ON ANNIHILATION OF ANTINUCLEONS

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The distribution of secondary particles in stars produced in annihilation of nucleon-antinucleon pairs is calculated, using the statistical theory of multiple production of particles. Exact expressions for the statistical weights, taking into account conservation of energy, momentum and isotopic spin, were used in the calculations. The calculations were carried out for several magnitudes of the effective volume in which the energy is distributed, and for several types of particles created in the annihilation. The mean multiplicity (3 — 5) of π-mesons obtained theoretically agrees with the available experimental data, whereas the number of K-mesons predicted by the theory is substantially higher than experimentally observed. The momentum distribution of secondary π-mesons in the center-of-mass system is also computed.

IT has been recently established that antiprotons which are greatly slowed down or stopped in photo emulsions produce stars, caused by the annihilation of nucleon-antinucleon pairs.[1-3] In view of the doubtfulness of perturbation theory calculations, it is of interest to use the statistical theory of Fermi[4] in this case

I. According to the statistical theory, the probability of production of n particles is given by the formula

$$S_n(E_0) = (V/8\pi^3\hbar^3)^{n-1} g_n(T)\, dQ_n(E_0)/dE_0, \qquad (1)$$

where V is an effective volume in which the energy E_0 of the annihilating particles is distributed, $g_n(T)$ is a factor which takes into account the conservation of isotopic spin[5] T and the identity of the particles produced in the annihilation,[6] and $Q_n(E_0)$ is the volume in momentum space corresponding to energy E_0. For the calculation of $dQ_0(E_0)/dE_0$ a method propsed by Rozental' and the author[7] was used, which made it possible to obtain exact values of this quantity, taking into account conservation of energy and momentum.

In the present article, calculations are given under various assumptions about the effective volume V and types of particles produced in the annihilation.

A. In the annihilation only π-mesons are produced:

$$\tilde{N} + N \rightarrow n\pi.$$

The following three variants were considered:

(1) The volume V was defined as

$$V = 4\pi r_0^3/3, \text{ where } r_0 = \hbar/\mu c = 1.4 \times 10^{-13} \text{ cm}. \qquad (2)$$

The calculation of such a variant was carried out earlier by Belen'kii and Rozental';[8] however, they assumed that all π-mesons produced were ultra-relativistic particles.

(2) With the same V the final-state interaction of the π-mesons was taken into account according to the ideas of Pomeranchuk,[9] which led to an increase of the volume by a factor of $(n-1)$ and to an additional factor $(n-1)^{n-1}$ in Eq. (1).

(3) The interaction of the π-mesons was also taken into account in accordance with Ref. 9, but r_0 was taken equal to 1.0×10^{-13} cm.

The calculated distributions of numbers n of π-mesons in annihilation stars are given in Table 1 (with probabilities of different n given in per cent) for these variants.

Starting from the law of conservation of isotopic spin and its projection, it is possible to obtain the distribution of numbers of charged particles in annihilation stars. The results of the work of Nikishov[10] were used here. These distributions are given in Table 2 (the probabilities are expressed in per cent). The mean number of all particles and the mean number of charged particles from a star were calculated on the basis of Tables I and II and are given in Table III.

TABLE I

Variant	Number of particles in the star	2	3	4	5	6	7
A(1)	$\tilde{p}-p$ or $\tilde{n}-n$	10.9	47.4	35.0	6.4	0.3	—
	$\tilde{p}-n$ or $\tilde{n}-p$	7.9	51.3	33.8	6.7	0.3	—
A (2)	$\tilde{p}-p$ or $\tilde{n}-n$	0.3	4.5	31.2	39.6	24.3	0.1
	$\tilde{p}-n$ or $\tilde{n}-p$	0.2	5.4	23.7	44.8	25.8	0.1
A (3)	$\tilde{p}-p$ or $\tilde{n}-n$	3,5	22,5	41,8	26,2	5,9	—
	$\tilde{p}-n$ or $\tilde{n}-p$	2.6	24,4	40,3	27,0	5,7	—

TABLE II

Variant	Number of charged particles in the star	0	1	2	3	4	5	6	7
A (1)	$\tilde{p}-p$ or $\tilde{n}-n$	10.0		71 9		18.0		0.1	
	$p-n$ or $\tilde{n}-p$		30.6		67.5		1.9		
A (2)	$\tilde{p}-p$ or $\tilde{n}-n$	2.4		36.8		56.6		4.1	
	$\tilde{p}-n$ or $\tilde{n}-p$		12.4		64,6		22.9		~0.05
A (3)	$\tilde{p}-p$ or $\tilde{n}-n$	5.4		55.9		37.8		0.9	
	$\tilde{p}-n$ or $\tilde{n}-p$		20.6		69,0		10,4		

B. In the annihilation, in addition to the process $\tilde{N} + N \rightarrow n\pi$, processes involving the production of K-meson pairs are possible

$$\tilde{N} + N \rightarrow 2K + n\pi.$$

Two variants for the choice of V were considered:

1) The effective radius, as previously, is $r_0 = 1.4 \times 10^{-13}$ cm.

2) The effective radius is $r_0 = 1.0 \times 10^{-13}$ cm and the correction of Pomeranchuk was taken into account, assuming K-mesons to interact just as π-mesons.

TABLE III

Вариант		A (1)	A (2)	A (3)
Mean number of all particles in a star	$\tilde{p}-p$ or $\tilde{n}-n$	3.38	4.86	4.08
	$\tilde{p}-n$ or $\tilde{n}-p$	3.41	4.97	4,14
Mean number of charged particles in a star	$p-p$ or $\tilde{n}-n$	2,17	3.25	2.68
	$\tilde{p}-n$ or $\tilde{n}-p$	2.43	3.20	2.80

TABLE IV

Variant	Final State	2π	3π	4π	5π	6π	$2K$	$2K+\pi$	$2K+2\pi$	$2K+3\pi$
B (1)	$\tilde{p}-p$ or $\tilde{n}-n$	3,8	16,7	12,3	2,2	0,1	6.6	26.4	27.7	4.2
	$\tilde{p}-n$ or $\tilde{n}-p$	2,9	18,8	12,3	2,5	0,1	4.9	26.3	27.7	4.5
B (2)	$\tilde{p}-p$ or $\tilde{n}-n$	1.2	7.9	14.6	9.1	2.4	2.2	13.0	32.7	16.9
	$\tilde{p}-n$ or $\tilde{n}-p$	0.9	8.7	14.3	9.7	2.4	1.6	12.2	32.3	17.9

The distributions of various possible final states in annihilation stars are given in Table 4; the mean number of all particles, the mean number of π-mesons, and the mean number of K-mesons in a star are in Table V.

Recently Sudarshan[11] considered analogous questions for case B, using approximate formulae for the calculation of $dQ_n(E_0)/dE_0$. Several details of the calculation in his work are unclear. Thus, for example, if one goes to a system of units in which $\hbar = M = c = 1$, Eq. (1) can be written

$$S_n(E_0) = (5{,}21V/V_0)^{n-1} g_n(T)\, dQ_n(E_0)/dE_0$$

(where V_0 is defined by Eq. (2), whereas in Ref. 11 the expression

$$S_n(E_0) = (0{,}945V/V_0)^{n-1} g_n(T)\, dQ_n(E_0)/dE_0.$$

is used.

The above results can be compared with the experimental data obtained by Segrè and collaborators.[3] They found in photographic emulsions about 30 annihilation stars caused by antiprotons. The mean number of charged π-mesons in a star was about 2.5, and the mean number of all π-mesons (taking into account neutral ones), about 5 ± 1. A substantial number of K-mesons were not observed among the particles produced as a result of annihilation. It is also well-known, that in the collision of particles of energy $10^9 - 10^{10}$ ev, the number of K-mesons among the secondary particles constitutes not more than $1-2\%$ of the number of π-mesons. From this one can conclude that, apparently, variants A corresponds more closely to reality than variants B (and, consequently, also the work of Ref. 11), in which it was found that K-mesons should be formed in $60-65\%$ of the cases of annihilation and should constitute $25-30\%$ of the secondary particles. It is possible that, within the framework of the statistical theory, K-mesons should be taken into account in a different way than π-mesons. Thus, for example, if it is postulated that the effective volume for formation of a K-meson differs from the effective volume for formation of π-mesons, and an effective radius $r = \hbar/M_K c = 0.4 \times 10^{-13}$ cm (M_K is the mass of the K-meson) is defined, then the number of K-mesons, according to the statistical theory, becomes negligible compared with the number of π-mesons. However, there are no sufficiently convincing arguments for the introduction in this fashion of two effective volumes.

As can be seen from Tables I — III, all variants A agree satisfactorily with existing experiments and it is not possible to choose between them.

The data given relate to the case of annihilation at rest. Calculations have shown that the energy-dependence of the mean number of all particles ($\bar{n}$), coming from a star is very accurately given by the formula

$$\bar{n} = 2.87\,(E/Mc^2)^{1/4},$$

where E is the total energy of the nucleon—antinucleon pair in the laboratory system.

TABLE V

Variant		B (1)	B (2)
Mean number of π-mesons in a star	$\tilde{p}-p$ or $\tilde{n}-n$	2,11	2,73
	$\tilde{p}-n$ or $\tilde{n}-p$	2,16	2,78
Mean number of K-mesons in a star	$\tilde{p}-p$ or $\tilde{n}-n$	1,29	1,28
	$\tilde{p}-n$ or $\tilde{n}-p$	1,27	1,27
Mean number of all particles in a star	$\tilde{p}-p$ or $\tilde{n}-n$	3,40	4,01
	$\tilde{p}-n$ or $\tilde{n}-n$	3,43	4,05

II. The momentum distribution of π-mesons produced as a result of annihilation was also calculated (only for case A).

The probability that a given particle created in the process of multiple production with energy E_0 in the center-of-mass system, has in this coordinate system,* a momentum in the interval from p to p + dp, is[12]

$$\omega_n(E_0, p)\,dp = 4\pi p^2 S_{n-1}\left(E_0 - \sqrt{p^2+\mu^2},\ p\right)dp. \tag{3}$$

Generalizing the formulae of Ref. 7 to the case $P_0 \neq 0$ and neglecting factors independent of p, we obtain the following formula†

$$\omega_n(E_0, p)\,dp = p^2\left(E_0 - \sqrt{p^2+\mu^2}\right)^{3n-7}\Big\{D_{n-1}^{(0)} + C_{n-1}^1 D_{n-1}^{(1)}\nu^2 \tag{4}$$
$$+ C_{n-1}^1 D_{n-1}^{(2)}\nu^4 \ln\frac{1}{\nu} + \left[\left(\frac{3}{4}C_{n-1}^1 + C_{n-1}^2\right)D_{n-1}^{(2)} + \frac{1}{2}C_{n-1}^1 F_{n-1}^{(2)}\right]\nu^4 + \cdots\Big\}.$$

Here

$$D_N^{(A)} = \frac{(1-k^2)^{N-2}}{2k}(-1)^A \sum_{r=0}^{N} C_N^r \left\{\frac{(1+k)^{N-r+1}(1-k)^r}{(2N-r-(A+1))!\,(N+r-(A+2))!} - \frac{(1+k)^{N-r}(1-k)^{r+1}}{(2N-r-(A+2))!\,(N+r-(A+1))!}\right\},$$

$$F_N^{(A)} = \frac{(1-k^2)^{N-2}}{2k}(-1)^A \sum_{r=0}^{N} C_N^r \left\{(1+k)^{N-r}(1-k)^{r+1}\left[\frac{\alpha(N+r-A)}{(2N-r-(A+2))!} + \frac{\alpha(2N-r-(A+1))}{(N+r-(A+1))!}\right] - (1+k)^{N-r+1}(1-k)^r\right.$$
$$\left.\times\left[\frac{\alpha(N+r-(A+1))}{(2N-r-(A+1))!} + \frac{\alpha(2N-r-A)}{(N+r-(A+2))!}\right]\right\}, \quad k = p/\left(E_0 - \sqrt{p^2+\mu^2}\right), \quad \nu = \mu\left[\left(E_0 - \sqrt{p^2+\mu^2}\right)^2 - p^2\right]^{-1/2};$$

$$\alpha(z) = \begin{cases} (-1)^{z-1}|z|! & \text{for } z = 0, -1, -2, -3\ldots \\ 0 & \text{for } z = 1 \\ \dfrac{1}{(z-1)!}\displaystyle\sum_{r=1}^{z-1}\frac{1}{r} & \text{for } z = 2, 3, 4, \ldots \end{cases}.$$

For the special case n = 3, Eq. (4) can be written in the simpler form

$$\omega_3(E_0, p)\,dp = p^2\left(E_0 - \sqrt{p^2+\mu^2}\right)^2 \times \left[1 - \tfrac{1}{3}k^2(1-4\nu^2)\right]\sqrt{1-4\nu^2}\,dp. \tag{4'}$$

The momentum distributions of π-mesons (case A) obtained according to Eqs. (4) and (4′), are given in Fig. 1.

In conclusion I should to thank I. L. Rozental' for continuing interest in this work and for discussion of the problems involved in it. I should also like to thank Z. S. Maksimov for carrying out numerical calculations.

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*In our case of annihilation at rest, the center-of-mass system coincides with the laboratory system.

†See Ref. 7 for the remaining terms of the series in the braces where Eqs. (5) and (6) should be taken for $D_N^{(A)}$ and $F_N^{(A)}$. The first term of the series, corresponding to the case of ultra-relativistic particles, was obtained earlier by Rozental'.[12]

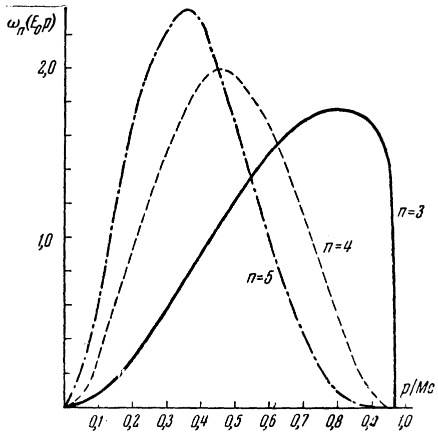

FIG. 1. Momentum distribution $\omega_n(E_0, p)$ of π-mesons for the process $N + N \rightarrow n\pi$ ($n = 3, 4, 5$). Here p is in units $Mc = 0.93$ Bev/c. The curves are normalized such that $\int \omega_n(E_0, p)\,dp = 1$.

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CONTRIBUTION TO THE PHENOMENOLOGICAL THEORY OF PARAMAGNETIC RELAXATION IN PARALLEL FIELDS

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The role of spin-lattice interaction in the phenomenological theory of complex paramagnetic susceptibility is taken into account in the case of parallel fields to a greater extent than was previously done in Ref. 3.

I. In the first work on the phenomenological theory of paramagnetic relaxation in parallel fields (Casimir and Du Pre[1] and others,(see Ref. 2) considered only the spin-lattice relaxation (see Ref. 2 for terminology). Later Shaposhnikov[3] (whose work will be designated hereafter by I) presented a phenomenological theory of complex paramagnetic susceptibility for the case of parallel fields, taking both spin-lattice and spin-spin relaxation into account, while Khutsishvili[4] has made a general phenomenological analysis of paramagnetic relaxation in a constant field using the Onsager principle, and has shown in particular under what assumptions the corresponding results of the theory given in I are obtained. Recently Yokota[5] repeated independently the examination of paramagnetic relaxation, previously carried out in I, generalizing somewhat the statement of the problem, and arriving at a final result (coinciding with the results of I) in only one particular case. In the present communication the theory of I is generalized with account of the work of Khutsishvili and Yokota. Here, as in I, we have in mind isotropic non-conducting paramagnetic materials in condensed state (for example, polycrystalline powders of paramagnetic salts, which are frequently used in experiments).