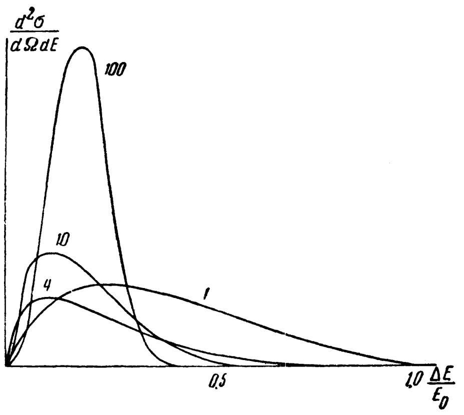

Energy distribution of neutrons scattered by a crystal over an angle of 90°. The numbers indicate the value of E/Θ. The scale of the curve for E/Θ = 1 is increased 10 times with respect to the other curves

a free nucleus the energy loss is simply by the angle of scattering from the laws of conservation of energy and momentum. For E/Θ = 100 the width of the curve is still fairly large. In a real crystal the approach to a δ-function will, apparently, occur fast owing to the possibility of knocking a nucleus out of the lattice, a process not considered in our paper.

The calculations, using Eq. (34), can be simplified by using the circumstances that for $x \lesssim 10$ the series (34) converges very rapidly, while for $x \gtrsim 10$ the asymptotic formula

$$\log f(x) = 0.8207 \cdot x^{1/3} + \frac{1}{6}\log x - 0.5875 - 0.1028\, x^{-1/3}. \qquad (46)$$

is correct with a large degree of accuracy.

In conclusion the author expresses his gratitude to A. S. Davydov and V. M. Agranovich for valuable discussions and suggestions.

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QUANTUM KINETIC EQUATION FOR PLASMA WITH ACCOUNT OF CORRELATION

IU. L. KLIMONTOVICH and S. V. TEMKO

Moscow State University

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A quantum kinetic equation has been obtained for a system of particles with Coulomb interaction. This equation differs from the known quantum kinetic equation by the fact that correlation of the mutual positions of charged particles has been taken into account.

A quantum kinetic equation for a set of interacting particles can be obtained by solution of the system of equations for the quantum distribution functions f_1 and f_2.[1,2] In this case the function f_3 which enters the equation for the distribution function f_2, is expressed approximately in terms of the functions f_1 and f_2. For a solution of this system of equations, quantum conditions for the vanishing of correlation at infinity are necessary. However, as was noted by N. N. Bogoliubov, the solution of the equation for the density matrix (or, correspondingly, for the quantum distribution function) can be reduced to a solution of the equation for the quantum function F_s with classical boundary conditions. Here $f_s = \gamma_s F_s$, γ_s is the symmetrization operator for s particles. In the case of systems with central interaction, the equations for F_1 and F_2 have the following form:

$$\frac{\partial F_1}{\partial t}+\frac{p}{m}\frac{\partial F_1}{\partial q}=\frac{ni}{(2\pi)^9\hbar}\int[\Phi(|q-q'-\tfrac{1}{2}\hbar(\tau-\tau')|)-\Phi(|q-q'+\tfrac{1}{2}\hbar(\tau-\tau')|)]\exp\{i\tau(\eta-p)$$
$$+i\tau'(\eta'-p')\}(1+P_{1,2})F_2(q,q',\eta,\eta';t)\,d\tau\,d\tau'\,d\eta\,d\eta'\,dq'\,dp'; \qquad (1)$$

$$\frac{\partial F_2}{\partial t}+\frac{p}{m}\frac{\partial F_2}{\partial q_1}+\frac{p_1}{m}\frac{\partial F_2}{\partial q_2}=\frac{i}{(2\pi)^6\hbar}\int\Big[\Phi\Big(|q_1-q_2-\tfrac{1}{2}\hbar(\tau_1-\tau_2)|\Big)-\Phi\Big(|q_1-q_2+\tfrac{1}{2}\hbar(\tau_1-\tau_2)|\Big)\Big]\exp\{i\tau_1(\eta_1-p_1)$$
$$+i\tau_2(\eta_2-p_2)\}F_2(q_1,q_2,\eta_1,\eta_2;t)\,d\tau_1\,d\tau_2\,d\eta_1\,d\eta_2+\frac{ni}{(2\pi)^6\hbar^4}\int\Big[\Phi\Big(|q_1-q'-\tfrac{1}{2}\hbar\tau_1|\Big)-\Phi\Big(|q_1-q'+\tfrac{1}{2}\hbar\tau_1|\Big)\Big]e^{i\tau_1(\eta_1-p_1)}$$
$$\times(1+P_{1,3}+P_{2,3})\,F_3(q_1,q_2,q',\eta_1,p_2,p';t)\,d\tau_1 d\eta_1 dq' dp'+\frac{ni}{(2\pi)^6\hbar^4}\int\Big[\Phi\Big(|q_2-q'-\tfrac{1}{2}\hbar\tau_2|\Big)$$
$$-\Phi\Big(|q_2-q'+\tfrac{1}{2}\hbar\tau_2|\Big)\Big]e^{i\tau_2(\eta_2-p_2)}(1+P_{1,3}+P_{2,3})F_3(q_1,q_2,q_1',p_1,\eta_2,p';t)\,d\tau_2 d\eta_2 dq' dp'. \qquad (2)$$

Here Φ is the energy of interaction of a pair of particles, n is the number of particles per unit volume, and P_{ik} is the permutation operator for a pair of particles. Limiting ourselves to consideration of pair correlation, we can write

$$F_2(q_1,q_2,p_1,p_2;\ t)=F_1(q,p_1;\ t)\,F_1(q_2,p_2;\ t)+g(q_1,q_2,p_1,p_2;\ t) \qquad (3)$$

and a similar expression for F_3. The function g is small in comparison with the first term for weak interactions.

In the case of short range forces or for a region in which we can restrict ourselves to pair interaction, we obtain a kinetic equation for the quantum distribution function F_1 corresponding to the equation for the density matrix which was obtained by Bogoliubov and Gurov.[3] Here F_3 is expressed only in terms of the derivative of the function F_1.

We have considered Eqs. (1) and (2) for a system of particles with Coulomb interaction. If the interaction is weak and the correlation distance, determined by the exchange interaction r_{ex} is less than the correlation distance of the Coulomb interaction of the particles r_D, then for the region $r > r_{ex}$ the equations for F_1 and F_2 are simplified and in the similar case take the following form:

$$\frac{\partial F_1(p,t)}{\partial t}=\frac{ni}{(2\pi)^9\hbar^4}\int\Big[\Phi\Big(|q-\tfrac{1}{2}\hbar(\tau_1-\tau_2)|\Big)-\Phi\Big(|q+\tfrac{1}{2}\hbar(\tau_1-\tau_2)|\Big)\Big]\exp\{i\tau_1(\eta_1-p_1)$$
$$+i\tau_2(\eta_2-p_2)\}(1+P_{12})\,g(q,\eta_1,\eta_2;F_1)\,d\tau_1\,d\tau_2\,d\eta_1\,d\eta_2\,dq\,dp_2; \qquad (4)$$

$$g(q,p_1,p_2;F_1)=-\frac{im}{(2\pi)^6\hbar}\iiint_0^\infty\Big[\Phi\Big(|q-(p_1-p_2)\theta-\tfrac{1}{2}\hbar(\tau_1-\tau_2)|\Big)-\Phi\Big(|q-(p_1-p_2)\theta+\tfrac{1}{2}\hbar(\tau_1-\tau_2)|\Big)\Big]$$
$$\times\exp\{i\tau_1(\eta_1-p_1)+i\tau_2(\eta_2-p_2)\}F_1(\eta_1,t)\,F_2(\eta_2,t)\,d\tau_1\,d\tau_2\,d\eta_1\,d\eta_2\,d\theta\mp\frac{nim}{(2\pi)^9\hbar^4}\iiint_0^\infty\Big[\Phi\Big(|q-q'-(p_1-p_2)\theta-\tfrac{1}{2}\hbar\tau|\Big)$$
$$-\Phi\Big(|q-q'-(p_1-p_2)\theta+\tfrac{1}{2}\hbar\tau|\Big)\Big]\exp\{iq'(\pi'-\pi'')/\hbar\}\Big(e^{i\tau(\eta-p_1)}\delta\Big(p_2-\frac{\pi'+\pi''}{2}\Big)F_1(\eta_1,t)\,F_1(\pi',t)\,F_1(\pi'',t)$$
$$-e^{i\tau(\eta-p_2)}\delta\Big(p_1-\frac{\pi'+\pi''}{2}\Big)F_1(\pi',t)\,F_1(\eta_1,t)\,F_1(\pi'',t)\Big)dp'\,d\theta\,dq'\,d\tau\,d\eta\,d\pi'\,d\pi''$$
$$-\frac{nim}{(2\pi)^9\hbar^4}\iiint_0^\infty\Big[\Phi\Big(|q-q'-(p_1-p_2)\theta-\tfrac{1}{2}\hbar\tau|\Big)-\Phi\Big(|q-q'-(p_1-p_2)\theta+\tfrac{1}{2}\hbar\tau|\Big)\Big]$$
$$\times\big(e^{i\tau(\eta_1-p_1)}F_1(\eta_1,t)\,g(-q',p_2,p';F_1)-e^{i\tau(\eta-p_2)}F_1(\eta_1,t)\,g(q',p_1,p';F_1)\big)\,dp'd\theta dq'd\tau d\eta. \qquad (5)$$

The function depends on the time only through $F_1(p,t)$. Equations (4) and (5) transform as $\hbar\to 0$, into the corresponding equations of Sec. 11 of the well-known monograph of Bogoliubov.[4]

To obtain the kinetic equation we must express the function g in terms of F_1 and substitute in Eq. (4). However, Eq. (5), the equation for g, is very difficult to solve exactly for F_1; therefore, for the solution of this equation we have made use of the method of expansion in terms of a small parameter. Here we have considered the case in which the motion of the isolated particle does not disturb the statistical equilibrium of the totality of charged particles being considered. We then get the kinetic equation for the function $F_1(p;t)$ from Eqs. (4), (5) [as a result of transformation of the brackets]:

$$\frac{\partial w(t;p)}{\partial t} = \frac{2\pi v}{(2\pi\hbar)^6\hbar}\int\left\{\frac{\nu(|p'-p|/\hbar)\,\nu(|p-p'|/\hbar)}{1+B((p'-p)/\hbar;(p'+p)/2)} \pm \frac{\nu(|p-p_1'|/\hbar)\,\nu(|p'-p_1|/\hbar)}{1+B((p-p_1')/\hbar;(p+p_1')/2)}\right\}\delta(p+p_1-p'-p_1')\,\delta(E+E_1-E'-E_1')$$
$$\times[(1\pm w(p))(1\pm w(p_1))\,w(p')\,w(p_1') - (1\pm w(p'))(1\pm w(p_1'))\,w(p)\,w(p_1)]\,dp_1\,dp'\,dp_1'. \qquad (6)$$

Here

$$w(t;p) = (2\pi)^3 nF_1(t;p);\ \nu(|k|) = \int e^{ikq}\Phi(|q|)\,dq;\ E(p) = p^2/2m;$$

$$B(k,p) = \left|\mathrm{Re}\,\frac{im\nu(|k|)}{(2\pi)^9\hbar^4}\iint\limits_0^\infty [e^{i\hbar k\tau/2} - e^{-i\hbar k\tau/2}]\exp\{i\theta k(p'-p) + i\tau(\eta-p')\}\,w(\eta)\,d\theta\,d\tau\,d\eta\,dp'\right|;\ v = 1/n. \qquad (7)$$

An expression is omitted in Eq. (6) which corresponds to consideration of the effect of the particles of the system upon one another. Equation (6) (for $B = 0$) goes over into the quantum kinetic equation obtained by Bogoliubov and Gurov.[3] If it is assumed that all the functions w in Eq. (6) depend on the time, then this equation can be regarded as the quantum analogue of the well known kinetic equation for a classical system with Coulomb interaction, obtained by Landau,[5] in which, however, the divergence at large distances is automatically eliminated by account of correlation. If we consider that the motion of the individual particle with momentum p (or of an outside charged particle moving in a plasma) does not disrupt the thermal equilibrium of the particles of the system surrounding it, then Eq. (6) for $\hbar \to 0$ goes over into the Fokker-Planck equation for the plasma.[6] In the stationary case, the solution of Eq. (6) coincides with the Fermi or Bose distribution.[7] For B in the case of systems obeying Fermi statistics, for complete degeneracy, we find after expanding in a series:

$$B(k,p) = \frac{1}{r_D^2 k^2} + \frac{1}{2r_D^2 k^2}\left(\frac{kp}{kp_0}\right)\ln\left|\frac{1-kp/kp_0}{1+kp/kp_0}\right|. \qquad (8)$$

Here $r_D = (p_0^2/12\pi mne^2)^{1/2}$ is the Debye radius for the Fermi distribution, and p_0 is the limiting momentum.

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