[9] B. A. Tavger and V. M. Zaitsev, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 564 (1956); Soviet Phys. JETP **3**, 430 (1956).

[10] Internationale Tabellen zur Bestimmung von Kristallstrukturen, Bd. 1 (Borntrager, Berlin, 1935).

[11] L. Landau and E. Lifshitz, *Quantum Mechanics* (M.–L., Gostekhizdat 1948).

[12] H. Bizette, J. phys. et radium **12**, 161 (1951).

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# Electron-Positron Pair Production in Collisions Between Fast π-Mesons and Nucleons

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The cross section is calculated for pair production in collisions between high-energy π-mesons and nucleons.

LANDAU AND POMERANCHUK[1] have considered gamma-radiation processes in the collision of high-energy π-mesons with nucleons. Their method can be used also for treating electron-positron pair production.

The cross section for this process, naturally, will be $\alpha^2$ times less than that for gamma-radiation. Owing, however, to the experimental difficulties in the detection and energy measurement of photons, it may be found experimentally simpler to deal with electron-positron pairs. Thus a theoretical investigation of electron-positron pair production is of interest.

The method used is based on the fact that at high π-meson energies the nucleon appears as an opaque sphere, so that the meson ψ-function outside the radius of action of the nuclear forces is a superposition of a plane wave and a diffracted wave. It turns out that in order to calculate the cross section for photon or pair production, it is sufficient to know the meson wave function outside the radius of action of the nuclear forces.

The two cases of pair production accompanying meson diffraction and absorption (stopping) must be considered separately. In the second case the final π-meson state is an ingoing wave, and we have a "sink" for particles. This leads to a situation in which the transition currents do not satisfy a continuity equation, and it becomes necessary to introduce an additional transition charge density in order to obtain charge conservation. This method was first applied by Tisza[2] to pair production in β-decay.

Landau and Pomeranchuk performed their calculation with the aid of a Green function. In the present work we make use of the transition-current method, since this makes it easier to account for the "disappearance" of charge. This "disappearance" effect does not have to be treated in considering gamma-radiation, since the interaction Hamiltonian of the meson and the electromagnetic field contains only the vector potential, so that the introduction of an additional transition charge density does not change its form.

The interaction of mesons with the electron-positron field is given by*

$$\hat{V} = e\,[\,(\boldsymbol{\alpha},\ \mathbf{A}(\mathbf{r})) - \varphi(\mathbf{r})], \tag{1}$$

where $\mathbf{A}$ and $\varphi$ are the meson transition potentials.

The matrix element corresponding to production of a pair with total momentum $\mathbf{k}_0 = \mathbf{k} + \mathbf{k}'$ and total energy $\varepsilon_0 = \varepsilon + \varepsilon'$, is

$$U = e\,[(u^*_{k'}\,\alpha \mathbf{A}_{k_0,\,\varepsilon_0} v_k) - (u^*_{k'} v_k)\,\varphi_{k_0,\,\varepsilon_0}], \tag{2}$$

where $\mathbf{A}_{k_0,\,\varepsilon_0}$ and $\varphi_{k_0,\,\varepsilon_0}$ are the Fourier components of the transition potentials, given by

$$\mathbf{A}_{k_0,\,\varepsilon_0} = \int \mathbf{A}_{\varepsilon_0}(\mathbf{r})\, e^{-i\mathbf{k}_0\mathbf{r}}\, d\mathbf{r}, \qquad \varphi_{k_0,\,\varepsilon_0} = \int \varphi_{\varepsilon_0}(r)\, e^{-i\mathbf{k}_0\mathbf{r}}\, d\mathbf{r}; \tag{3}$$

The index $\varepsilon_0$ indicates that the transition takes

* Throughout we have set $\hbar = c = 1$.

place between states whose energy difference is $\varepsilon_0$, and $u_{k'}$ and $v_k$ are the electron and positron spinor amplitudes, respectively. The transition potentials are given in terms of the transition currents by the expression

$$\mathbf{A}_{k_0,\varepsilon_0} = 4\pi \mathbf{j}_{k_0,\varepsilon_0}/(\varepsilon_0^2 - k_0^2);$$
$$\varphi_{k_0,\varepsilon_0} = 4\pi\rho_{k_0,\varepsilon_0}/(\varepsilon_0^2 - k_0^2). \quad (4)$$

Finally,

$$\mathbf{j} = \frac{e}{4\pi i}(\nabla\psi_{p'}^*\psi_p - \psi_{p'}^*\nabla\psi_p),$$
$$\rho = \frac{e}{4\pi i}\left(\psi_{p'}^*\frac{\partial\psi_p}{\partial t} - \frac{\partial\psi_{p'}^*}{dt}\psi_p\right), \quad (5)$$

where $\psi_p$ and $\psi_{p'}$ are the wave functions of the initial and final meson states with momenta $\mathbf{p}$ and $\mathbf{p}'$, respectively.

The probability for pair production is given by the expression (normalized for unit volume)

$$dw = 2\pi |U|^2 (2\pi)^{-9} d\mathbf{k}\, d\mathbf{k}'\, d\mathbf{p}'\delta(E - E' - \varepsilon - \varepsilon'), \quad (6)$$

where $E$ and $E'$ are the initial and final meson energies. In order to integrate over the direction at which the electron is omitted, let us separate out the invariant $|U|^2 d\mathbf{k}'$ and go over to the center-of-mass system of the pair, where this integration is easily performed. Integrating over $d\mathbf{k}'$ and summing over spins, we obtain

$$dw = \frac{e^2}{3(2\pi)^7}\sqrt{1 - \frac{4m^2}{\varepsilon_0^2 - k_0^2}}\left(1 + \frac{2m^2}{\varepsilon_0^2 - k_0^2}\right)$$
$$\times (\varepsilon_0^2 - k_0^2)(A_{k_0,\varepsilon_0}^2 - \varphi_{k_0,\varepsilon_0}^2)\, d\mathbf{k}_0 p'E'd\varepsilon_0 d\Omega_{p'}. \quad (7)$$

Let us first consider pair production with diffraction of the meson. Here

$$\psi_p = \sqrt{\frac{2\pi}{E}}\left[e^{i\mathbf{pr}} - \frac{p}{2\pi i}\int\limits_{\rho^2 \leqslant R^2} \frac{e^{ip|\mathbf{r}-\rho|}}{|\mathbf{r}-\rho|}d\rho\right]e^{-iEt},$$
$$\psi_{p'} = \sqrt{\frac{2\pi}{E}}\left[e^{i\mathbf{p'r}} + \frac{p'}{2\pi i}\int\limits_{\rho^2 \leqslant R^2} \frac{e^{-ip'|\mathbf{r}-\rho|}}{|\mathbf{r}-\rho|}d\rho\right]e^{-iE't}. \quad (8)$$

The integrals are taken over planes passing through the nucleus and perpendicular to $\mathbf{p}$ and $\mathbf{p}'$, respectively; $R$ is the radius of the nucleon. The calculation is exactly the same as that of Landau and Pomeranchuk,[1] who used the Green function, and we shall therefore present only the result:

$$d\sigma = \frac{e^4R^2}{3\pi^3}\frac{p'}{p}\sqrt{1 - \frac{4m^2}{x}}\left(1 + \frac{2m^2}{x}\right)\frac{dx}{x}\frac{dk_0}{k_0}\frac{J_1^2(|p'\theta' + k_0\theta|R)}{|p'\theta' + k_0\theta|^2}$$
$$\times\left\{\frac{(p'\theta' + k_0\theta)^2}{(\alpha^2 + \theta^2)(\alpha'^2 + (\theta - \theta')^2)} - \frac{\mu^2}{p^2p'^2}\left[\frac{p^2}{\alpha^2 + \theta^2} - \frac{p^2}{\alpha'^2 + (\theta' - \theta)^2}\right]^2\right.$$
$$\left. + \frac{k_0(p + p')}{2pp'}\left[\frac{p'^2}{\alpha^2 + \theta^2} - \frac{p^2}{\alpha'^2 + (\theta' - \theta)^2}\right]\right\} d\theta\, d\theta'. \quad (9)$$

Here we have introduced the notation

$$\alpha = \mu/p, \quad \alpha' = \mu/p', \quad x = \varepsilon_0^2 - k_0^2,$$
$$\mathbf{p}' = (\mathbf{pp}')\mathbf{p}/p^2 + p'\theta', \quad \mathbf{p} = (\mathbf{pk}_0)\mathbf{p}/p^2 + k_0\theta. \quad (10)$$

The variable $x$ characterizes the angle of divergence of the pair, and is related to it by the expression

$$x = kk'\left(\frac{m^2k_0^2}{k^2k'^2} + \vartheta^2\right).$$

In calculating the matrix elements, the distance

$$r_{\mathrm{eff}} \sim pp'/\mu^2 k_0;$$

is of importance, and when $p \gg \mu$ and the energy of the pair is not too large, $r_{\mathrm{eff}} \gg R$, which verifies the validity of using free-particle wave functions.

The spectral distribution of the pairs produced is given by the expression

$$d\sigma = \frac{2e^4R^2}{3\pi}\frac{p'}{p}\left(\ln\frac{k_0}{m} - \frac{5}{6}\right)\frac{dk_0}{k_0}G(\mu R), \quad (11)$$

where $G$ is defined by the integral

$$G(\mu R) = \int\limits_0^\infty \frac{J_1^2(2\mu Rx)}{x^2}\left\{[\sinh^{-1}(x(4x^2 + 3)) + \sinh^{-1}x]\frac{1 + 2x^2}{\sqrt{1 + x^2}} - 4x\right\}dx$$
$$G(\mu R) \simeq \frac{2{,}6}{\mu R} \quad (\text{for } \mu R \gg 1); \quad G(1) = 2. \quad (12)$$

Finally, we present the total cross section for pair production:

$$\sigma = \frac{e^4 R^2}{3\pi} G(\mu R)\left(\ln \frac{p}{m}\right)^2\left[1 - \frac{11}{3\ln(p/m)}\right] \quad (13)$$

Let us now consider pair production with stopping of the meson. For the initial wave function let us take a plane wave, since the whole process takes place in the region of space in front of the nucleus. We shall describe the final state as a converging wave; since dimensions much larger than the nuclear radius are of importance, it is natural to consider only $s$-waves:

$$\psi_p = \sqrt{\frac{2\pi}{E}}\, e^{i\mathbf{pr}} e^{-iEt}, \quad \psi_{p'} = \sqrt{\frac{2\pi}{E'}}\, \frac{e^{-ip'r}}{4\pi r} e^{-iE't}. \quad (14)$$

The Fourier components of the transition current are given by

$$\mathbf{j}_{k_0, \varepsilon_0} = \frac{2\pi e}{\sqrt{EE'}} \frac{2\mathbf{p} - \mathbf{k}_0}{|\mathbf{p} - \mathbf{k}_0|^2 - p'^2},$$

$$\rho_{k_0, \varepsilon_0} = \frac{2\pi e}{\sqrt{EE'}} \frac{2E - \varepsilon_0}{|\mathbf{p} - \mathbf{k}_0|^2 - p'^2}. \quad (15)$$

We shall show that the model we have chosen leads to exactly the same results as the Green function calculation in the case of photon production.

The interaction Hamiltonian of mesons and the electromagnetic field is

$$\hat{V} = (\mathbf{A}_{\text{phot}}\, \mathbf{j}_{\text{mes}}).$$

The matrix element for photon production is

$$U = \sqrt{2\pi / k}\, (\mathbf{e}\mathbf{j}_k),$$

where $\mathbf{e}$ is the polarization vector of the photon, and $\mathbf{j}_k$ is the Fourier component of the meson transition current. The photon production probability is given by

$$dw = 2\pi\, |U|^2 (2\pi)^{-3} d\mathbf{k} - (2\pi)^{-3} 4\pi p'^2 dp' \delta(E),$$

and the cross section by

$$d\sigma = \frac{e^2}{4\pi} R^2 \frac{(\mathbf{pe})^2}{(|\mathbf{p} - \mathbf{k}|^2 - p'^2)} \frac{k\,dk}{pp'}\, d\theta,$$

where $\theta$ is the angle between $\mathbf{k}$ and $\mathbf{p}$. This expression is exactly the same as Equation (45) of Landau and Pomeranchuk.[1]

The transition current and charge as given by expression (15) do not satisfy a continuity equation. Let us calculate the additional transition charge density $\rho'$ which must be introduced to preserve charge conservation. To do this let us integrate the continuity equation

$$\operatorname{div} \mathbf{j} - i\varepsilon_0 (\rho + \rho') = 0 \quad (16)$$

over a sphere surrounding the origin and then go to the limit as the radius of the sphere approaches zero. Here j and $\rho$ are given by Equation (5), where $\psi_p$ and $\psi_{p'}$ are taken from (14). The only point at which $\psi_p$ has a singularity is the origin, so that $\rho' = \rho_0' \delta(r)$. Since $\int \rho dr = 0$ when $R \to 0$, we have

$$\rho_0' = \frac{1}{i\varepsilon_0} \int \mathbf{j}\, dS = -\frac{2\pi e}{\varepsilon_0 \sqrt{EE'}} \qquad (R \to 0).$$

The corrected expressions for the transition current and charge are

$$\mathbf{j}_{k_0, \varepsilon_0} = \frac{2\pi e}{\sqrt{EE'}} \frac{2\mathbf{p} - \mathbf{k}_0}{|\mathbf{p} - \mathbf{k}_0|^2 - p'^2},$$

$$\rho_{k_0, \varepsilon_0} = \frac{2\pi e}{\sqrt{EE'}} \left\{\frac{2E - \varepsilon_0}{|\mathbf{p} - \mathbf{k}_0|^2 - p'^2} - \frac{1}{\varepsilon_0}\right\}. \quad (17)$$

The differential cross section for pair production with stopping of the meson is given by

$$d\sigma = \frac{2e^4 R^2}{3\pi^2}\left(1 - \frac{k_0}{p}\right)\left(\ln \frac{k_0}{m} - \frac{5}{6}\right) \times \frac{dk_0}{k_0}\left\{\frac{1}{(\mu^2/p^2) + \theta^2} - \frac{\mu^2/p^2}{(\mu^2/p^2 + \theta^2)^2} - \frac{1}{4}\right\} d\theta; \quad (18)$$

where $\theta$ is the angle between $\mathbf{k}_0$ and $\mathbf{p}$.

The pair spectrum is given by

$$d\sigma = \frac{4e^4 R^2}{3\pi}\left(1 - \frac{k_0}{p}\right)\left(\ln \frac{k_0}{m} - \frac{5}{6}\right)\left(\ln \frac{p}{\mu} - 1\right)\frac{dk_0}{k_0}. \quad (19)$$

Finally, the total cross section is

$$\sigma = \frac{2e^4 R^2}{3\pi} \ln \frac{p}{\mu}\left(\ln \frac{p}{m}\right)^2\left(1 - \frac{11}{3\ln(p/m)}\right) \times \left(1 - \frac{1}{\ln(p/\mu)}\right). \quad (20)$$

Landau and Pomeranchuk note that the interaction of a meson with the nucleon background should lead to the "smearing out" of the charge over a region of dimensions $\sim 1/\mu$. This leads to the occurrence of a form factor $F$, which accounts for the finite dimensions of the $\pi$-meson, in all the expressions for the cross sections. Comparison of experiment and the theory based on the assumption of a point meson should give some information on the "structure" of the particle interacting with the vacuum.

It was shown by Landau and Pomeranchuk that if $k_0 \ll p$, $\theta' < \mu/2p$ and $\theta \ll \mu^2/pk_0\theta'$, the form-factor depends only on $k_0\theta^2$, and can be written

$$F = F\,(pk_0\,\theta^2\,/\,2\mu^2).$$

The form-factor is equal to unity when its argument is small, and approaches zero as its argument increases.

In conclusion, the author takes this opportunity to express his gratitude to Professor I. Ia. Pomeranchuk for his constant attention and direction.

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[1] L. D. Landau, I. Ia. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) **24**, 5 (1953).

[2] L. Tisza, Physik. Z. Sowjetunion **11**, 425 (1937).

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