USE OF THE HARTREE-FOCK EQUATIONS FOR A SYSTEM OF QUASI-PARTICLES

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Submitted to JETP editor April 6, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 289-291 (July, 1957)

THE state of a system of many strongly interacting particles in states close to some ground state can be described in terms of a set of quasi-particles. In a weakly "nonideal" system, the number of quasi-particles in states close to the ground state may be considered insignificant, and their interaction can be ignored. In strongly "nonideal" systems, the number of quasi-particles may be quite large and their interaction can no longer be ignored. We present an attempt to account for the interaction between quasi-particles by means of a self-consistent field.

The time-dependent Hartree equations for a system having on the average N quasi-particles per unit volume can be written

$$i\hbar\dot{\psi}_j = -\frac{\hbar^2}{2m}\Delta\psi_j + \int G(|\mathbf{r}-\mathbf{r}'|)\sum_i |\psi_i(\mathbf{r}')|^2 d\mathbf{r}'\,\psi_j, \qquad (1)$$

where $G(|\mathbf{r}-\mathbf{r}'|)$ is the interaction kernel for quasiparticles. Writing $\psi_j = \rho_j^{1/2}\exp\{iS_j/h\}$, these equations can be written in the hydrodynamic form

$$\dot{S}_j + \frac{1}{2m}(\nabla S_j)^2 + \int G(|\mathbf{r}-\mathbf{r}'|)\sum_i \rho_i(\mathbf{r}')\,d\mathbf{r}' - \frac{\hbar^2}{4m}\frac{\Delta\sqrt{\rho_j}}{\rho_j} = 0, \quad \dot{\rho}_j + \operatorname{div}(\rho_j\nabla S_j) = 0. \qquad (2)$$

This system has an exact solution with $\rho_j^0 = \text{const} = 1/V$ (where V is the volume of the system, assumed equal to unity) and

$$S_j^0 = m(\nabla_p\varepsilon_j^0)\,\mathbf{r} - \varepsilon_j^0 t + \text{const}, \qquad (3)$$

where ϵ_j^0 is the energy and $\mathbf{p}$ is the momentum of a quasi-particle in a state of constant density. For a weakly "nonideal" system, $\nabla_p\epsilon_j^0$ can be set equal to the velocity $\mathbf{v}_j^0$. The energy spectrum of a system in a state close to the constant-density state may be found by linearizing Eqs. (2). Calculations similar to those performed previously[1] lead to the dispersion equation

$$1 = \frac{1}{m}G(k)k^2\sum_j\left[(\omega - \mathbf{k}\nabla_p\varepsilon_j^0)^2 - \frac{\hbar^2k^4}{4m^2}\right]^{-1}, \qquad (4)$$

where G(k) is the Fourier component of the interaction kernel, and k is the wave number. For small k this equation has the solution

$$\omega^2 = \frac{N}{m}G(k)k^2 + \overline{(\mathbf{k}\cdot\nabla_p\varepsilon)^2}. \qquad (5)$$

Suitable choice of the dependence of ϵ_j on $\mathbf{p}_j$ makes it possible to account for nonlocal interactions.

If exchange interactions are accounted for within the framework of the Hartree-Fock equation, we obtain instead of (4)

$$1 - \frac{k^2}{m}G(k)\sum_i F(\omega, \mathbf{kp}_i) + \left(\sum_i\rho_i\right)^{-1}\sum_{j,i}G\left(\frac{1}{\hbar}|\mathbf{p}_j+\mathbf{p}_i|\right)F(\omega,\mathbf{kp}_i)\rho_i = 0, \quad F(\omega,\mathbf{kp}_i) = \left[\left(\frac{\omega-\mathbf{kp}_i}{m}\right)^2 - \frac{\hbar^2k^4}{4m^2}\right]^{-1}. \qquad (6)$$

When $\omega \gg \langle \mathbf{kp}/m\rangle_{av}$, the function $G(|\mathbf{p}_i+\mathbf{p}_j|)$ can be taken outside of the summation over j and evaluated at some intermediate point $\mathbf{p}'$ between zero and the maximum value of the Fermi momentum. The last term in (6) then becomes

$$-(k^2/m)\sum_i G(|\mathbf{p}_i+\mathbf{p}'|/\hbar).$$

The maximum and minimum values of the correction for the exchange interaction differ from each other by a factor of two. For small k the maximum value of this exchange correction leads to

$$\omega^2 = \omega_0^2 + (\overline{p^2}/m^2)k^2 - 3\omega_0^2(\hbar k/p_{\max})^2.$$

This equation was first obtained by Silin[2] with a somewhat different numerical coefficient on the last term.

The method here described is generalized by indicating a general way of finding the quantity $(\overline{\nabla_p \epsilon})^2$, which enters into the dispersion equation (5). Since this quantity is given in terms of quantities characterizing the ground state, it is sufficient to write the energy $\epsilon = (1/2)m(\nabla_p \epsilon)^2$ of a single particle for this state as a function of the momentum, accounting for all possible interactions by single-particle wave functions (which are plane waves in the constant-density state).

We take this opportunity to thank V. P. Silin for correcting a serious error in the first version of this communication.

[1] P. S. Zyrianov and V. M. Eleonskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 592 (1956); Soviet Phys. JETP **3**, 620 (1956).

[2] V. P. Silin, Физика металлов и металловедение, (Physics of Metals and Metal Reseach) **3**, 193 (1956).

Translated by E. J. Saletan
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NEW FORM OF ISOMERISM IN EU[152]

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Submitted to JETP editor April 7, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 291-293 (July, 1957)

EXPERIMENTAL data on the β-decay of Eu^{152} indicates that the β and γ spectra of its ground state (T = 13 years) and isomeric state (T = 9.2 hr) are very complex.[1-4] Certain details of the decay schemes, however, are quite well established and can be used to determine the spin of the isomer. These are shown in Fig. 1. The log τf value of 7.6 for the 9.2 hr β^- transition of Eu^{152} to the ground state 0+ of Gd^{152} is characteristic for singly forbidden β^- transitions and indicates that the spin of the Eu^{152} isomer is 1—.

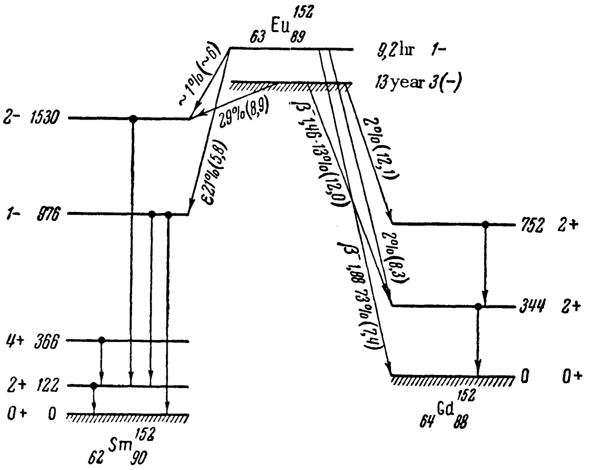

FIG. 1

The value 0— is excluded, since K-capture is observed to the 1537 kev 2— level of Sm^{152}. The value 2— is less likely, since in this case the β^- transition would be "unique" and one would expect to have log $\tau f \geq 7.6$.

The spin of 13-year Eu^{152} was directly measured by the paramagnetic resonance method,[5] and was found to be 3. These experimental values of 3 (—) and 1— for the spins of the ground state and isomeric state of Eu^{152} lead to several serious difficulties:

1. The spin difference of these two states is 2. Therefore the γ-transition between the isomeric and ground states of Eu^{152} must be* E2. Nevertheless, this transition is never observed. If it does exist, it is retarded by a factor of more than 10^{12} compared to the usual E2 transitions. One is not able[6,8] to explain this in terms of the selection rules for any of the known quantum numbers (for deformed nuclei) I, K, N, n_z, Λ, or Σ. The isomerism of Eu^{152}

*From a more detailed analysis of the decay scheme it follows that the parity of 13-year Eu^{152} would seem to be the same as that of 9.2-hour Eu^{152}. It is, however, impossible to eliminate completely the possibility that 13-year Eu^{152} has positive parity.