_e.$$

In considering the electron spectrum at a fixed π meson energy, it is convenient to write (2) in the form

$$W(\varepsilon) = \Phi_S + \Phi_V [\varepsilon_0^2 - (1 - \varepsilon)^2] + \Phi_T (1 - \varepsilon)^2 + \Phi_{ST} (1 - \varepsilon). \quad (3)$$

Here the $\Phi_{S,V,T,ST}$ depend only on E_π, and are independent of the E_e, and

$$\varepsilon = 2E_e / (M - E_\pi), \quad \varepsilon_0 = k_\pi / (M - E_\pi), \quad 1 - \varepsilon_0 \leqslant \varepsilon \leqslant 1 + \varepsilon_0.$$

Equation (3) is equivalent to Eq. (8) of Pais and Treiman.[2] However, the choice of the electron energy E_e as the variable (rather than the angle between the electron and π meson) makes Eq. (3) clearer. It follows from (2) that $\Phi_{S,V,T} > 0$, with the sign and magnitude of Φ_{ST} being determined by the relative phases of g_S and g_T. If time (combined) parity is conserved, all the g are real.* If $\Phi_{ST} = 0$, it is seen from (3) that $W(\epsilon)$ is symmetric about $\epsilon = 1$. Lack of such symmetry would indicate the presence both of the S and the T interactions. As is also seen from (3), the presence of a maximum at $\epsilon = 1$ in the spectrum would indicate the presence of the V interaction, whereas a minimum would indicate the T interaction. If it were to turn out that the experimental data is not consistent with (3), this would indicate that the weak lepton interaction is nonlocal.

3. A measurement of the electron spectrum for fixed E_π that would give complete information on the type of interaction is, however, a difficult experimental problem. In this connection it is of interest to obtain expressions for the electron and π meson spectra $W(E_e)dE_e$ and $W(E_\pi)dE_\pi$, which are obtained by integrating Eq. (2) over E_π and E_e, respectively. The integration over E_π can be performed

*We note that the function g_T in Eq. (1) differs by a factor of i from f_T of Pais and Treiman.[2] Therefore their assertion that invariance under time reversal corresponds to real $f_{S,V,T}$ is incorrect.

The author is grateful to B. L. Ioffe and I. M. Smushkevich for discussing this question.

only under certain assumptions as to the form of $g(E_\pi)$. Furuichi et al.[1] have performed this integration and obtained an expression for $W(E_e)dE_e$ on the assumption that $g(E_\pi) = \text{const}$. Matinian[3] has also obtained an expression for $W(E_e)dE_e$ for the S interaction. Comparing their formulas with the experimental data, the authors[1] conclude that $g_T \neq 0$.

The integration over E_e, which can be performed without any assumptions as to the form of $g(E_\pi)$, gives

$$W(E_\pi)\,dE_\pi = \{|g_S|^2 (M^2 + m_\pi^2 - 2ME_\pi)\,k_\pi + |g_V|^2\, 2k_\pi^3/3$$
$$+ |g_T|^2 (M^2 + m_\pi^2 - 2ME_\pi)\,k_\pi^3/3M^2\}\,dE_\pi/32\pi^3 M^3, \quad m_\pi \leqslant E_\pi \leqslant (M^2 + m_\pi^2)/2M = E_{\pi\,\max}. \tag{4}$$

(For the S interaction, Matinian[3] has previously an expression for $W(E_\pi)dE_\pi$.) From Eq. (4) it follows in particular, that by measuring the π meson spectrum near its upper limit one can establish the presence or absence of the V interaction even without knowing the form of $g_{S,V,T}(E_\pi)$. Indeed, for the S and T interactions, $W(E_{\pi\,\max}) = 0$ in all cases, whereas for the V interaction $W(E_\pi \max) = 0$ only if $g_V = 0$ (it is unlikely that $g_V(E_\pi)$ vanishes at this point accidentally).

The author is grateful to Professor I. Ia. Pomeranchuk and Academician L. D. Landau for constant interest in the work, and to Academician A. I. Alikhanov for discussing the possible experimental measurements of the K_{e3} decay spectra.

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[2] A. Pais and S. B. Treiman, Phys. Rev. **105**, 5 (1957).

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Translated by E. J. Saletan
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