

obtained earlier\* [cf. Ref. 10, Eq. (27)]. Similarly, we can obtain the particular forms for  $\sigma_p^{(9)}$  found by Feld.<sup>4</sup>

3. An analysis of the experimental data on the angular distribution of mesons produced by photons on nucleons in the energy region  $E_\gamma \leq 400$  Mev has been carried out in a paper by Watson et al. (this work is considered in greater detail in Ref. 2). This analysis has shown that in the indicated energy region it is impossible to obtain good agreement between the theory and the experimental data if meson production in only s- and p-states is taken into account. In this connection it would be of interest to analyze the indicated experimental data using the method used in Ref. 1 but taking account of meson production in d-states using the expression given in (1).

In conclusion the author wishes to express his gratitude to M. A. Markov for discussion of this material.

<sup>1</sup>Watson, Keck, Tollestrup and Walker, Phys. Rev. **101**, 1159 (1956).

<sup>2</sup>N. F. Nelipa, Usp. Fiz. Nauk (in preparation).

<sup>3</sup>K. A. Ter-Martirosian, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 895 (1951).

<sup>4</sup>B. T. Feld, Phys. Rev. **89**, 330 (1953).

<sup>5</sup>K. M. Watson, Phys. Rev. **95**, 228 (1954).

<sup>6</sup>Morita, Sugie and Yoshida, Prog. Theor. Phys. **12**, 713 (1954).

<sup>7</sup>Biedenharn, Blatt and Rose, Rev. Mod. Phys. **24**, 249 (1954).

<sup>8</sup>Simon, Vander Sluis and Biedenharn, Oak Ridge National Laboratory, ORNL-1679 (1955).

<sup>9</sup>L. C. Biedenharn, Oak Ridge National Laboratory, ORNL-1501 (1953).

<sup>10</sup>M. Gell-Mann and K. Watson, Usp. Fiz. Nauk **59**, 399 (1956); [Annual Review of Nuclear Sciences **4**, 219 (1954)].

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\*It is necessary to make the following substitutions for the unknown matrix elements  $E_{11} \rightarrow E_{11}$ ,  $M_{11} \rightarrow -M_{11}$ ,  $M_{13} \rightarrow -M_{13}$ ,  $E_{23} \rightarrow E_{23}/2\sqrt{3}$ .

## PHASE OF A SCATTERED WAVE

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WE consider here the application of one form of the method of variation of constants for determining the phase of the scattered wave in a spherically-symmetric one-particle problem in quantum mechanics. We have the equation

$$d^2G/d\rho^2 + [1 - l(l+1)/\rho^2 - U(\rho)]G = 0, \quad (1)$$

where  $l = 0, 1, 2, \dots$ , while  $U(\rho)$  satisfies the condition  $\int_0^\infty U(\rho) d\rho < C$  and may be given as  $U(\rho) = \gamma(\rho)/\rho$ , where  $\gamma(\rho) = \gamma_0 + \gamma_1\rho + \dots$ .

We seek a solution of Eq. (1) which may be represented in the following form when  $\rho \rightarrow 0$

$$G = A_0\rho^{l+1} \quad (2)$$

and which takes on the following asymptotic form at large values of  $\rho$

$$G = \text{const} \cdot \sin\left(\rho - \frac{\pi l}{2} + \delta_l\right); \quad \delta_l = \text{const}. \quad (3)$$

If we try a solution of Eq. (1) in the form

$$G = A(\rho) \sin(\rho + \delta(\rho)) \quad (4)$$

imposing the additional condition

$$dG/d\rho = A(\rho) \cos(\rho + \delta(\rho)), \quad (5)$$

on the functions  $A(\rho)$  and  $\delta(\rho)$ , we obtain from Eq. (1), in view of Eqs. (4) and (5), the following expressions for  $A(\rho)$  and  $\delta(\rho)$ :

$$dA/d\rho = A[l(l+1)\rho^{-2} + U(\rho)] \sin(\rho + \delta) \cos(\rho + \delta), \quad (6)$$

$$d\delta/d\rho = -[l(l+1)\rho^{-2} + U(\rho)] \sin^2(\rho + \delta). \quad (7)$$

We may note that since  $d \ln G/d\rho = \cot[\rho + \delta(\rho)]$ , in view of Eq. (2), when  $\rho \rightarrow 0$

$$\sin(\rho + \delta(\rho)) = \rho/(l+1). \quad (8)$$

Using the last expression and Eq. (7) we find

$$\delta(\rho) = -\frac{l}{l+1}\rho - \frac{l}{(l+1)^2} \int_0^\rho \rho \gamma(\rho) d\rho.$$

Using Eq. (8) we find the following expression from Eq. (6):

$$A(\rho) = A_0 \rho^l (1 + \gamma_0 \rho / (l+1) + \dots),$$

which applies for values of  $\rho$  close to zero.

If the limitations imposed on the function  $U(\rho)$  are satisfied it is obvious that

$$\delta(\rho) \rightarrow -\pi l/2 + \delta_l \text{ for } \rho \rightarrow \infty.$$

It should be noted that in the case  $\gamma(\rho) = \gamma_0$ , in view of the fact that  $\sin^2(\rho + \delta)$  is bounded, it follows from Eq. (7) that at large values of  $\rho$

$$\delta(\rho)_j = -\pi l/2 + \delta_l - \alpha \ln \rho,$$

where  $\alpha$  is a constant.

Because of the monotonic variation of  $\delta(\rho)$  at large values of  $\rho$  it is possible to integrate Eqs. (6) and (7) numerically with high accuracy and to determine the quantity  $\delta_1$  — the phase of the scattered wave. It is obvious that this form of the method of variation of arbitrary constants can be easily extended to the case in which the function  $U(\rho)$  is complex.

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## NUCLEAR SUBSHELLS AND DEFORMATION IN THE REGION PAST LEAD

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**N**UCLEAR deformation causes a change in the shape of the nuclear potential well. As a result the usual sequence for filling levels, described by Mayer,<sup>1</sup> becomes inaccurate and the scheme proposed by Nilsson must be employed.<sup>2</sup> An interesting feature is the fact that nuclear deformation is not a gradual effect; rather, it is found that the onset of deformation occurs suddenly at certain critical nucleon num-