ON THE PRODUCTION OF π MESONS BY PROTONS ON NUCLEI OF VARIOUS ELEMENTS

A. A. ANSEL'M and V. M. SHEKHTER

Leningrad Physico-Technical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor February 21, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 481-484 (August, 1957)

The problem considered is that of the dependence of the probability of π meson production in proton-nucleus collisions on the number A of nucleons in the nucleus. The deviation from the $A^{2/3}$ law is explained by the effective attenuation of the proton beam penetrating into the nucleus.

THE dependence of the cross-section for production of π^+ mesons on the atomic number in collisions of protons with nuclei of various elements has been studied experimentally in a number of papers.[1-5] It has been found that in the heavy-element region the observed yield of π^+ mesons increases with A more slowly that would follow from an $A^{2/3}$ law (whereas for the case of photoproduction of π-mesons the $A^{2/3}$ law is obeyed over a very wide range of variation of A).

As an explanation of this fact it is natural to suggest[4] that in addition to the large probability for absorption of a π meson produced inside the nucleus, which gives the $A^{2/3}$ law (only the surface nuclei play an effective part in the production), there is also an appreciable absorption of the protons penetrating into the nucleus. By absorption one obviously means here an effective attenuation of the beam of protons capable of producing π mesons. Some other possible causes of the deviation from proportionality to $A^{2/3}$ will be considered below

If we assume that for a given energy of the incident protons there are produced per unit volume of the nucleus, independently of its atomic number, the number $w(\theta)\sin\theta d\theta d\varphi$ of mesons emitted in the direction (θ, φ) relative to the incident beam, then the cross-section for π meson production from the nucleus can be written in the form

$$d\sigma/d\Omega = w(\theta)\int e^{-\varkappa s_1-\eta s_2}\,dV, \tag{1}$$

where s_1 and s_2 are the paths of the meson and proton inside the nucleus, and κ and η are their respective absorption coefficients. The integration is taken over the volume of the nucleus. This does not take account of effects of elastic scattering of meson and proton and of Coulomb interaction and the detailed momentum distribution of the nucleons in the nucleus.

In order of magnitude

$$w(\theta) \approx \sigma_0(\theta)\, A\,/\,{}^4/_3\pi R^3 = 3\sigma_0(\theta)\,/\,4\pi R_0^3, \tag{2}$$

where $\sigma_0(\theta)$ is the cross-section for single-nucleon production of π mesons and $R = R_0 A^{1/3}$ is the radius of the nucleus.

In the absence of absorption of the proton ($\eta = 0$), Eq. (1) gives[6]

$$\frac{d\sigma}{d\Omega} = w(\theta)\,\frac{\pi R^2}{\varkappa}\left[1-\frac{1}{2(\varkappa R)^2}+\frac{e^{-2\varkappa R}}{2(\varkappa R)^2}(1+2\varkappa R)\right], \tag{3}$$

from which we have for $\kappa R \gg 1$: $d\sigma/d\Omega \sim R^2 \sim A^{2/3}$.

The validity of the $A^{2/3}$ law for small A means that we can regard the meson absorption as the larger through practically the entire periodic table. Under the condition $\kappa R \gg 1$ (for all values of R in question) and $\kappa \gg \eta$, Eq. (1) reduces to the form:

$$d\sigma/d\Omega = C(\theta)\,F(\eta R;\theta),\quad C(\theta) \equiv w(\theta)\,\pi/\varkappa\eta^2 \approx 3\sigma_0(\theta)/4\varkappa\eta^2 R_0^3,\quad F(\eta R;\theta) = (\eta R)^2\sin^2\frac{\theta}{2}+\cos\theta\left[\frac{1}{4}-e^{-2\eta R}\left(\eta R+\frac{1}{2}\right)\right]$$

$$+I(\eta R;\theta)\quad \left(\theta \leqslant \frac{\pi}{2}\right),\quad F(\eta R;\theta) = (\eta R)^2\sin^2\frac{\theta}{2}+\frac{1}{4}\cos\theta+I(\eta R;\theta)\quad\left(\theta \geqslant \frac{\pi}{2}\right),$$

$$I(\eta R;\theta)=\frac{\cos^2\theta}{2\pi}\int\limits_0^{\pi/2}\frac{\exp\{-2\eta R\sin\theta\sin\alpha\}}{1-\sin^2\theta\sin^2\alpha}\,d\alpha+\frac{\sin\theta\cos^2\theta}{\pi}(\eta R)\int\limits_0^{\pi/2}\frac{\exp\{-2\eta R\sin\theta\sin\alpha\}\sin\alpha}{1-\sin^2\theta\sin^2\alpha}\,d\alpha$$
$$+\frac{2\sin^2\theta}{\pi}(\eta R)^2\int\limits_0^{\pi/2}\exp\{-2\eta R\sin\theta\sin\alpha\}\cos^{-2}\alpha\,d\alpha. \tag{4}$$

The structure of the formula (4) is obvious. Under the condition $\kappa R \gg 1$ only a thin layer of thickness $\sim 1/\kappa$ at the surface of the nucleus makes any contribution to the cross-section. The part of the surface of the nucleus which is in front in relation to the incident protons and visible at the angle θ, on which the protons fall directly, is responsible for the term $(\eta R)^2\sin^2\theta/2$ (this part of the surface of the sphere is contained between two planes passing through the center of the nucleus and perpendicular to the directions of the incident proton beam and the propagation of the mesons). The other terms are due to the production of π mesons near that part of the surface reached by a partly attenuated beam of protons.

For the angles $\theta = 0, \pi/2$, and π, Eq. (4) gives

$$F(\eta R,0)=\frac{1}{2}-e^{-2\eta R}\left(\eta R+\frac{1}{2}\right),\quad F\left(\eta R,\frac{\pi}{2}\right)=\frac{1}{2}(\eta R)^2+\frac{2}{\pi}(\eta R)^2\int\limits_0^{\pi/2}e^{-2\eta R\sin\alpha}\cos^2\alpha\,d\alpha,\quad F(\eta R;\pi)=(\eta R)^2. \tag{5}$$

For $\eta R \lesssim 1$:

$$F(\eta R;\theta)=(\eta R)^2\Big\{1-(\eta R)\frac{4}{3\pi}[(\pi-\theta)\cos\theta+\sin\theta]+(\eta R)^2\cos^4\frac{\theta}{2}-(\eta R)^3\frac{8}{15\pi}\Big[(\pi-\theta)\cos\theta+\sin\theta\Big(1-\frac{\sin^2\theta}{3}\Big)\Big]$$
$$+(\eta R)^4\frac{2}{9}\cos^6\frac{\theta}{2}\Big(1+\sin^2\frac{\theta}{2}\Big)+\cdots\Big\}. \tag{6}$$

For $\eta R\sin\theta \gg 1$:

$$F(\eta R;\theta)=(\eta R)^2\sin^2\frac{\theta}{2}+\eta R\frac{\sin\theta}{\pi}+\frac{1}{4}\cos\theta+\cdots. \tag{7}$$

Figure 1 shows curves of the function $F(\eta R, \theta)$ plotted against $(\eta R)^2$ for $\theta = 0, \pi/4, \pi/2, 3\pi/4$, and π.

Before going on to the comparison of our results with the experimental data, we note one more circumstance.[4] If the cross-section for production of π^+ mesons on protons is much larger than that for the production on neutrons, then in the expression given in Eq. (4) for $C(\theta)$ we should insert the additional factor Z/A, where Z is the number of protons in the nucleus. At the beginning of the periodic table this factor is about 1/2 and is almost unchanged as A varies, but for the heavy elements Z can be considerably smaller than $A/2$. To take account of this effect there are plotted in Figs. 2–4, besides the experimental values of the cross-sections (circles), also the values of the cross-sections multiplied by A/Z (crosses). Good agreement with experiment can be regarded as existing when the theoretical curve passes somewhere between the circles and the crosses.

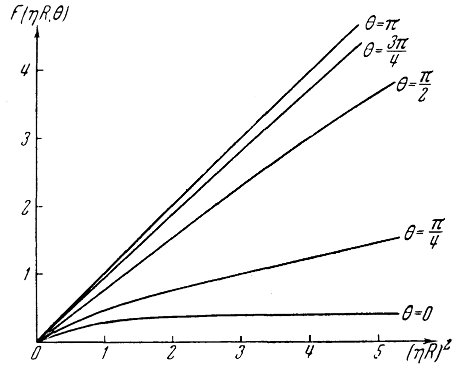

FIG. 1

Figure 2 shows the data from the work of Meshkovskii and others.[5] Ordinates are values of the total (with respect to the energies of the π^+ mesons produced) cross-section for π-meson production, measured in millibarns per steradian. The measurements were carried out at angle 45° with the direction of incidence of the proton beam, with initial proton energy 660 Mev. The errors for the values of the cross-sections multiplied by $A/2Z$ are not

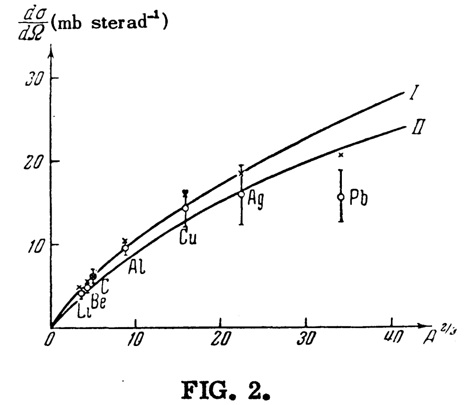

FIG. 2.

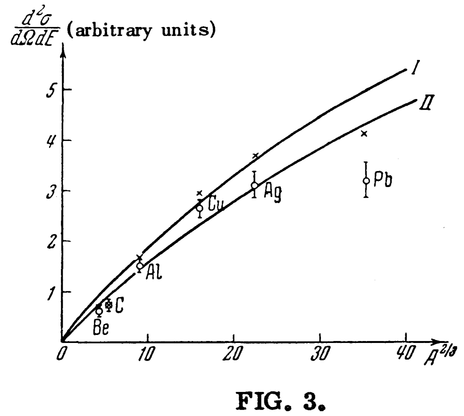

FIG. 3.

drawn in, for greater clearness in reading the diagram. Figure 3 shows the data of Sagane and others.[3] Ordinates are values of $d^2\sigma/d\Omega dE$ at $\theta = 90°$ and energy $E_{\pi^+} = (33 \pm 3)$ Mev; the proton energy used in the experiment was $E_p = 340$ Mev. Figure 4 shows the data of Clark;[1] $\theta = 130 - 150°$, $E_{\pi^+} = 40$ Mev, $E_p = 240$ Mev. The theoretical curve is drawn for the angle 135°. In all the diagrams the theoretical curve I corresponds to the value $\eta R_0 = 0.28$, and curve II to $\eta R_0 = 0.32$. The value of $C(\theta)$ in Eq. (4) was chosen so that curve I should go through the cross for Ag. As can be seen from the diagrams, within the limits of the present errors the theoretical results are in adequate agreement with experiment. The best agreement is given by curve I compared with the cross-sections multiplied by A/2Z. Comparisons with the data of Merritt and Hamlin[4] ($\theta = 0°$) and of Block and others[2] ($\theta = 90°$) were not carried out because of the small number of experimental points in the region of heavy nuclei. Some systematic reduction of the cross-section for Pb is possibly explained by an increased density of nuclear matter in lead (doubly magic nucleus).

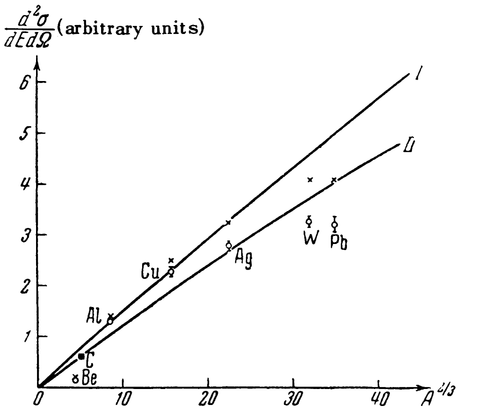

FIG. 4.

We note that according to these curves the deviation from the $A^{2/3}$ law exists already for the lightest elements, so that even if one draws a straight line through the first few points (Li — Al) it lies at an appreciable angle with the line that one would have in the absence of attenuation of the proton beam.

In conclusion we express our deep gratitude to K. A. Ter-Martirosian for proposing the problem and for discussions.

[1] D. L. Clark, Phys. Rev. 87, 157 (1952).

[2] Passman, Block, and Havens, Phys. Rev. 88, 1239 (1952).

[3] R. Sagane and W. Duziak, Phys. Rev. 92, 212 (1953).

[4] J. Merritt and D. A. Hamlin, Phys. Rev. 99, 1523 (1955).

[5] Meshkovskii, Pligin, Shalamov, and Shchebanov, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1328 (1957); Soviet Phys. JETP 5, 1085 (1957).

[6] Brueckner, Serber, and Watson, Phys. Rev. 84, 258 (1951).

Translated by W. H. Furry
90