TIME REVERSAL AND POLARIZATION PHENOMENA IN N + N ⇄ d + π REACTION

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By an analysis of the polarization phenomena in the reactions N + N ⇄ d + π it is shown how the condition of invariance under time reversal leads to relations between the polarization phenomena in the direct and inverse reactions. A generalization of Wolfenstein's theorem to this binary reaction is obtained. Expressions for the various observable quantities in the reactions N + N ⇄ d + π are given both when the treatment is confined to s, p, and d waves with $J \leq 2$, and also in the general case.

INTRODUCTION

As is well known, the requirement of invariance under change of the sign of the time leads[1-3] to the condition of symmetry of the S matrix (notation of Blatt and Bidenharn[2] with an emendation from Ref. 3)

$$S^{J}_{\alpha's'l';\,\alpha sl} = S^{J}_{\alpha sl;\,\alpha's'l'}. \tag{1}$$

For the interaction of spinless particles this condition leads to the equality of the differential cross-sections of direct and inverse reactions (apart from a factor of the ratio of the squares of the relative momenta in the direct and inverse reactions). For particles with spin, for which there appears a difference between the time-reversed and inverse processes, this same condition leads to the equality of the averages over spins of the cross-sections of the direct and inverse reactions. But in the case of interaction of particles with spin not equal to zero, one can express in terms of the elements of the S matrix not only the reaction cross-section of unpolarized particles, but also the average values of the spin operators in the direct and inverse processes. Because of the existence of the condition (1) it is of interest to consider its consequences for the polarization phenomena in direct and inverse processes and to try to find relationships analogous to the connections between the cross-sections for the reactions between unpolarized beams and unpolarized targets. Such a connection has been discussed in the literature in a consideration of polarization phenomena in elastic scattering. Dalitz[4] and Wolfenstein and Ashkin[5] have shown that in the case of elastic scattering of particles with spin 1/2 from a target with arbitrary spin, for which the principle of detailed balancing reduces to an identity, the requirement of invariance under change of sign of the time has as a consequence a connection between the polarization $\langle\hat{\boldsymbol{\sigma}}_1\rangle_f$ of particles appearing from the interaction of unpolarized particles and the azimuthal asymmetry I_p in the scattering cross-section of polarized particles from an unpolarized target. This connection is of the form

$$I_p = \langle\hat{\boldsymbol{\sigma}}_1\rangle_f \mathbf{N} I_0(\theta), \tag{2}$$

where $I_0(\theta)$ is the scattering cross-section of unpolarized particles and **N** is the direction of the polarization of the beam.

As has been shown by Lakin,[6] a similar relation holds also for the elastic scattering of particles with spin 1.

In the present paper a phenomenological analysis of the polarization phenomena is carried out for the reactions

$$p + p \to d + \pi^+ \ (A), \qquad n + p \to d + \pi^0 \ (C),$$
$$\pi^+ + d \to p + p \ (B), \qquad \pi^0 + d \to n + p \ (D)$$

in order to find out from this example what consequences arise in virtue of the condition (1) for the more general case of binary reactions (nuclear reactions with two particles in the initial and in the final states).

The main result of this work is a generalization of the Wolfenstein theorem (2) to the case of the reactions here considered. Incidentally we obtain general expressions for the various observable quantities in reactions (A) and (B) in terms of the interaction amplitude, which may prove useful in determining it from experimental data.

THE S MATRIX

We denote the relative angular momenta in the initial and final states of reaction (B) by ℓ' and ℓ, and chose as the z axis the direction of the incident π mesons. Denoting, further, the Clebsch-Gordan coefficients by $C^{j_1 j_2}_{m_1 m_2}$, the four spin functions of the N — N system by ψ_M (M = + 1, 0, — 1 for the triplet, M = c for the singlet) and the spin functions of the deuteron by φ_m, we write the S matrix for the reaction (B) in the form

$$S_B = \sum_{m,\,M} \psi_M \left\{ \sum_{Jll'} (2l'+1)^{1/2} (l \,|\, S^B_J \,|\, l')\, C^{lsJ}_{m-M,M} C^{l'1J}_{0m} Y^{m-M}_{l}(\theta, \varphi) \right\} \varphi^+_m \tag{3}$$

(s is the spin of the system of nucleons). Then the S matrix for reaction (A) takes the form

$$S_A = \sum_{m,\,M} \varphi_m \left\{ \sum_{Jll'} (2l+1)^{1/2} (l' \,|\, S^A_J \,|\, l)\, C^{l'1J}_{M-m,\,m} C^{lsJ}_{0M} Y^{M-m}_{l'}(\theta, \varphi) \right\} \psi^+_M. \tag{4}$$

The condition (1) requires that $(\ell|S^B_J|\ell') = (\ell'|S^A_J|\ell) = (\ell'|S_J|\ell)$. (Here it is essential that the elements of the S matrix be defined to correspond with Ref. 3.)

If we confine ourselves to including the interactions with mesons in only s, p, and d states, then the laws of conservation of total angular momentum and of parity permit transitions $^3P_0 \to {}^1S_0$ (amplitude $(0|S_0|1) \equiv a_0$), $^3S_1 \to {}^3P_1$ (amplitude a_1), $^3P_2 \to {}^1D_2$ (a_2), $^3D_1 \to {}^3P_1$ (a_3), $^3D_2 \to {}^3P_2$ (a_4), $^3D_2 \to {}^3F_2$ (a_5), $^3D_3 \to {}^3F_3$ (a_6). In the further analysis we drop the transition $^3D_3 \to {}^3F_3$ with J = 3, keeping the transitions with J ≤ 2.* Then the S matrices of reactions (A) and (B) can be put in the form

$$\begin{aligned} 4\sqrt{4\pi}\, S_B = {} & \psi_c \{-\sqrt{2} a_0 (\mathbf{k}_0 \boldsymbol{\varphi}^+) + \sqrt{10} a_2 [3t(\mathbf{k}\boldsymbol{\varphi}^+) - (\mathbf{k}_0 \boldsymbol{\varphi}^+)]\} + i\sqrt{6} a_1 (\mathbf{k}[\boldsymbol{\psi}\boldsymbol{\varphi}^+]) \\ & + i\sqrt{3} a_3 \{3t(\mathbf{k}_0[\boldsymbol{\psi}\boldsymbol{\varphi}^+]) - (\mathbf{k}[\boldsymbol{\psi}\boldsymbol{\varphi}^+]) - 2[(\boldsymbol{\psi}\mathbf{n})(\mathbf{k}_0\boldsymbol{\varphi}^+) + (\boldsymbol{\psi}\mathbf{k}_0)(\mathbf{n}\boldsymbol{\varphi}^+)]\} + i\sqrt{5} a_4 \{3t(\mathbf{k}_0[\boldsymbol{\psi}\boldsymbol{\varphi}^+]) - (\mathbf{k}[\boldsymbol{\psi}\boldsymbol{\varphi}^+]) + [(\boldsymbol{\psi}\mathbf{n})(\mathbf{k}_0\boldsymbol{\varphi}^+) \\ & + (\boldsymbol{\psi}\mathbf{k}_0)(\mathbf{n}\boldsymbol{\varphi}^+)]\} + i\sqrt{10} a_5 \{(5t^2-1)(\mathbf{k}[\boldsymbol{\psi}\boldsymbol{\varphi}^+]) - 2t(\mathbf{k}_0[\boldsymbol{\psi}\boldsymbol{\varphi}^+]) + [(\boldsymbol{\psi}\mathbf{n})(\mathbf{k}_0\boldsymbol{\varphi}^+) + (\boldsymbol{\psi}\mathbf{k}_0)(\mathbf{n}\boldsymbol{\varphi}^+)] - 5t[(\boldsymbol{\psi}\mathbf{n})(\mathbf{k}\boldsymbol{\varphi}^+) + (\boldsymbol{\psi}\mathbf{k})(\mathbf{n}\boldsymbol{\varphi}^+)]\}; \end{aligned} \tag{3'}$$

$$\begin{aligned} 4\sqrt{4\pi}\, S_A = {} & \{-\sqrt{2} a_0 (\mathbf{k}\boldsymbol{\varphi}) + \sqrt{10} a_2 [3t(\mathbf{k}_0\boldsymbol{\varphi}) - (\mathbf{k}\boldsymbol{\varphi})]\} \psi_c^+ + i\sqrt{6} a_1 (\mathbf{k}_0[\boldsymbol{\varphi}\boldsymbol{\psi}^+]) + i\sqrt{3} a_3 \{3t(\mathbf{k}[\boldsymbol{\varphi}\boldsymbol{\psi}^+]) - (\mathbf{k}_0[\boldsymbol{\varphi}\boldsymbol{\psi}^+]) \\ & - 2[(\boldsymbol{\varphi}\mathbf{n})(\mathbf{k}\boldsymbol{\psi}^+) + (\boldsymbol{\varphi}\mathbf{k})(\mathbf{n}\boldsymbol{\psi}^+)]\} + i\sqrt{5} a_4 \{3t(\mathbf{k}[\boldsymbol{\varphi}\boldsymbol{\psi}^+]) - (\mathbf{k}_0[\boldsymbol{\varphi}\boldsymbol{\psi}^+]) + [(\boldsymbol{\varphi}\mathbf{n})(\mathbf{k}\boldsymbol{\psi}^+) + (\boldsymbol{\varphi}\mathbf{k})(\mathbf{n}\boldsymbol{\psi}^+)]\} \\ & + i\sqrt{10} a_5 \{(5t^2-1)(\mathbf{k}_0[\boldsymbol{\varphi}\boldsymbol{\psi}^+]) - 2t(\mathbf{k}[\boldsymbol{\varphi}\boldsymbol{\psi}^+]) + [(\mathbf{k}\boldsymbol{\varphi})(\mathbf{n}\boldsymbol{\varphi}^+) + (\mathbf{n}\boldsymbol{\varphi})(\mathbf{k}\boldsymbol{\psi}^+)] - 5t[(\mathbf{k}_0\boldsymbol{\varphi})(\mathbf{n}\boldsymbol{\psi}^+) + (\mathbf{n}\boldsymbol{\varphi})(\mathbf{k}_0\boldsymbol{\psi}^+)]\}, \end{aligned} \tag{4'}$$

where $\mathbf{k}_0$ and $\mathbf{k}$ are unit vectors giving the directions of the relative momenta in the initial and final states, $t = (\mathbf{k}_0\mathbf{k}) = \cos\theta$, $\mathbf{n} = [\mathbf{k}_0\mathbf{k}]$, and the vectors $\boldsymbol{\psi}$ and $\boldsymbol{\varphi}$ are constructed from the ψ_m and φ_m respectively (M ≠ c). The quantities $a_0, a_1, a_2, \ldots$ in Eqs. (3) and (4) differ from the corresponding quantities in Ref. 8 by the factor $(2\ell+1)^{1/2}$.

In the general case the S matrix for the reaction (A) can be put in the form

$$\begin{aligned} \sqrt{4\pi}\, S_A = {} & [A(\mathbf{k}\boldsymbol{\varphi}) + B(\mathbf{k}_0\boldsymbol{\varphi})]\psi_c^+ + i\{C[(\boldsymbol{\varphi}\mathbf{n})(\mathbf{k}\boldsymbol{\psi}^+) + (\boldsymbol{\varphi}\mathbf{k})(\mathbf{n}\boldsymbol{\psi}^+)] \\ & + D[(\boldsymbol{\varphi}\mathbf{n})(\mathbf{k}_0\boldsymbol{\psi}^+) + (\boldsymbol{\varphi}\mathbf{k}_0)(\mathbf{n}\boldsymbol{\psi}^+)] + E(\mathbf{k}[\boldsymbol{\varphi}\boldsymbol{\psi}^+]) + F(\mathbf{k}_0[\boldsymbol{\varphi}\boldsymbol{\psi}^+])\} \end{aligned} \tag{5}$$

(The $\sqrt{4\pi}$ and the factor i are retained for closer correspondence with Eq. (4)). In virtue of the condition (1) we get analogously for reaction (B)

$$\begin{aligned} \sqrt{4\pi}\, S_B = {} & \psi_c [A(\mathbf{k}_0\boldsymbol{\varphi}^+) + B(\mathbf{k}\boldsymbol{\varphi}^+)] + i\{C[(\mathbf{k}_0\boldsymbol{\psi})(\mathbf{n}\boldsymbol{\varphi}^+) + (\mathbf{n}\boldsymbol{\psi})(\mathbf{k}_0\boldsymbol{\psi}^+)] \\ & + D[(\mathbf{k}\boldsymbol{\psi})(\mathbf{n}\boldsymbol{\varphi}^+) + (\mathbf{n}\boldsymbol{\psi})(\mathbf{k}\boldsymbol{\varphi}^+)] + E(\mathbf{k}_0[\boldsymbol{\psi}\boldsymbol{\varphi}^+]) + F(\mathbf{k}[\boldsymbol{\psi}\boldsymbol{\varphi}^+])\}. \end{aligned} \tag{6}$$

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*Expansion in terms of the conserved quantity J seems more reasonable than expansion in terms of ℓ. The use of the latter can only lead to misunderstanding. For example, in the paper of Clementel and Villi,[7] where expansion in terms of ℓ is used for the analysis of the p — p scattering, it was erroneously found that the mixing coefficients occur in the expression for the integrated cross-section, which cannot be correct (cf. Ref. 2), and which happened because of the incomplete consideration of the states with different values of ℓ for a given J.

With the limitation to s, p, and d waves with $J \leq 2$:

$$2A = -(2a_0 + \sqrt{10}a_2);\ 2B = 3\sqrt{10}a_2 t;\ 4C = -2\sqrt{3}a_3 + \sqrt{5}a_4 + \sqrt{10}a_5;$$
$$4D = -5\sqrt{10}a_5 t;\ 4E = (3\sqrt{3}a_3 + 3\sqrt{5}a_4 - 2\sqrt{10}a_5)\,t;\ 4F = \sqrt{6}a_1 - \sqrt{3}a_3 - \sqrt{5}a_4 + \sqrt{10}a_5(5t^2 - 1). \tag{7}$$

In the general case

$$A(-t) = A(t);\ B(-t) = -B(t);\ C(-t) = C(t);\ D(-t) = -D(t);\ E(-t) = -E(t);\ F(-t) = F(t). \tag{7'}$$

REACTION CROSS-SECTION OF UNPOLARIZED PARTICLES

The equality of the reaction cross-sections for unpolarized particles

$$4I^{(A)}(\theta) = 3I_0^{(B)}(\theta),$$

follows directly from Eqs. (5) and (6), and (with the ratio of the squares of the relative momenta omitted)

$$16\pi I_0^{(A)}(\theta) = 12\pi I_0^{(B)}(\theta) = |A|^2 + |B|^2 + 2\,\mathrm{Re}\,[(A^+B) + 2(E^+F)]\cos\theta$$
$$+ 2|E|^2 + 2|F|^2 + 2\sin^2\theta\,[|C|^2 + |D|^2 + \cos\theta\cdot 2\mathrm{Re}\,(C^+D)]. \tag{8}$$

With the limitation to s, p, and d waves with $J \leq 2$ we get from Eqs. (7) and (8)

$$64\pi I_0^{(A)}(\theta) = 48\pi I_0^{(A)}(\theta) = 4\rho_0^2 + 12\rho_1^2 + 10\rho_2^2(1 + 3\cos^2\theta) + [6(\sqrt{2}\omega_{13} + \sqrt{10}\omega_{14} + \sqrt{5}\omega_{34}) - 4\sqrt{10}\omega_{02} + 4\sqrt{15}\omega_{15}$$
$$+ 2\sqrt{30}\omega_{35}](3\cos^2\theta - 1) + 3[\rho_3^2(5 - 3\cos^2\theta) + 5\rho_4^2(1 + \cos^2\theta)] + 10\rho_5^2(1 + 6\cos^2\theta - 5\cos^4\theta)$$
$$+ 10\sqrt{6}\omega_{45}(10\cos^4\theta - 9\cos^2\theta + 1), \quad a_m = \rho_m e^{i\alpha_m}; \quad \omega_{mn} = \rho_m\rho_n\cos(\alpha_m - \alpha_n), \tag{8'}$$

which, apart from differences of notation, agrees with results previously obtained.[8,9]

Before going on to the comparison of the polarization phenomena in reactions (A), we note that the study of reaction (B) at total meson energy E_π provides a possibility of obtaining information about the reaction (A) at a (kinetic) energy W of the protons given by

$$W = \frac{M_d}{M}E_\pi + \frac{\mu}{M}\mu c^2 - \varepsilon\left(\frac{M_d}{M} + 1\right) \approx 2E_\pi \tag{9}$$

(M_i, M, μ are the masses of the deuteron, nucleon, and π meson, and ϵ is the binding energy of the deuteron).

From Eq. (9) it follows, for example, that by the use of a π^+ meson beam with energy E_π = 320 + 140 = 460 Mev (from the 680 Mev accelerator at the Joint Institute for Nuclear Research) the study of reaction (B) gives a possibility of getting information about the formation of mesons in reaction (A) at proton energies about 920 Mev, which considerably exceeds the energy of the particles in the accelerator. Such a "gain of energy" is due to the fact that in reaction (A) a considerable part of the whole energy is expanded in motion of the center of mass of the system, whereas in the inverse reaction the energy in the center-of-mass system does not differ by much from the value of the energy in the laboratory system of coordinates.

POLARIZATION PHENOMENA IN THE DIRECT AND INVERSE REACTIONS

1. We now examine the information obtainable from a study of the polarization phenomena. Because of isotopic invariance we can consider all at once the general process N + N ⇄ d + π, where N is a nucleon and π is a π^+ or π^0 meson. The cross-section for meson production by polarized protons in reaction (A) can be put in the form

$$I^{(A)}(\theta, \varphi) = I_0^{(A)}(\theta) + p\cos\varphi P(\theta) = I^{(A)}(\theta) + pI_p \tag{10}$$

(p is the polarization of the nucleon beam, with the direction of polarization taken along the y axis).

Using the formula

$$(\hat{\sigma}_1\mathbf{p})\,S = S_c(\boldsymbol{\psi}\mathbf{p}) + (\mathbf{S}\mathbf{p})\,\psi_c + i([\mathbf{p}\boldsymbol{\psi}]\,\mathbf{S}),\ \ S = S_c\psi_c + (\mathbf{S}\boldsymbol{\psi});$$

we get

$$16\pi I_{p_1}(\theta)\cos\varphi = 4\pi\,\mathrm{Sp}\,S_A\,(\hat{\sigma}_1\mathbf{p})\,S_A^+ = (\mathbf{np})\,P_1(\theta);$$
$$P_1(\theta) = 2\,\mathrm{Im}\,\{[C^+(A + B\cos\theta) + D^+(A\cos\theta + B) + (B^+E) + (AF^+)]$$
$$+ [\sin^2\theta\,(CD^+) + (F^+E) + C(E + F\cos\theta)^+ + D(E\cos\theta + F)^+]\}. \tag{10'}$$

In the expression for $P_1(\theta)$ the first four terms, which give the contribution of singlet-triplet transitions in reaction (A), do not change sign with the replacement $\theta \to \pi - \theta$, i.e., they are symmetric with respect to 90°; the remaining terms, which give the contribution of triplet-triplet transitions, change sign with the replacement $\theta \to \pi - \theta$, i.e., they vanish at $\theta = 90°$.

From a comparison of the formula

$$(\hat{\sigma}_2 \mathbf{p}) S = -[S_c(\hat{\psi}\mathbf{p}) + (S\mathbf{p})\psi_c] + i([\mathbf{p}\psi] S)$$

with the one we had before for $(\hat{\sigma}_1 \mathbf{p})\mathbf{S}$, it can be seen that the expression for the azimuthal asymmetry, when the target is polarized, differs from (10′) by the sign of the first four terms

$$16\pi I_{p_2}(\theta)\cos\varphi = 4\pi \operatorname{Sp} S_A (\hat{\sigma}_2\mathbf{p}) S_A^+ = (\mathbf{np}) P_1'(\theta);$$
$$P_1'(\theta) = 2\operatorname{Im}\{-[C^+(A + B\cos\theta) + D^+(A\cos\theta + B) + (B^+E) + (AF^+)] + [\sin^2\theta(CD^+) + (F^+E) + C(E + F\cos\theta)^+ + D(E\cos\theta + F)^+]\}. \quad (10'')$$

On the other hand, for the polarization of the nucleons in reaction (B), when the deuteron target is unpolarized, we get from Eq. (6)

$$12\pi I_0^{(B)}(\theta)\langle\hat{\sigma}_1\rangle_f = 4\pi \operatorname{Sp} S_B^+ \hat{\sigma}_1 S_B = \mathbf{n} P_1(\theta); \quad (11)$$

$P_1(\theta)$ is given by Eq. (10′). For the polarization of the other nucleon we get

$$12\pi I_0^{(B)}(\theta)\langle\hat{\sigma}_2\rangle_f = \mathbf{n} P_1'(\theta), \quad (11')$$

$P_1'(\theta)$ is given by Eq. (10″). From Eqs. (7′), (10′), (10″), (11′) and (8):

$$\langle\hat{\sigma}_1(\theta, \varphi)\rangle = \langle\hat{\sigma}_2(\pi - \theta, \pi + \varphi)\rangle;$$

and from Eqs. (10) and (11) there follows the validity of the equation:

$$4 I_p^{(A)} = 3 I_0^{(B)}(\theta)\langle\hat{\sigma}\rangle_f N \quad (12)$$

(the ratio of the momenta is omitted).

Thus a knowledge of the polarization of the nucleons in reaction (B) gives just the same information as the cross-section for the production of mesons by polarized nucleons in reaction (A).

Additional information can be obtained by studying the polarization of the deuterons produced in reaction (A). If in dealing with the interaction of unpolarized particles we limit ourselves to just the polarization vector, then for it we get by means of Eq. (4′) and the formula $\hat{\mathbf{S}}(S_m\varphi_m) = i[\mathbf{S}\varphi]$ the result

$$16\pi I_0^{(A)}(\theta)\langle\hat{S}\rangle_f = \mathbf{n} P_2(\theta); \quad P_2(\theta) = 2\operatorname{Im}\{(AB^+) + \sin^2\theta(D^+C) + (C + F)^+E + (D^+E) + (C^+F + D^+E)\cos\theta\}. \quad (13)$$

With the limitation to waves with $J \leq 2$ the angular dependence takes the form

$$2\sin\theta P_2(\theta) = \sin\theta\cos\theta\{6\sqrt{10}\,\varepsilon_{02} + 9\sqrt{2}\,\varepsilon_{13} + 3\sqrt{10}\,\varepsilon_{14} + 2\sqrt{15}\,\varepsilon_{15} - 6\sqrt{5}\,\varepsilon_{34} - 2\sqrt{30}\,\varepsilon_{35}\},$$

where $\epsilon_{mn} = \rho_m\rho_n \sin(\alpha_m - \alpha_n)$. And in the general case, as can be seen from Eq. (7′), there remains a proportionality to the product of $\sin 2\theta$ by an even function of $\cos\theta$.

Let us now consider the expression for the cross-section of reaction (B) on a polarized target. On the assumption that the target is polarized in such a way that only the average value of the spin vector of the deuteron is different from zero, the cross-section can be put in the form (10). For $\langle\mathbf{S}\rangle \neq 0$ the averages $\langle D_{ik}\rangle$ are also different from zero, but we consider only the contribution of the term proportional to $\langle\mathbf{S}\rangle$.) Then from Eq. (6) we have

$$3 I_p^{(B)} = 12\pi \operatorname{Sp} S_B(\hat{S}\mathbf{p}) S_B^+ = (\mathbf{np}) P_2(\theta). \quad (14)$$

From a comparison of Eqs. (13) and (14) we get still another consequence of invariance under time reversal

$$3 I_p^{(B)} = 4 I_0^{(A)}\langle\hat{S}\rangle_f \mathbf{N}. \quad (15)$$

The equations (12) and (15) are a generalization of the relation (2) to the case of the binary reaction under consideration.

2. As is well known,[4,6] the state of polarization of a deuteron is characterized not only by the average value of the spin vector $\hat{\mathbf{S}}$ of the deuteron but also by a tensor of the second rank which can be constructed from the spin vector. In the literature one finds the tensor

$$\hat{D}_{ik} = \frac{1}{2}(\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i) - \frac{2}{3}\delta_{ik} = \hat{T}_{ik} - \frac{2}{3}\delta_{ik},$$

introduced by Dalitz, and also the tensor $T_{2,m}$ with the components

$$2T_{2,\pm 2} = \sqrt{3}\{(\hat{S}_x^2 - \hat{S}_y^2) \pm i(\hat{S}_x\hat{S}_y + \hat{S}_y\hat{S}_x)\}, \quad 2T_{2,\pm 1} = \mp\sqrt{3}\{(\hat{S}_x\hat{S}_z + \hat{S}_z\hat{S}_x) \pm i(\hat{S}_z\hat{S}_y + \hat{S}_y\hat{S}_z)\}, \tag{16}$$
$$\sqrt{2}T_{2,0} = 3\hat{S}_z^2 - 2.$$

For the elastic scattering of particles with spin 1, as follows from the results of Cheishvili[10] and as has been shown by Lakin,[6] relations of the type (2) hold not only between the average value of the polarization vector and the corresponding term in the cross-section, but also for the tensor terms. The correspondence of the vector terms for reactions (A) and (B) has been established above. Let us now make the comparison, for our binary reaction, between the expression for the polarization tensor of the deuteron in reaction (A), for the interaction of unpolarized protons, and the cross-section of reaction (B) for an arbitrary polarized deuterium target.

From a calculation using the formula

$$[\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i](S_m\varphi_m) = 2\delta_{ik}(S_m\varphi_m) - [S_i\varphi_k + S_k\varphi_i]$$

we have for the average values of the tensor $\hat{D}_{ik}$

$$\begin{aligned}16\pi I_0^{(A)}(\theta)\{{}^1/_3\,\delta_{ik} - \langle\hat{D}_{ik}\rangle\} &= \delta_{ik}\{|E|^2 + |F|^2 + \cos\theta\cdot 2\operatorname{Re}(E^+F)\} + k_ik_k\{|A|^2 + \sin^2\theta\cdot|C|^2 - |E|^2\\ &+ 2\operatorname{Re}[C^+(E\cos\theta + F)]\} + k_{0i}k_{0k}\{|B|^2 + |D|^2\sin^2\theta - |F|^2 - 2\operatorname{Re}[D^+(E + F\cos\theta)]\} + n_in_k\{|C|^2\\ &+ |D|^2 + 2\operatorname{Re}[C^+(D\cos\theta - F) + D^+E]\} + (k_{0i}k_k + k_ik_{0k})\operatorname{Re}\{(AB^+) + (C^+D)\sin^2\theta - (EF^+) - [C^+(E + F\cos\theta)]\\ &+ [D^+(E\cos\theta + F)]\} = D_1(\theta)\delta_{ik} + D_2(\theta)k_ik_k + D_3(\theta)k_{0i}k_{0k} + D_4(\theta)n_in_k + D_5(\theta)(k_{0i}k_k + k_{0k}k_i).\end{aligned} \tag{17}$$

From eqs. (17) and (7′) there follow for $D_1, D_2, \ldots$ as functions of $t = \cos\theta$ the symmetry properties

$$D_1(-t) = D_1(t); \quad D_2(-t) = D_2(t); \quad D_3(-t) = D_3(t); \quad D_4(-t) = D_4(t); \quad D_5(-t) = -D_5(t). \tag{17′}$$

It can be verified without difficulty that the condition (1) has as a consequence that the expression for the corresponding term in the cross-section differs from (17) by the replacement $D_2 \rightleftarrows D_3$. Consequently, a study of all the components of the polarization tensor in reaction (A) gives the same information as the study of the cross-section of reaction (B) for a polarized deuterium target.

As can be seen from Eq. (17), for certain contractions of the polarization tenson, for example for $\langle\hat{D}_{ik}\rangle n_in_k$, a relation of the type of Eqs. (12) and (15) follows from the requirement of symmetry with respect to change of the sign of the time.

The expressions for $\langle T_{2,m}\rangle$ are found from Eq. (17), if we use the relations following from Eq. (16),

$$2T_{2,\pm 2} = \sqrt{3}\{(\hat{T}_{xx} - \hat{T}_{yy}) \pm 2i\hat{T}_{xy}\} = \sqrt{3}\{(\hat{D}_{xx} - \hat{D}_{yy}) \pm 2i\hat{D}_{xy}\}, \quad T_{2,\pm 1} = \mp\sqrt{3}(\hat{T}_{xy} \pm i\hat{T}_{yz}) = \mp\sqrt{3}(\hat{D}_{xy} \pm i\hat{D}_{yz}),$$
$$\sqrt{2}T_{2,0} = 3\hat{T}_{zz} - 2 = \hat{D}_{zz}. \tag{16′}$$

The availability of polarized proton beams makes it possible to study the polarization of the deuterons in reaction (A) under the action of polarized protons. We have

$$\langle\hat{D}_{ik}\rangle = {}^1/_4\operatorname{Sp}[S_A(1 + \mathbf{p}\hat{\sigma}_1)S_A^+\hat{D}_{ik}] / {}^1/_4\operatorname{Sp}[S_A(1 + \mathbf{p}\hat{\sigma}_1)S_A^+], \tag{18}$$

where the denominator contains the expression for the cross-section of reaction (A), $I^{(A)}(\theta, \varphi)$, as given in Eqs. (8) and (10). The polarization of the incident protons is denoted by **p**; the expression for the interaction of unpolarized particles is obtained from Eq. (18) for $p = 0$. Thus for polarized protons one needs only to consider the changes arising on account of the second term in the numerator, and to replace $I_0(\theta)$ by $I(\theta, \varphi)$ in the expressions given previously.

For the added term in the polarization vector of the deuteron we get:

$$\begin{aligned}4\pi\operatorname{Sp}(\hat{\sigma}_1\mathbf{p})S_A^+\hat{S}S_A &= \mathbf{n}(\mathbf{np})2\operatorname{Re}[(BC^+) - (AD^+)] + [\mathbf{np}]2\operatorname{Re}\{C^+(E + F\cos\theta) + D^+(F + E\cos\theta)\}\\ &+ \mathbf{p}\cdot 2\operatorname{Re}\{A^+(E + F\cos\theta) + B^+(E\cos\theta + F)\} - \mathbf{k}\cdot 2\operatorname{Re}\{[(\mathbf{pk})(A + E) + (\mathbf{pk}_0)(B + F)]E^+\}\\ &- \mathbf{k}_0\cdot 2\operatorname{Re}\{[(\mathbf{pk})(A + E) + (\mathbf{pk}_0)(B + F)]F^+\} + \mathbf{l}\cdot 2\operatorname{Re}\{(\mathbf{pk})(BC^+) + (\mathbf{pk}_0)(BD^+) + (\mathbf{pl})|D|^2\\ &+ (\mathbf{pm})(C^+D)\} + \mathbf{m}\cdot 2\operatorname{Re}\{(\mathbf{pk})(AC^+) + (\mathbf{pk}_0)(AD^+) + (\mathbf{pm})|C|^2 + (\mathbf{pl})(C^+D)\},\end{aligned} \tag{19}$$

where $\mathbf{m} = [\mathbf{kn}]$; $\boldsymbol{\ell} = [\mathbf{k}_0\mathbf{n}]$.

Analogously we have for the polarization of the protons in reaction (B) with a polarized deuterium target (only $\langle\hat{\mathbf{S}}\rangle \neq 0$)

$$4\pi\,\mathrm{Sp}\,\hat{S}S_B^+(\hat{\sigma}_1\mathbf{p})S_B = \mathbf{n}(\mathbf{np})\,2\,\mathrm{Re}\,[(B^+C)-(A^+D)] + \mathbf{p}\,2\,\mathrm{Re}\,\{[A^+(E+F\cos\theta)]+[B^+(F+E\cos\theta)]\}$$
$$-[\mathbf{np}]\,2\,\mathrm{Re}\,\{[C^+(E+F\cos\theta)]+[D^+(F+E\cos\theta)]\} + \mathbf{l}2\,\mathrm{Re}\,\{(\mathbf{pm})(C^+D)+|C|^2(\mathbf{pl})-(A^+D)(\mathbf{pk})-(\mathbf{pk}_0)(A^+C)\}$$
$$+\mathbf{m}2\,\mathrm{Re}\,\{(\mathbf{pl})(C^+D)+|D|^2(\mathbf{pm})-(B^+C)(\mathbf{pk}_0)-(B^+D)(\mathbf{pk})\} - \mathbf{k}2\,\mathrm{Re}\,\{F^+[(\mathbf{pk}_0)(A+E)+(\mathbf{pk})(B+F)\} \quad (20)$$
$$-\mathbf{k}_0 2\,\mathrm{Re}\,\{E^+[(\mathbf{pk}_0)(A+E)+(\mathbf{pk})(B+F)]\},$$

from which the correspondence with Eq. (19) can be seen. The analogous correspondence also remains valid for the tensor terms. Therefore we shall present only the expressions for the additions to the polarization of the deuterons in reaction (A). From Eqs. (5) and (18) we have

$$4\pi\{{}^1/_3\,\delta_{ik}\,\mathrm{Sp}\,(\hat{\sigma}_1\mathbf{p})S_A^+S_A - \mathrm{Sp}\,(\hat{\sigma}_1\mathbf{p})S_A^+\hat{D}_{ik}S_A\} = (\mathbf{np})\,\mathrm{Im}\,\{2\,[n_in_k(CD^+)+k_ik_k(AC^+)+k_{0i}k_{0k}(BD^+)]+(k_{0i}k_k+k_{0k}k_i)[(AD^+)$$
$$+(BC^+)]\} + (n_ik_k+n_kk_i)\,\mathrm{Im}\,\{(\mathbf{pk})[C^+(A+2E)]+(\mathbf{pk}_0)[(AD^+)+2(C^+F)]+(\mathbf{pl})(C^+D)\} + (n_ik_{0k}+n_kk_{0i})\,\mathrm{Im}\,\{(\mathbf{pk})[(BC^+)$$
$$+2(D^+E)]+(\mathbf{pk}_0)[D^+(B+2F)]+(\mathbf{pm})(C^+D)\} + (p_in_k+n_ip_k)\,\mathrm{Im}\,\{[C(E+F\cos\theta)^+]+D(E\cos\theta+F)^+ + (EF^+)\}$$
$$+\mathrm{Im}\,(AE^+)(q_ik_k+q_kk_i)+\mathrm{Im}\,(AF^+)(q_{0i}k_k+k_iq_{0k})+\mathrm{Im}\,(BE^+)(q_ik_{0k}+k_{0i}q_k)+\mathrm{Im}\,(BF^+)(q_{0i}k_{0k}+q_{0k}k_{0i}), \quad (21)$$

where

$$\mathbf{q}_0 = [\mathbf{pk}_0], \quad \mathbf{q} = [\mathbf{pk}].$$

The expression for the term proportional to δ_{ik} is the same as Eq. (10′).

3. As a final example we shall compare the expressions for the correlation of the polarizations of the protons (nucleons) in reaction (B), when the target is polarized, and for the changes in the cross-section for production of mesons by reaction (A) when both the beam and the target are polarized. By means of the formula

$$(\hat{\sigma}_2\mathbf{b})\psi_c = -(\mathbf{b}\boldsymbol{\psi}); \quad (\hat{\sigma}_2\mathbf{b})(\boldsymbol{\psi}\mathbf{p}) = -(\mathbf{pb})\psi_c + i(\boldsymbol{\psi}[\mathbf{pb}])$$

it is not hard to get for the correlation of the polarizations

$$12\pi I_0^{(B)}(\theta)\langle(\hat{\sigma}_2\mathbf{b})(\hat{\sigma}_1\mathbf{p})\rangle = 4\pi\,\mathrm{Sp}\,S_B^+(\hat{\sigma}_2\mathbf{b})(\hat{\sigma}_1\mathbf{p})S_B = (\mathbf{bp})\{12\pi I_0^{(B)}(\theta) - 2[|A|^2+|B|^2+2\,\mathrm{Re}\,(A^+B)\cos\theta+|E|^2+|F|^2$$
$$-2\,\mathrm{Re}\,(EF^+)\cos\theta]\} + (\mathbf{n}[\mathbf{pb}])\,2\,\mathrm{Re}\,\{[A^+(C+F+D\cos\theta)]+[B^+(C\cos\theta+D-E)]\} - (\mathbf{np})(\mathbf{nb})\{|C|^2+|D|^2$$
$$+4\,\mathrm{Re}\,[C^+(D+F)-(D^+E)]\} - 2(\mathbf{k}_0\mathbf{b})(\mathbf{k}_0\mathbf{p})\{|C|^2\sin^2\theta-|E|^2\} - 2(\mathbf{kb})(\mathbf{kp})\{|D|^2\sin^2\theta-|F|^2\}+[(\mathbf{kb})(\mathbf{k}_0\mathbf{p}) \quad (22)$$
$$+(\mathbf{kp})(\mathbf{k}_0\mathbf{b})]\,2\,\mathrm{Re}\,[(E^+F)-(C^+D)\sin^2\theta] + [(\mathbf{pk}_0)(\mathbf{bl})+(\mathbf{bk}_0)(\mathbf{pl})]\,2\,\mathrm{Re}\,(C^+E) + [(\mathbf{pk}_0)(\mathbf{bm})+(\mathbf{bk}_0)(\mathbf{pm})]\,2\,\mathrm{Re}\,(C^+F)$$
$$+[(\mathbf{pk})(\mathbf{bl})+(\mathbf{bk})(\mathbf{pl})]\,2\,\mathrm{Re}\,(D^+E)+[(\mathbf{pk})(\mathbf{bm})+(\mathbf{bk})(\mathbf{pm})]\,2\,\mathrm{Re}\,(D^+F),$$

where

$$S_B = S_c\psi_c + (\mathbf{S}\boldsymbol{\psi}).$$

The expression for $I_{pp}(\theta, \varphi)$ is obtained from Eq. (22) by the interchange $\mathbf{k} \rightleftarrows \mathbf{k}_0$. Thus, just as for the other tensor quantities, it follows from the condition (1) that the study of all the components of the two quantities gives identical information, and for certain contractions, for example for $\langle(\hat{\sigma}_2\mathbf{n})(\hat{\sigma}_1\mathbf{n})\rangle$, a relation of the type of Eq. (12) holds.

From a comparison of Eq. (22) with Eq. (17) it follows that the study of the correlation of the polarizations of the protons in reaction (B) gives even more extensive information that the study of the polarization of the deuterons in reaction (A).

All results so far obtained for reactions (A) and (B) transfer directly to the reactions

$$n + p \rightleftarrows d + \pi^0. \quad (\mathrm{C-D})$$

Furthermore, isotopic invariance provides the possibility of setting up a connection between properties of the cross-sections of reactions (A — B) and (C — D), since, as is well known,

$$I(p + p \rightleftarrows d + \pi^+) = 2I(n + p \rightleftarrows d + \pi^0). \quad (23)$$

This relation is valid for the reaction cross-sections of both unpolarized[11] and polarized[10] particles. For the case of interaction of polarized particles one can prove additional relations. The primary relation of this sort is of the form

$$\langle\hat{\sigma}\rangle_f I_0^{(A)}(\theta) = 2\langle\hat{\sigma}\rangle_f I_0^{(D)}(\theta), \quad (24)$$

the existence of which follows from Eqs. (23), (10), and (1). From Eq. (24), in virtue of the validity of Eq. (23) for the interaction of unpolarized particles, there follows the equality of the polarizations of the corresponding nucleons in the reactions $d + \pi^0 \to n + p$ and $d + \pi^+ \to p + p$. Similar equalities hold for the polarization of the deuterons and the correlation of the polarizations of the nucleons.

DISCUSSION

The existence of the condition (1) makes it evident from the very beginning that the total information obtainable from the study of the direct reaction coincides with the total information obtainable from the study of the inverse reaction. The results of the present paper show that for the binary reaction considered there appear, in virtue of the condition (1), relations of the types of Eqs. (12) and (15) between the azimuthal asymmetry and the polarization, similar to the relation between the unpolarized cross-sections. On consideration of the tensor terms it turns out that for some contractions, for ecample $\langle \hat{D}_{ik} \rangle n_i n_k$, relations of the type of Eq. (12) remain valid. Taken as a whole, the study of all the components of the tensor quantities in reactions (A) and (B) gives identical sets of information.

If in the expressions (5) and (6) one sets C = D = E = F = 0, leaving only the quantities A and B different from zero, the resulting formulas will relate to a process in which the interaction of two spinless particles results in the formation of a spinless particle of the opposite parity and a particle with spin 1. As can be seen from Eq. (13), a relation of the type (15) remains valid in this case also. The expression for the tensor $\langle \hat{D}_{ik} \rangle$ takes the form

$$4\pi \{{}^1\!/_3\delta_{ik} - \langle \hat{D}_{ik} \rangle\} I_0(\theta) = |A|^2 k_i k_k + |B|^2 k_{0i} k_{0k} + \mathrm{Re}(AB^+)(k_{0i} k_k + k_{0k} k_i).$$

The opposite case with A = B = 0 and C, D, E, and F different from zero corresponds to the conversion of a scalar particle into a pseudoscalar particle in a reaction with a particle of spin 1. The symmetry properties (7′) do not hold in these cases.

The present case of a binary reaction with the spins of the particles different in the initial and final states is a rather general one. The generalization of Wolfenstein's theorem to the case of binary reactions in which the spins of the particles do not change (and the intrinsic parities of the particles are unchanged) is carried through without difficulty. For the simplest case, in which the values of the spins in the initial and final states are 0 and 1/2, the result follows at once from the fact that the transition amplitude can be put in the form

$$M = a(\theta) + b(\theta)(\hat{\sigma}\mathbf{n}),$$

if the intrinsic parities of the spinless particles in the initial and final states are the same, and

$$M = b_1(\theta)(\hat{\sigma}\mathbf{k}_1) + b_2(\theta)(\hat{\sigma}\mathbf{k}_2)$$

($\mathbf{k}_1$ and $\mathbf{k}_2$ are relative momenta) when, for example, in interacting with a nucleon a scalar particle turns into a pseudoscalar particle.

For the case in which the spins of the particles are $s_1 = s_2 = 1/2$, for example for the reaction

$$p + \mathrm{T} \rightleftarrows n + \mathrm{He}_2^3, \tag{E}$$

the validity of the consequences of invariance under time reversal that have been established for the elastic scattering of nucleons by nucleons will be obvious if we show that in this case also the expression for the amplitude M does not differ from the case of elastic scattering. This last assertion can be proved either by requiring, instead of the invariance of M, the invariance of the matrix MM^+, or by considerations like those of Wright[13] on the possible transitions in the reaction (E). Either of these procedures makes it possible to transfer the consequences of the time reversal from the elastic scattering to the case of binary reactions in which there is no change of the spins (and parities) of the particles. For binary reactions different from elastic scattering, in which the particles in the initial state are different from those in the final state, there is a peculiar "doubling" of the relations of the type of Eq. (2).

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Note added in proof (June 29, 1957). While this paper was in press the writer learned that Baz' [(J. Exptl. Theoret. Phys. 32, 628 (1957), Soviet Phys. JETP 5, 521 (1957)] has generalized Wolfenstein's theorem to the case of a reaction of the type of (E) by another method.

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30

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ELECTRON DENSITY FLUCTUATIONS AND THE SCATTERING OF RADIO WAVES IN THE IONOSPHERE

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A description is given of results from a determination of electron density fluctuations obtained by measuring the "turbidity coefficient" of the ionosphere and the energy scattered at very high frequencies. The linear dimensions ξ of inhomogeneities at $\sim$ 80 km, which are effective in uhf scattering, were also determined. The formulas employed were based on an expression for the scattering cross section σ which was obtained with the auto correlation coefficient $\rho(r) \sim \exp\{-(r/\xi)^2\}$. The author concludes that when the ionosphere is sounded at frequencies below the critical frequency, the received signal comprises in addition to the "specularly" reflected wave also waves which are principally scattered forward and latter reflected at higher ionospheric levels. In oblique distant uhf transmission through the ionosphere scattering from inhomogeneities of optimum size makes the largest contribution.

1. INTRODUCTION

It is known that one of the principal characteristics of the "calm" unperturbed ionosphere is its "statistical inhomogeneity," the mechanisms of which are still unknown.[1,2] It has been argued that turbulence and in some instances plasma oscillations and waves participate in these processes. However, all such discussions are of a very tentative nature since the theory of the phenomena is still relatively undeveloped. There is also very little reliable experimental information available to serve as a basis for any theoretical model.

Up to the present time the following parameters have been determined experimentally:

1. The ranges of linear dimensions ξ_s of the inhomogeneities, principally at altitudes $z \approx 100-120$ km and $\approx 250-350$ km. From experiments with vertical sounding of the ionosphere the most frequently encountered values are $\xi_0 \sim 200-300$ m.

2. The ranges of random velocities v_s of the inhomogeneities; the values $v_0 \sim 1-3$ m/sec have been obtained.