FINE STRUCTURE IN THE ALPHA DECAY OF ODD NUCLEI

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The excitation of rotational levels in the α-decay of non-spherical odd nuclei is considered.

As is well known, every odd nucleus is characterized by an angular momentum number K that is different from zero. For this reason the system formed upon α-decay of a nucleus — the daughter nucleus plus an α-particle — is characterized not only by the total energy of decay, E, but also by the total angular momentum, **K**, and its projection on a fixed axis, K_z. On the other hand, the spectrum of emitted particles cannot depend on the orientation of the total system in space. For this reason, without loss of generality, we can set $K_z = K$. Thus we obtain for the wave function of the daughter nucleus plus α-particle system the following equations:

$$\nabla^2\Psi - \frac{m}{I}[\mathbf{j}^2 - \Lambda(\Lambda+1)]\Psi + \frac{2m}{\hbar^2}(E-U)\Psi = 0, \quad \mathbf{K}^2\Psi = K(K+1)\Psi, \quad K_z\Psi = K\Psi. \tag{1}$$

Here **j** is the operator for the angular momentum of the daughter nucleus, Λ is the projection of the angular momentum of the daughter nucleus on its axis of symmetry, I is the moment of inertia of the nucleus, m is the reduced mass, and U is the potential energy of the electrostatic interaction of the α-particle with the nucleus. The angular momentum of the daughter nucleus can be expressed in terms of the total angular momentum **K** and the orbital angular momentum of the α-particle, **l**, through the equation:

$$\mathbf{j}^2 = (\mathbf{K} - \mathbf{l})^2 = \mathbf{K}^2 - 2\mathbf{l}\mathbf{K} + \mathbf{l}^2.$$

It is convenient to solve Eq. (1) in a system of coordinates rotating with the nucleus. Introducing into this system the spherical coordinates of the α-particle, namely r, $\mu = \cos\vartheta$, and φ, and likewise the Euler angles $\nu = \cos\theta$, and ϕ, which determine the orientation of the rotating axes relative to the stationary ones, we obtain the following expressions for the momentum operators that interest us:[1]

$$\mathbf{K}^2 = -\frac{\partial}{\partial\nu}\left[(1-\nu^2)\frac{\partial}{\partial\nu}\right] - \frac{1}{1-\nu^2}\frac{\partial^2}{\partial\phi^2} + \frac{2i\nu}{1-\nu^2}\left(\Lambda - i\frac{\partial}{\partial\varphi}\right)\frac{\partial}{\partial\phi} + \frac{1}{1-\nu^2}\left(\Lambda - i\frac{\partial}{\partial\varphi}\right)^2;$$

$$\mathbf{l}\mathbf{K} = \sqrt{1-\nu^2}\,\frac{\partial}{\partial\nu}\left(\sqrt{1-\mu^2}\sin\varphi\cdot\frac{\partial}{\partial\mu} - \frac{\mu}{\sqrt{1-\mu^2}}\cos\varphi\cdot\frac{\partial}{\partial\varphi}\right) + \frac{1}{\sqrt{1-\nu^2}}\frac{\partial}{\partial\phi}\left(\sqrt{1-\mu^2}\cos\varphi\cdot\frac{\partial}{\partial\mu} + \frac{\mu}{\sqrt{1-\mu^2}}\sin\varphi\cdot\frac{\partial}{\partial\varphi}\right) \tag{2}$$

$$- \frac{i\nu}{\sqrt{1-\nu^2}}\left(\sqrt{1-\mu^2}\cos\varphi\cdot\frac{\partial}{\partial\mu} + \frac{\mu}{\sqrt{1-\mu^2}}\sin\varphi\cdot\frac{\partial}{\partial\varphi}\right)\left(\Lambda - i\frac{\partial}{\partial\varphi}\right) - i\frac{\partial}{\partial\varphi}\left(\Lambda - i\frac{\partial}{\partial\varphi}\right);$$

$$\mathbf{l}^2 = -\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial}{\partial\mu}\right] - \frac{1}{1-\mu^2}\frac{\partial^2}{\partial\varphi_2}; \qquad K_z = -i\frac{\partial}{\partial\phi}.$$

We will look for a particular solution to (1) which, at large distances, describes an α-particle with momentum ℓ, leaving the nucleus, and a daughter nucleus in an angular momentum state j. This has the form

$$\Psi_{jl}(r, \mu, \varphi, \nu, \phi) = N_{jl}^{-1/2} e^{iK\phi} Y_{jl}(\nu, \mu, \varphi)\, e^{\sigma(r, \mu, \varphi, \nu)}. \tag{3}$$

The factor $e^{iK\phi}Y_{j\ell}$ represents the angular part of the wave function and, therefore, satisfies the equations

$$\mathbf{K}^2 e^{iK\phi}Y_{jl} = K(K+1)e^{iK\phi}Y_{jl}, \quad \mathbf{l}^2 Y_{jl} = l(l+1)Y_{jl}, \quad \mathbf{l}\mathbf{K}e^{iK\phi}Y_{jl} = \frac{1}{2}[K(K+1) - j(j+1) + l(l+1)]e^{iK\phi}Y_{jl}. \tag{4}$$

The solutions of equation (4) have the form

$$Y_{jl}(\nu, \mu, \varphi) = \sum_m A_m^{jl}(1+\nu)^{(K+\Lambda+m)/2}(1-\nu)^{(K-\Lambda-m)/2} p_l^m(\mu) e^{im\varphi}, \quad p_l^m(\mu) = \sqrt{\frac{(l-m)!}{(l+m)!}}P_l^m(\mu);$$

$$P_l^m(\mu) = \frac{(1-\mu^2)^{m/2}}{2^l l!}\frac{d^{l+m}(\mu^2-1)^l}{d\mu^{l+m}}. \tag{5}$$

The constants $A_m^{j\ell}$ are determined from the recursion relations

$$i(K-\Lambda-m+1)\sqrt{(l+m)(l-m+1)}\,A_{m-1}^{jl} - [K(K+1) - j(j+1) + l(l+1) - 2m(\Lambda+m)]A_m^{jl}$$

$$- i(K+\Lambda+m+1)\sqrt{(l-m)(l+m+1)}\,A_{m+1}^{jl} = 0. \tag{6}$$

The normalization for the angular functions

$$N_{jl} = \int |Y_{jl}(\nu, \mu, \varphi)|^2 d\nu\, d\mu\, d\varphi = \frac{2^{2K+1}}{(2K+1)!} \frac{4\pi}{2l+1} \sum_m (K+\Lambda+m)!\,(K-\Lambda-m)!\, |A_m^{jl}|^2 \tag{7}$$

is introduced into Eq. (3) for convenience.

It is reasonable that σ should not depend on the angular variables μ, φ, ν at large r. Substituting (3) in the first of the equations (1), and taking into consideration (2) and (4), we get

$$\begin{aligned}
&Y_{jl}\left[\left(\frac{\partial\sigma}{\partial r}\right)^2 + \frac{\partial^2\sigma}{\partial r^2} + \frac{2}{r}\frac{\partial\sigma}{\partial r}\right] + \left(\frac{m}{I} + \frac{1}{r^2}\right)\Big\{(1-\mu^2)\Big|\Big\{Y_{jl}\left[\left(\frac{\partial\sigma}{\partial\mu}\right)^2 + \frac{\partial^2\sigma}{\partial\mu^2}\right] + 2\frac{\partial Y_{jl}}{\partial\mu}\frac{\partial\sigma}{\partial\mu}\Big\} - 2\mu Y_{jl}\frac{\partial\sigma}{\partial\mu} \\
&+ \frac{1}{1-\mu^2}\Big\{Y_{jl}\left[\left(\frac{\partial\sigma}{\partial\varphi}\right)^2 + \frac{\partial^2\sigma}{\partial\varphi^2}\right] + 2\frac{\partial Y_{jl}}{\partial\varphi}\frac{\partial\sigma}{\partial\varphi}\Big\}\Big\} + 2\frac{m}{I}\Big[\sqrt{1-\nu^2}\Big\{\sqrt{1-\mu^2}\sin\varphi\cdot\left[Y_{jl}\left(\frac{\partial\sigma}{\partial\mu}\frac{\partial\sigma}{\partial\nu} + \frac{\partial^2\sigma}{\partial\mu\,\partial\nu}\right) + \frac{\partial Y_{jl}}{\partial\mu}\frac{\partial\sigma}{\partial\nu} + \frac{\partial Y_{jl}}{\partial\nu}\frac{\partial\sigma}{\partial\mu}\right] \\
&- \frac{\mu}{\sqrt{1-\mu^2}}\cos\varphi\cdot\left[Y_{jl}\left(\frac{\partial\sigma}{\partial\varphi}\frac{\partial\sigma}{\partial\nu} + \frac{\partial^2\sigma}{\partial\varphi\,\partial\nu}\right) + \frac{\partial Y_{jl}}{\partial\varphi}\frac{\partial\sigma}{\partial\nu} + \frac{\partial Y_{jl}}{\partial\nu}\frac{\partial\sigma}{\partial\varphi}\right]\Big\} + \frac{iK}{\sqrt{1-\nu^2}}\Big[\sqrt{1-\mu^2}\cos\varphi\cdot Y_{jl}\frac{\partial\sigma}{\partial\mu} \\
&+ \frac{\mu}{\sqrt{1-\mu^2}}\sin\varphi\cdot Y_{jl}\frac{\partial\sigma}{\partial\varphi}\Big] - \frac{\nu}{\sqrt{1-\nu^2}}\Big\{\sqrt{1-\mu^2}\cos\varphi\cdot\Big\{Y_{jl}\left[\frac{\partial\sigma}{\partial\varphi}\frac{\partial\sigma}{\partial\mu} + \frac{\partial^2\sigma}{\partial\varphi\partial\mu}\right] + \left(\frac{\partial Y_{jl}}{\partial\varphi} + i\Lambda Y_{jl}\right)\frac{\partial\sigma}{\partial\mu} + \frac{\partial Y_{jl}}{\partial\mu}\frac{\partial\sigma}{\partial\varphi}\Big\} \\
&+ \frac{\mu}{\sqrt{1-\mu^2}}\sin\varphi\cdot\Big\{Y_{jl}\left[\left(\frac{\partial\sigma}{\partial\varphi}\right)^2 + \frac{\partial^2\sigma}{\partial\varphi^2}\right] + \left(2\frac{\partial Y_{jl}}{\partial\varphi} + i\Lambda Y_{jl}\right)\frac{\partial\sigma}{\partial\varphi}\Big\}\Big\} - \Big\{Y_{jl}\left[\left(\frac{\partial\sigma}{\partial\varphi}\right)^2 + \frac{\partial^2\sigma}{\partial\varphi^2}\right] + \left(2\frac{\partial Y_{jl}}{\partial\varphi} + i\Lambda Y_{jl}\right)\frac{\partial\sigma}{\partial\varphi}\Big\}\Big] \\
&+ \left\{\frac{2m}{\hbar^2}(E-U) - \frac{l(l+1)}{r^2} - \frac{m}{I}[j(j+1) - \Lambda(\Lambda+1)]\right\} Y_{jl} = 0.
\end{aligned} \tag{8}$$

We will express the quasi classical expansion for $\sigma(r, \mu, \varphi, \nu)$ in the form $\sigma = \sigma_{-1} + \sigma_0 + \sigma_1 + \ldots$; we then get an infinite set of equations for the successive approximations $\sigma_{-1}, \sigma_0, \sigma_1, \ldots$ The solution of these equations is made easier in the case of a slightly non-spherical daughter nucleus. In this case the equation of the surface of the nucleus is represented conveniently in the form

$$R(\mu) = R_0 + \xi(\mu) = R_0\left\{1 + \sum_n \alpha_n P_n(\mu)\right\}. \tag{9}$$

The quantities σ_{-1}, σ_0 and $\sigma_1\ldots$ are expanded in series in powers of ξ:

$$\sigma_{-1} = \sigma_{-1}^{(0)} + \sigma_{-1}^{(1)} + \ldots; \quad \sigma_0 = \sigma_0^{(0)} + \sigma_0^{(1)} + \ldots; \quad \sigma_1 = \sigma_1^{(0)} + \sigma_1^{(1)} + \ldots$$

etc. The electrostatic potential produced by the daughter nucleus is easily calculated in terms of an analogous series. As a result we get

$$\sigma_{-1}^{(0)} = i\int_{a_{jl}}^{r} k_{jl}\,dr; \quad \sigma_0^{(0)} = -\ln\left(\sqrt{-ik_{jl}r}\right); \quad \sigma_{-1}^{(1)} = \frac{3}{2}i\varkappa_b^2 \sum_n \frac{\alpha_n}{2n+1} P_n(\mu)\int_r^\infty \left(\frac{R_0}{r}\right)^{n+1}\frac{dr}{k_{jl}}. \tag{10}$$

Here

$k_{jl} = \sqrt{k_\varepsilon^2 - \varkappa_b^2 R_0/r - l(l+1)/r^2}$, $\quad \varkappa_b^2 = 4mZe^2/\hbar^2 R_0$; $\quad k_\varepsilon^2 = 2m\varepsilon/\hbar^2$; $\quad \varepsilon \equiv \varepsilon_j = E - (\hbar^2/2I)[j(j+1) - \Lambda(\Lambda+1)]$.

The turning point of a_{jl} is determined by the equation

$$k_{jl}^2(a_{jl}) = k_\varepsilon^2 - \varkappa_b^2 R_0/a_{jl} - l(l+1)/a_{jl}^2 = 0.$$

The formulae (10) are valid also at $r < a_{jl}$ if it is kept in mind that in this region

$$k_{jl} = i\varkappa_{jl} = i\sqrt{l(l+1)/r^2 + \varkappa_b^2 R_0/r - k_\varepsilon^2}.$$

Further terms of the double series for the function $\sigma(r, \mu, \varphi, \nu)$ are sufficiently small to be neglected.

Introducing (10) into (32) and examining the behavior as $r \to \infty$ it is seen that the corresponding flux through the closed sphere does not depend on j and ℓ. Therefore, if we express the wave function in the form $\Psi = \sum_{j\ell} c_{j\ell}\Psi_{j\ell}$, then the relative probability of exciting a rotational level with angular momentum j, accompanying the emission of the α-particle with angular momentum ℓ, is expressible as $w_{j\ell} = |c_{j\ell}|^2$, and the total probability of exciting a level with angular momentum j will be $w_{j\ell} = \sum_\ell w_{j\ell}$.

The coefficients $c_{j\ell}$ can be expressed in terms of the values of the wave function at the surface of the nucleus $\Psi_S(\mu, \varphi, \nu, \phi)$. Thus we get, in the required approximation,

$$\begin{aligned}
\sigma_{-1}^{(0)}(S) &= \sigma_{-1}^{(0)}(R_0)\big|_{j=\Lambda,\, l=0} - \varkappa_{\Lambda 0}(R_0)\,\xi + \frac{\gamma_j}{2}[j(j+1) - \Lambda(\Lambda+1)] + \frac{\gamma_l}{2}l(l+1); \\
\sigma_{-1}^{(1)}(S) &= \sigma_{-1}^{(1)}(R_0)\big|_{j=\Lambda,\, l=0}; \quad \sigma_0^{(0)}(S) = \sigma_0^{(0)}(R_0)\big|_{j=\Lambda,\, l=0}.
\end{aligned} \tag{11}$$

In these equations

$$\gamma_j = 2\frac{\partial\sigma_{-1}^{(0)}}{\partial j(j+1)}(R_0)\Big|_{j=\Lambda,\, l=0} = \frac{m}{Ik^2}\left(\frac{\varkappa_b^2 R_0}{k}\operatorname{arc\,tg}\frac{\varkappa}{k} + \varkappa R_0\right); \quad \gamma_l = 2\frac{\partial\sigma_{-1}^{(0)}}{\partial l(l+1)}(R_0)\Big|_{j=\Lambda,\, l=0} = \frac{2\varkappa}{\varkappa_b^2 R_0},$$

$$k \equiv k_\varepsilon = \sqrt{\frac{2mE}{\hbar^2}}, \qquad \varkappa \equiv \varkappa_{\Lambda 0}(R_0) = \sqrt{\varkappa_b^2 - k^2}. \tag{12}$$

Substituting (11) into (3) and keeping in mind Eq. (10) we get

$$\sqrt{\varkappa}\, R_0 \exp\{-\sigma_{-1}^{(0)}| + \varkappa\xi - \sigma_{-1}^{(1)}|\}\Psi_S(\mu, \varphi, \nu, \phi)$$

$$= \sum_{j,l} c_{jl} \exp\left\{\frac{\gamma_j}{2}[j(j+1) - \Lambda(\Lambda+1)] + \frac{\gamma_l}{2}\, l(l+1)\right\} N_{jl}^{-1/2} e^{iK\phi} Y_{jl}(\nu, \mu, \varphi). \tag{13}$$

The vertical lines to the right of quantities in the equation have the meaning that they should be taken at $r = R_0$, $j = \Lambda$, and $l = 0$. As is seen, Eqs. (10), (9), (4), and (2), show that the function Ψ_S is not completely arbitrary, but satisfies the equations

$$\mathbf{K}^2\Psi_S = K(K+1)\Psi_S, \quad K_z\Psi_S = K\Psi \tag{14}$$

and therefore has the form

$$\Psi_S(\mu, \varphi, \nu, \phi) = e^{iK\phi}\psi_S(\nu, \mu, \varphi), \quad \psi_S(\nu, \mu, \varphi) = \sum_{m'} \chi_{m'}(\mu)(1+\nu)^{(K+\Lambda+m')/2}(1-\nu)^{(K-\Lambda-m')/2} e^{im'\varphi}. \tag{15}$$

This result does not depend on the approximations made, but follows strictly from Eqs. (9), (2), and the two last equations of (1). Using the orthogonality of the functions Y_{jl}, and likewise (7) and (5), and normalizing the probability by the condition $w_\Lambda = 1$, we finally obtain

$$w_j = e^{-\gamma_j[j(j+1)-\Lambda(\Lambda+1)]} \frac{\sum_{l=|j-K|}^{j+K} \frac{2l+1}{S_{jl}} e^{-\gamma_l l(l+1)} \left|\int \psi_S e^{\varkappa\xi - \sigma_{-1}^{(1)}|} Y_{jl}^* \, d\nu\, d\mu\, d\varphi\right|^2}{\sum_{l=|\Lambda-K|}^{\Lambda+K} \frac{2l+1}{S_{\Lambda l}} e^{-\gamma_l l(l+1)} \left|\int \psi_S e^{\varkappa\xi - \sigma_{-1}^{(1)}|} Y_{\Lambda l}^* \, d\nu\, d\mu\, d\varphi\right|^2} = e^{-\gamma_j[j(j+1)-\Lambda(\Lambda+1)]}$$

$$\times \frac{\sum_{l=|j-K|}^{j+K} \frac{2l+1}{S_{jl}} e^{-\gamma_l l(l+1)} \left|\sum_m A_m^{jl*}(K+\Lambda+m)!\,(K-\Lambda-m)! \int_{-1}^{1} \chi_m(\mu) e^{\varkappa\xi-\sigma_{-1}^{(1)}|} P_l^m(\mu)\, d\mu\right|^2}{\sum_{l=|\Lambda-}^{\Lambda+K} \frac{2l+1}{S_{\Lambda l}} e^{-\gamma_l l(l+1)} \left|\sum_m A_m^{\Lambda l*}(K+\Lambda+m)!\,(K-\Lambda-m)! \int_{-1}^{1} \chi_m(\mu) e^{\varkappa\xi-\sigma_{-1}^{(1)}|} P_l^m(\mu)\, d\mu\right|^2}, \tag{16}$$

where

$$S_{jl} = \sum_m (K+\Lambda+m)!\,(K-\Lambda-m)!\,|A_m^{jl}|^2.$$

Further simplifications can be made if it is assumed that, immediately after the emission of the α-particle, the direction of the axis of the daughter nucleus is the same as the direction of the axis of the mother nucleus just before the decay. The rotational wave function of the mother nucleus has the form

$$e^{iK\varphi}(1+\nu)^{(K+\Lambda_0)/2}(1-\nu)^{(K-\Lambda_0)/2},$$

where Λ_0 is the projection of the angular momentum of the mother nucleus on the axis of symmetry of the nucleus. In the absence of dispersion of the direction of the axis of the daughter nucleus relative to the original direction of the axis of the mother nucleus, only the term $m' = \Lambda_0 - \Lambda$ in the sum appearing in Eq. (15) will be different from zero.* Introducing, in analogy to the case of even even nuclei,[2] the notation $\chi_{\Lambda_0-\Lambda}(\mu) = \Psi(S)$ and substituting into (16), we get

$$w_j = e^{-\gamma_j[j(j+1)-\Lambda(\Lambda+1)]} \frac{\sum_{l=|j-K|}^{j+K} (2l+1) \frac{|A_{\Lambda_0-\Lambda}^{jl}|^2}{S_{jl}} e^{-\gamma_l l(l+1)} \left|\int_{-1}^{1} \Psi(S) e^{\varkappa\xi-\sigma_{-1}^{(1)}|} P_l^{\Lambda_0-\Lambda}(\mu)\, d\mu\right|^2}{\sum_{l=|\Lambda-K|}^{\Lambda+K} (2l+1) \frac{|A_{\Lambda_0-\Lambda}^{\Lambda l}|^2}{S_{\Lambda l}} e^{-\gamma_l l(l+1)} \left|\int_{-1}^{1} \Psi(S) e^{\varkappa\xi-\sigma_{-1}^{(1)}|} P_l^{\Lambda_0-\Lambda}(\mu)\, d\mu\right|^2}. \tag{17}$$

Especially intense are the so-called favored transitions,[3] i.e., transitions with $\Lambda = \Lambda_0 = K$. For such transitions it is natural, analogously to the case in even even nuclei,[2] to assume $\Psi(S) = \text{const}$. It is easy to see that in favored transitions the sums in Eqs. (5) and (7) do not have terms with positive m. Substituting

*The assumption that has beem made has no analogue in the case of even even nuclei.[2] In that case, owing to the complete spherical symmetry of the state with zero angular momentum, the wave function does not depend on the Eulerian angles, and the question of the influence of possible dispersion of the direction of the axis has no meaning. The validity of this assumption is examined below.

$A^{j\ell}_{-|m|} = a^{j\ell}_{|m|}$, changing signs, and keeping in mind that in heavy nuclei the most important is the quadrupole deformation $\xi = R_0\, \alpha_2 P_2(\mu)$, we get

$$w_j = e^{-\gamma_j [j(j+1) - K(K+1)]} \frac{\sum\limits_{l=|j-K|}^{j+K}{}' \frac{2l+1}{S_{jl}} e^{-\gamma_l l(l+1)} \left| \int\limits_0^1 e^{\beta P_2(\mu)} P_l(\mu)\, d\mu \right|^2}{\sum\limits_{l=0}^{2K}{}' \frac{2l+1}{S_{Kl}} e^{-\gamma_l l(l+1)} \left| \int\limits_0^1 e^{\beta P_2(\mu)} P_l(\mu)\, d\mu \right|^2}, \tag{18}$$

in which

$$S_{jl} = \sum_{m=0}^{\min\{{2K \atop l}\}} m!\,(2K-m)!\,|a_m^{jl}|^2; \quad i(m+1)\sqrt{(l-m)(l+m+1)}\, a_{m+1}^{jl} - [K(K+1) - j(j+1) + l(l+1)$$
$$+ 2m(K-m)]\, a_m^{jl} - i(2K-m+1)\sqrt{(l+m)(l-m+1)}\, a_{m-1}^{jl} = 0; \quad a_0^{jl} = 1. \tag{19}$$

The primed summation sign in Eq. (18) means that the summation should be carried out only over the values of the orbital angular momentum ℓ. Just as in the previous work,[2] we have

$$\beta = \left[\frac{4}{5}\varkappa R_0\left(1 - \frac{k^2}{2\varkappa_b^2}\right) - i\frac{2}{5}\frac{k^3 R_0}{\varkappa_b^2}\right]\alpha_2.$$

The deformation α_2 was calculated from the experimental values of w_{K+1} for three odd nuclei with $K = 5/2$. The deformations calculated according to (18) are presented in Table I along with the quadrupole moments, $Q_0 = 6/5\, ZR_0^2 \alpha_2 10^{24}$. The results in the table differ but little from the corresponding values in neighboring even even nuclei.[2]

TABLE I

$10^{13} A^{-1/3} R_0 =$	α_2 1.0	α_2 1.4	Q_0 1.0	Q_0 1.4
Th^{229}	0.25	0.21	10	17
Np^{237}	0.20	0.16	9	13
Np^{239}	0.18	0.15	8	13

The assumption made above concerning the absence of "rocking" of the axis of the nucleus at the instant of α emission can apparently be checked experimentally. In fact, from formula (17) it is easy to see that

$$w_{jl} / w_{j'l} = (S_{j'l} / S_{jl}) \exp\{-\gamma_j [j(j+1) - j'(j'+1)]\}, \tag{20}$$

where, for favored transitions, the sums $S_{j\ell}$ are normalized according to Eq. (19). In the case of $K = 5/2$, formula (20) gives, in particular*

$$w_{9/2,2} / w_{7/2,2} = (7/20)\, e^{-9\gamma_j}. \tag{21}$$

In the case of the three nuclei mentioned above, this quantity practically coincides with $w_{9/2}/w_{7/2}$, since, according to (18), the calculated contribution of α-particles with momentum equal to 4 does not exceed 15%. In Table II the experimental values of the ratio $w_{9/2}/w_{7/2}$ are compared with those calculated from formula (21). As is seen from the table all the calculated values deviate somewhat from the experimental ones. The contribution α-particles with $\ell = 4$ cannot be important and could only increase the discrepancy. It is possible that this disagreement could be explained by actually assuming some dispersion of the direction of the axis of the nucleus to take place upon α decay.

TABLE II

Nucleus	$w_{9/2,2}/w_{7/2,2}$ theor.	$(w_{9/2}/w_{7/2})$ exptl.
Th^{229}	0.14	0.11
Np^{237}	0.16	0.10
Np^{239}	0.16	0.10

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*In the derivation of (20) and (21) it was not assumed that the quadrupole deformation and the behavior of the wave function at the surface of the nucleus were not affected. It has only been assumed that there is no dispersion of the direction of the axis of the nucleus at the instant of α emission.