CONSERVATION OF PARITY IN THE THEORY OF ELEMENTARY PARTICLES

V. L. POKROVSKII and IU. B. RUMER

West-Siberian Branch, Academy of Sciences, U.S.S.R.

Submitted to JETP editor March 14, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 277-279 (July, 1957)

THERE have been a number of recent experiments which indicate that parity is not conserved in the decay of elementary particles. In view of the fact that the requirement for invariance of physical effects under spatial reflections does not now seem to be self-evident it is natural to wonder why parity is conserved in one class of effects (strong and electromagnetic interactions), while in another (weak interactions, following the Gell-Mann designation) there is the possibility of parity non-conservation.

We wish to call attention to the fact that the system of five-dimensional optics[1] proposed by one of the present authors gives a natural method for classifying effects in which parity is conserved and is not conserved.

Five-optics starts from the recently discovered deep-lying symmetry of the equations of classical and quantum mechanics in space, time, and action. In 5-optics the coordinates, time, and action are combined in a five-dimensional metric space which is topologically bounded in the action coordinate with a period equal to the Planck constant h. Correspondingly, the momentum energy, and charge are combined in a five-dimensional vector for which a single conservation law is formulated. In five-dimensional optics the action, like a coordinate, is an algebraic quantity. A change in the sign of the action is equivalent to ordinary charge conjugation.

The interaction Lagrangian in five-dimensional optics, as in four-dimensional optics, must be formed from wave functions of the interacting particles. We consider all possible products of the components of the spinors $\bar{\xi}_\alpha \xi_\beta$ and $\xi_\alpha \eta_\beta$. All these can be broken down into irreducible groups.[2]

(1) Scalars: $\bar{\xi}\xi$ and $\xi\Omega\gamma_5\eta$.

(2) 4-Vectors: $\bar{\xi}\gamma_k\xi$ and $\xi\Omega\gamma_5\gamma_k\eta$.

(3) Anti-symmetric tensors of the second rank $\bar{\xi}\gamma_i\gamma_k\xi$, $\xi\Omega\gamma_i\gamma_k\eta$ (6 components).

(4) 4-pseudovectors: $\bar{\xi}\gamma_5\gamma_k\xi$ and $\xi\Omega\gamma_k\eta$.

(5) Pseudoscalars $\bar{\xi}\gamma_5\xi$ and $\xi\Omega\eta$, where the matrix Ω is defined by the relation:[3] $\gamma_k^T = \Omega\gamma_k\Omega^{-1}$.

In five-dimensional space a spinor is also a four-component quantity and the products $\bar{\xi}_\alpha \xi_\beta$ and $\xi_\alpha\eta_\beta$ can be broken down into the following irreducible groups:

(1) Pseudoscalar $\bar{\xi}\gamma_5\xi$; scalar $\xi\Omega\gamma_5\eta$.

(2) 5-vector $(\bar{\xi}\gamma_k\xi, \bar{\xi}\xi)$, 5-pseudovector $(\xi\Omega\gamma_k\eta, \xi\Omega\eta)$.

(3) Anti-symmetric pseudotensor of the second rank $(\bar{\xi}\gamma_i\gamma_k\xi, \xi\gamma_5\gamma_k\xi)$, anti-symmetric tensor of the second rank $(\xi\Omega\gamma_i\gamma_k\xi, \xi\Omega\gamma_5\gamma_k\xi)$ (10 components).

In particular, we wish to emphasize that in five-dimensional space it is impossible to form a bilinear conbination having the properties of a scalar and pseudovector from $\bar{\xi}$ and ξ and that it is impossible to form a pseudoscalar and a vector from ξ and η.

Following Pauli[3] we designate the following wave functions for bosons: Φ scalar, φ pseudoscalar, Φ_k vector, φ_k pseudovector.

We now consider two types of effects.

A. Emission of a boson by a fermion (pair production). The interaction Lagrangians for pseudoscalar and vector bosons are respectively:

$$L = \varphi\,(\bar{\xi}\gamma_5\xi),\quad L = \Phi_k\,(\bar{\xi}\gamma_k\xi) + \Phi_5\,(\bar{\xi}\,\xi)$$

(we do not consider interaction Lagrangians which contain derivatives). We see that parity is conserved in the emission of a pseudoscalar or vector boson in the five-dimensional theory. This fact is of particular interest in connection with the emission of a π-meson by a nucleon and a photon by an electron. The interaction Lagrangians can be pseudoscalar only for scalar and pseudovector mesons. Hence, if scalar and pseudovector mesons exist, parity cannot be conserved in emission processes.

B. Decay of a boson into two fermions. The interaction Lagrangian for a pseudoscalar boson is $L = \varphi^*(\xi\Omega\gamma_5\eta)$ while for a vector boson it is $L = \Phi_k^*(\xi\Omega\gamma_k\eta)$. We see that parity cannot be conserved in the decay of a pseudoscalar or vector boson into two fermions. In particular, parity is not conserved in the reaction $\pi^{\pm} \rightarrow \mu^{\pm} + \nu$. If there are scalar and pseudovector mesons which can decay into two fermions, parity cannot be conserved in this decay.

Since the action coordinate is on equal footing with the other spatial coordinates in five-dimensional theory, simultaneous spatial reflection and change of sign of the action leave the Lagrangian invariant in all product combinations.

Consequently, even a theory which is not invariant under spatial reflection becomes invariant with respect to the combined inversion proposed by Landau,[4] i.e., simultaneous spatial reflection and charge conjugation.

---

[1] Iu. B. Rumer, Исследования по 5-оптике, (Investigation of 5-Optics), GTTI 1956; Usp. Mat. Nauk **8**, 6 (1953).

[2] E. Cartan, Theory of Spinors (Russ. Transl.), GIIL 1948.

[3] W. Pauli, Exclusion Principle, Lorentz Group and Reflection of Space-Time and Charge. (In the collection "Niels Bohr and the Development of Physics," London, Pergamon Press, 1955).

[4] L. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 405 (1957); Soviet Phys. JETP **5**, 336 (1957); T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

Translated by H. Lashinsky
47

POLARIZATION IN DOUBLE SCATTERING OF ELECTRONS

IU. V. ANISHCHENKO and A. A. RULCHADZE

P. N. Lebedev Institute of Physics, Academy of Sciences, U.S.S.R.

Submitted to JETP editor March 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 279-280 (July, 1957)

As is well known, β-decay electrons are polarized longitudinally. This effect can be observed in double scattering of β-electrons. The differential cross section for double scattering of a longitudinally polarized beam of electrons is given as follows:[1]

$$I^{(2)} = (|f_1|^2 + |g_1|^2)(|f_2|^2 + |g_2|^2)(1 + \delta_1 \cos\varphi + \delta_2 \sin\varphi), \tag{1}$$

where

$$\delta_1 = (f_1 g_1^* - f_1^* g_1)(f_2^* g_2 - f_2 g_2^*) / (|f_1|^2 + |g_1|^2)(|f_2|^2 + |g_2|^2), \tag{2}$$

$$\frac{\delta_2}{\delta_1} = i\eta \frac{\sin\vartheta_1 (|g_1|^2 - |f_1|^2) + \cos\vartheta_1 (f_1^* g_1 + f_1 g_1^*)}{f_1^* g_1 - g_1^* f_1}, \tag{3}$$

where $f_1 \equiv f(\vartheta_1)$, $f_2 \equiv f(\vartheta_2)$ etc. and the angles ϑ_1 and ϑ_2 are the angles associated with the first and second scattering respectively, φ is the azimuthal angle measured from the plane of the first scattering, and η is the degree of longitudinal electron polarization. In the Born approximation Eqs. (2) and (3) assume the following form:

$$\delta_1 = -\frac{Z_1 Z_2 e^4}{(\hbar c)^2} 4 \frac{v^2}{c^2}\left(1 - \frac{v^2}{c^2}\right) \frac{\sin^4(\vartheta_1/2)\sin^4(\vartheta_2/2)}{\sin\vartheta_1 \cdot \sin\vartheta_2} \frac{\ln \operatorname{cosec}^2(\vartheta_1/2) \cdot \ln \operatorname{cosec}^2(\vartheta_2/2)}{[1-(v/c)^2 \sin^2(\vartheta_1/2)][1-(v/c)^2 \sin^2(\vartheta_2/2)]}; \tag{4}$$

$$\delta_2/\delta_1 = 2\eta \frac{\hbar c}{Z_1 e^2} \frac{c}{v} \frac{\cot^2(\vartheta_1/2)}{\ln \operatorname{cosec}^2(\vartheta_1/2)}. \tag{5}$$

It is apparent from Eq. (5) that $\delta_2 \gg \delta_1$, so that the effect of azimuthal asymmetry is much greater