

regardless of the specific character of the intermediate state during the attenuation of the current, only an insignificant part of the volume of the superconductor is in the normal state.

<sup>1</sup> A. A. Galkin and B. G. Lazarev, Dokl. Akad. Nauk. SSSR 59, 1017 (1948).

<sup>2</sup> Galkin, Kan, and Lazarev, J. Exptl. Theoret. Phys. (U.S.S.R.) 20, 865 (1950).

Translated by E. J. Rosen

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### Calculation of the Polarization of Neutrons on the Basis of the Diffused Edge Nuclear Model

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(Submitted to JETP editor March 16, 1957)  
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1583  
(June, 1957)

**T**HE RESULTS OF EXPERIMENTS on the polarization of 400-kev neutrons have been examined by Adair<sup>1</sup> on the basis of the optical nuclear model with a rectangular potential well. In the examination the spin-orbit potential was assumed to be constant inside the nucleus. However, this examination does not take into account the basic properties of spin-orbit interaction which must actually exist only on the surface of the nucleus, as may be concluded from analogy with electromagnetic interaction.

In view of the above it is logical to assume that the spin-orbit interaction is proportional to

$$\frac{1}{r} \frac{dV}{dr} (l\sigma), \quad (1)$$

where  $V$  is the depth of the potential well inside the nucleus. In this case, as can readily be seen, the magnitude of the spin-orbit interaction depends on the dimensions of the nucleus; this circumstance could not be taken into account in Adair's calculations.

In our calculations the selected form of the potential was

$$\begin{aligned} V &= V_0 (1 + i\zeta) = \text{const} \quad \text{for } r \leq r_0, \\ V &= V_0 e^{-\alpha(r-r_0)} (1 + c\alpha/r_0) \quad \text{for } r \geq r_0. \end{aligned} \quad (2)$$

With this choice of potential there is a small discontinuity at  $r = r_0$ ; this however is unimportant inasmuch as  $c\alpha/r_0 < 1$ . For the purpose of simplifying the solution, the factor  $1/r$  in (1) was replaced by the constant  $1/r_0$ , which is a satisfactory approximation for heavy nuclei in which the thickness of the boundary layer is  $d \ll r_0$ . This substitution greatly facilitates calculations, since it makes it possible to use the procedure developed in Ref. 2.

The spin-orbit coupling coefficient  $c$  was assumed equal to  $3.3 \times 10^{-27}$  cm<sup>2</sup> in accordance with the data of Levintov<sup>3</sup> obtained in investigating light nuclei.

The polarization was computed according to familiar formulas which were derived on the assumption that the spin of the target nucleus is zero. It was assumed that the cross section for elastic scattering is due to optical elastic scattering and resonance elastic scattering. The latter makes no contribution to the polarization since the individual levels with different values of  $l$  do not interfere with each other.

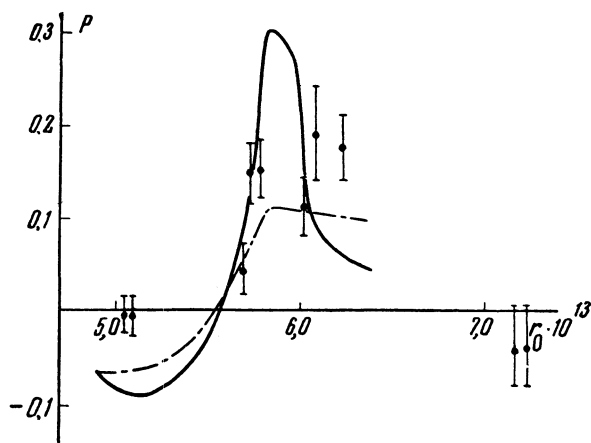
The calculations were carried out on the assumption that only the  $s$ - and  $p$ -phases differ from zero. Inasmuch as we considered nuclei close to the maximum of the  $p$ -wave ( $A \sim 100$ ), the  $d$ -phase in this region had a minimum as a function of  $A$  and, according to the calculations of one of the present authors, was close to zero.

In the calculations we assumed the following parameters:

$$\begin{aligned} r_0 &= 1.28 A^{1/2}, \quad V_0 = 42 \text{ Mev}; \quad \zeta = 0.05 \text{ and } \zeta = 0.1; \\ 1/\alpha &= 0.7 \cdot 10^{-13} \text{ cm.} \end{aligned}$$

We investigated polarization with scattering at an angle of  $90^\circ$  to the direction of the incident beam.

The results are shown in the accompanying figure.



Dependence of the polarization of neutrons on the nuclear radius.

As may be seen from the figure, the experimental points lie in the region of the maximum between the theoretical curves, which would indicate that the

closest agreement with experiments is obtained with  $0.05 < \zeta < 0.1$ . This conclusion is also in good agreement with data on the values of the parameter  $\zeta$ , obtained in investigating<sup>2</sup> the total cross sections and  $\Gamma_n/D$ .

Thus it has been shown that using one and the same value of the spin-orbit interaction constant one can describe scattering both from light and from heavy nuclei. This coefficient is somewhat smaller than that given by Ross and coworkers in connection with their investigation of the splitting of the ground level of magic number nuclei.

<sup>1</sup>R. K. Adair, S. E. Darden, and K. E. Fields, Phys. Rev. **96**, 503 (1954).

<sup>2</sup>P. E. Nemirovskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1143 (1957); Soviet Phys. JETP **5**, 932 (1957).

<sup>3</sup>I. I. Levintov, Dokl. Akad. Nauk SSSR, **107**, 240 (1956).

Translated by E. J. Rosen

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## Unitarity Relationships for Elastic Collisions of Particles with Arbitrary Spins

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(Submitted to JETP editor March 20, 1957)

J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1584-1585  
(June, 1957)

LET US CONSIDER the collisions of particles without spin. Let the energy of the impinging particles be such that only elastic scattering is possible. Then as Glauber and Schomaker<sup>1</sup> have shown the scattering amplitude  $f(\vartheta)$  obeys the following integral relation

$$4\pi \operatorname{Im} f(\vartheta) = k \int f^*(\vartheta'') f(\vartheta') d\omega(\mathbf{k}'') \quad (1)$$

Here  $\vartheta$  is the angle between the vectors of the initial  $\mathbf{k}$  and final  $\mathbf{k}'$  momenta;  $\vartheta'$  is the angle between  $\mathbf{k}$  and a variable vector  $\mathbf{k}''$ ;  $\vartheta''$  is the angle between  $\mathbf{k}'$  and  $\mathbf{k}''$  ( $\mathbf{k} = \mathbf{k}' = \mathbf{k}''$ , c. m. s.); and the integration is carried out over all directions of  $\mathbf{k}''$ . From (1) with  $\vartheta \rightarrow 0$ , we obtain the so-called optical theorem: the relation between the imaginary part of the scattering amplitude forward and the total cross section  $\sigma$ . Below we shall generalize (1), extending it to the case of elastic collision of

particles with arbitrary spins  $s_1$  and  $s_2$  and shall show that in addition to the optical theorem there can be deduced from (1) a series of other relations connecting scattering matrix elements that do not vanish when  $\mathbf{k}' \rightarrow \mathbf{k}$  with different spin characteristics.

The process of elastic scattering is fully described by the matrix  $M(\mathbf{k}, \mathbf{k}')$  in  $(2s_1 + 1)(2s_2 + 1)$ -dimensional spin space, determining the amplitude of the scattered wave

$$\Psi_{\mathbf{k}}(\mathbf{r} \rightarrow \infty) \sim \varphi_{\mathbf{k}'}^{\text{incid}} + \varphi_{\mathbf{k}}^{\text{scat}} = e^{i\mathbf{k}\mathbf{r}}\chi + M(\mathbf{k}, \mathbf{k}')\chi e^{i\mathbf{k}'\mathbf{r}}/r. \quad (2)$$

The indices of the total spin of the system and its projection at the spin functions  $\chi$  are omitted for simplicity of notation. In view of unitarity of  $S$ -matrix the wave functions  $\Psi_{\mathbf{k}}$  satisfy the same requirements of orthogonality and normalization as the initial functions of the incident wave and form, when  $r \rightarrow \infty$ , a complete system of functions with respect to the angular variables

$$\int \varphi_{\mathbf{k}'}^{+\text{incid}} \varphi_{\mathbf{k}}^{\text{incid}} d\omega = \int \Psi_{\mathbf{k}'}^+ \Psi_{\mathbf{k}} d\omega. \quad (3)$$

The + sign, as usual, denotes Hermitian conjugates.

Utilizing expression (2), the asymptotic representation of plane wave