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Application of the Dirac-Fock-Podol'skii Method to a Mechanical Many-Body Problem

I. G. FIKHTENGOL'TS

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The general form of all the classical integrals of the motion and the generalized expression for the inertial mass of a body, in the relativistic many-body problem, are established with the aid of the many-time formalism. The results are applied to a system of interacting electric charges and to a system of bodies interacting gravitationally.

THE TREATMENT of the relativistic many-body problem is physically more coherent if the motion of the bodies is described by four-dimensional rather than by three-dimensional vectors. In view of this, we shall describe each of the n bodies of the system not merely by its three spatial coordinates x_i, y_i, z_i ($i = 1, 2, \ldots, n$) but also by its time coordinate t_i. Correspondingly, we shall also apply the term "four-dimensional" to all functions and relations in which the motion of the bodies is described by four-dimensional vectors. Such a method was applied first in quantum electrodynamics in the work of Dirac, Fock, and Podol'skii,[1] and later in classical electrodynamics in the work of Markov.[2] In the present article this method is applied to the mechanical n-body problem.

The system of bodies is supposed isolated, and only the translational motion of the bodies is considered; no account is taken of the dependence of this motion on their shape and other parameters (the bodies are supposed spherically symmetric, and the distances between them are supposed much greater than their linear dimensions).

Treatment of the many-body problem from a purely mechanical point of view is naturally approximate, and permissible only when radiation may be neglected. Therefore we assume that the speeds of the mechanical motion are small in comparison with the speed of light, and we retain only quantities of order $\dot{\mathbf{r}}_i^2/c^2$ in the case of electrical interaction, and only quantities of order $\dot{\mathbf{r}}_i^4/c^4$ in the case of gravitational interaction between the bodies ($\dot{\mathbf{r}}_i^2$ is the square of the velocity of translational motion of the ith body).

1. THE FOUR-DIMENSIONAL EQUATIONS OF MOTION OF A SYSTEM OF BODIES, AND THEIR INTEGRALS

The equations of motion in which each body is described by means of its own time can be written in the form

$$d\mathscr{P}_{\nu i}/dt = \mathscr{F}_{\nu i}\,(\nu = 0, 1, 2, 3;\ i = 1, 2, \ldots, n). \quad (1)$$

Here

$$\mathscr{P}_{0i} = -c^{-1}\partial\mathscr{L}/\partial\dot{x}_{0i},\ \mathscr{P}_{ji} = \partial\mathscr{L}/\partial\dot{x}_{ji}, \quad (2)$$

$$\mathscr{F}_{0i} = -c^{-1}\partial\mathscr{L}/\partial x_{0i},\quad \mathscr{F}_{ji} = \partial\mathscr{L}/\partial x_{ji}\,(j = 1, 2, 3), \quad (3)$$

where $\mathscr{L}$ is the four-dimensional Lagrangian function (to be determined later), dependent on the variables t, $x_{\nu i}$, and $\dot{x}_{\nu i}$; t is the independent variable, for which we use the proper time of the coordinate system; $x_{0i} = t_i$ is the time coordinate, and $x_{1i} = x_i$, $x_{2i} = y_i$, $x_{3i} = z_i$ are the spatial coordinates, of the ith body in the chosen coordinate system. A superior dot indicates differentiation with respect to the variable t.

To establish relations between the general integrals of the four-dimensional equations (1) and the groups of transformations with respect to which these equations are invariant, we change from the variables t and $x_{\nu i}$ in Eqs. (1) to new variables τ

and $q_{\nu i}$, by setting

$$\tau = \tau(t, \alpha), \; q_{\nu i} = q_{\nu i}(t, x_{\nu i}, \alpha), \tag{4}$$

where α is a parameter, and where

$$(\tau)_{\alpha=0} = t, \; (q_{\nu i})_{\alpha=0} = x_{\nu i}. \tag{5}$$

By following the references,[3–6] it is not hard to show that when the four-dimensional equations (1) are invariant with respect to a one-parameter group of transformations, determined by relations (4) and (5), the equations (1) have the following integral corresponding to this group:

$$\left(\sum_{i=1}^{n}\sum_{\nu=0}^{3}\frac{\partial\mathscr{L}}{\partial\dot{x}_{\nu i}}\dot{x}_{\nu i} - \mathscr{L}\right)\left(\frac{\partial\tau}{\partial\alpha}\right)_{\alpha=0} - \sum_{i=1}^{n}\sum_{\nu=0}^{3}\frac{\partial\mathscr{L}}{\partial\dot{x}_{\nu i}}\left(\frac{\partial q_{\nu i}}{\partial\alpha}\right)_{\alpha=0} + \left(\frac{\partial f}{\partial\alpha}\right)_{\alpha=0} = \text{const}, \tag{6}$$

where the function f is determined by the relation

$$\mathscr{L}\left(\tau, q_{\nu i}, \frac{dq_{\nu i}}{d\tau}\right)\frac{d\tau}{dt} - \mathscr{L}(t, x_{\nu i}, \dot{x}_{\nu i}) = \frac{d}{dt} f(t, x_{\nu i}, \alpha), \text{ with } \tau = \tau(t, \alpha), \; q_{\nu i} = q_{\nu i}(t, x_{\nu i}, \alpha), \tag{7}$$

in accordance with the form of the function $\mathscr{L}$ and of the transformation (4). In formula (6) it is understood that $\mathscr{L}$ is the untransformed function $\mathscr{L}(t, x_{\nu i}, \dot{x}_{\nu i})$.

Existence of a function f that satisfies the condition (7) is equivalent to the requirement that the four-dimensional equation (1) be invariant with respect to the group of transformations (4). In particular, such a function f obviously exists when the function $\mathscr{L}$ itself is invariant with respect to the group of transformations being considered; for then the left side of Eq. (7) is by definition zero, and therefore f = const. It follows that when the function $\mathscr{L}$ is invariant with respect to a certain group of transformations, the function f may be set equal to zero in the integral (6) corresponding to this group of transformations.

We assume that the sets of quantities $\mathfrak{P}_{\nu i}$ and $\mathfrak{J}_{\nu i}$ ($\nu = 0, 1, 2, 3$) constitute four-dimensional vectors with respect to the Lorentz transformations; we deduce that the Lagrangian function $\mathscr{L}$ in which we are interested, and in which the motion of the bodies is described by four-dimensional vectors, must be invariant with respect to these transformations. It is of importance that the function $\mathscr{L}$ must also be invariant with respect to a change of the origin and scale of the proper time of the coordinate system. By setting first $\tau = t + \alpha$, $q_{\nu i} = x_{\nu i}$ and then $\tau = (1 + \alpha)t$, $q_{\nu i} = x_{\nu i}$ in the integral (6), we deduce that invariance of the four-dimensional $\mathscr{L}$ with respect to a change of origin and scale of the proper time of the coordinate system is equivalent to the supposition that the function $\mathscr{L}$ identically satisfies the equation

$$\sum_{i=1}^{n}\sum_{\nu=0}^{3}(\partial\mathscr{L}/\partial\dot{x}_{\nu i})\dot{x}_{\nu i} - \mathscr{L} = 0, \tag{8}$$

i.e., is a homogeneous function of the first degree in the variables $\dot{x}_{\nu i}$. From (1) and (8) it also follows that the four-dimensional function $\mathscr{L}$ must not depend explicitly on the variable t.

Together with the conditions already imposed on the four-dimensional function $\mathscr{L}$, it is necessary that when every $x_{0i} = t$ this function reduce to the ordinary Lagrangian function L of the mechanical system under consideration, *i.e.*, that the following equality be satisfied:

$$\mathscr{L}|_{x_{0i}=t} = L. \tag{9}$$

We shall call the function L, as distinguished from the function $\mathscr{L}$, three-dimensional (we shall apply the term "three-dimensional" to all functions and relations in which the motion of the bodies is described by three-dimensional vectors).

It is not hard to show that conditions (8) and (9) and the requirement that the function $\mathscr{L}$ be invariant with respect to Lorentz transformations completely determine this function. However, indeterminacy of the three-dimensional function L entails a corresponding indeterminacy of its four-dimensional analog $\mathscr{L}$.

In what follows, we shall consider only such functions as satisfy conditions (8) and (9) and are Lorentz-invariant. In accordance with this, all integrals of the four-dimensional equations (1) that are consequences of the invariance of the equations with respect to Lorentz transformations can be put into the following very simple form, in conformity with formula (6):

$$\sum_{i=1}^{n}\sum_{\nu=0}^{3}\frac{\partial\mathscr{L}}{\partial\dot{x}_{\nu i}}\left(\frac{\partial q_{\nu i}}{\partial\alpha}\right)_{\alpha=0} = \text{const}. \tag{10}$$

By starting from relations (10) and, in the Lorentz transformations, making each of the ten parameters in turn different from zero (and the rest equal to zero), we get the four-dimensional analogs of all ten of the usual integrals of motion of an isolated system of bodies. From the invariance of the function $\mathfrak{L}$ with respect to a shift of origin of the time coordinates x_{0i}, a shift of origin of the spatial coordinates x_{1i}, x_{2i}, x_{3i}, a rotation of the spatial axes, and a change to a reference system moving with constant velocity with respect to the original system, there follow respectively: the energy integral

$$\sum_{i=1}^{n} \partial\mathscr{L}/\partial\dot{x}_{0i} = \text{const}, \tag{11}$$

three momentum integrals

$$\sum_{i=1}^{n} \partial\mathscr{L}/\partial\dot{x}_{ji} = \text{const} \ (j = 1, 2, 3), \tag{12}$$

three angular-momentum integrals

$$\sum_{i=1}^{n} \left(\frac{\partial\mathscr{L}}{\partial\dot{x}_{ji}} x_{hi} - \frac{\partial\mathscr{L}}{\partial\dot{x}_{hi}} x_{ji}\right) = \text{const} \quad (j = 1, 2, 3;\ h = 1, 2, 3) \tag{13}$$

and three integrals of motion of the center of inertia

$$\sum_{i=1}^{n} \left(\frac{\partial\mathscr{L}}{\partial\dot{x}_{ji}} x_{0i} + \frac{1}{c^2}\frac{\partial\mathscr{L}}{\partial\dot{x}_{0i}} x_{ji}\right) = \text{const} \ (j = 1, 2, 3) \tag{14}$$

for the four-dimensional equations of motion (1).

Thus, by constructing the four-dimensional Lagrangian function $\mathfrak{L}$ of the mechanical problem under consideration, it is possible to find for each body, according to formula (2), its energy-momentum vector $\mathfrak{P}_{\nu i}$ ($\nu = 0, 1, 2, 3$); the set of quantities

$$\mathscr{P}_{\nu} = \sum_{i=1}^{n} \mathscr{P}_{\nu i} \ (\nu = 0, 1, 2, 3)$$

constitutes the conserved energy-momentum vector of the whole system of bodies, and the set of quantities

$$\mathscr{N}_{\mu\nu} = \sum_{i=1}^{n} (x_{\mu i}\overset{*}{\mathscr{P}}_{\nu i} - x_{\nu i}\overset{*}{\mathscr{P}}_{\mu i}) \times (\mu = 0, 1, 2, 3;\ \nu = 0, 1, 2, 3),$$

where

$$\overset{*}{\mathscr{P}}_{0i} = \mathscr{P}_{0i}/c,\ \overset{*}{\mathscr{P}}_{ji}\mathscr{P}_{ij} \ (j = 1, 2, 3;\ i = 1, 2, \ldots, n)$$

constitutes the conserved antisymmetric tensor of angular momentum and of the motion of the center of inertia.

Since $\mathfrak{P}_{0i}$ is the time component of the four-dimensional energy-momentum vector of the ith body, we deduce that the quantities

$$\mathscr{E}_i = c\mathscr{P}_{0i} \text{ and } \mathscr{M}_i = \mathscr{P}_{0i}/c \tag{15}$$

may be interpreted respectively as the four-dimensional analogs of the total energy and of the inertial mass of the ith body.

Upon setting every $x_{0i} = t$ in formulas (15), we get generalized expressions for the total energy

$$E_i = -\partial\mathscr{L}/\partial\dot{x}_{0i}\big|_{x_{01}=x_{02}\ldots x_{0n}=t} \tag{16}$$

and for the inertial mass

$$M_i = -\frac{1}{c^2}\frac{\partial\mathscr{L}}{\partial\dot{x}_{0i}}\bigg|_{x_{01}=x_{02}=\cdots=x_{0n}=t} \tag{17}$$

of the ith body in the ordinary (three-dimensional) treatment of the many-body problem.

Thus for an isolated body moving with velocity $\dot{\mathbf{r}}$, the four-dimensional Lagrangian function has the form

$$\mathscr{L} = -mc^2\sqrt{\dot{x}_0^2 - \dot{\mathbf{r}}^2/c^2},$$

where m is the rest mass of the body and x_0 is its time coordinate; accordingly,

$$E = mc^2/\sqrt{1 - \dot{\mathbf{r}}^2/c^2},\ M = m/\sqrt{1 - \dot{\mathbf{r}}^2/c^2}.$$

In the more general case, when the motion of a system of interacting bodies is considered, the relation (17) gives a generalized expression for the inertial mass of a body. Like the energies E_i, the inertial masses M_i are functions of time and depend in an essential way on the nature of the interaction of the bodies. We note that despite the indeterminacy of the function $\mathfrak{L}$, which is connected with the indeterminacy of the function L, relation (17) uniquely defines the quantity M_i.

Along with the establishment of generalized expressions for the inertial mass of a body, the passage to a many-time formalism gives a possibility of pointing out the general form of all the classical integrals of motion in the ordinary (three-dimensional) treatment of the many-body problem. Upon setting every $x_{0i} = t$ in the integrals (11) to (14), we have: the energy integral

$$\sum_{i=1}^{n} E_i = \text{const}, \tag{11*}$$

the momentum integrals

$$\sum_{i=1}^{n} P_{ji} = \text{const}, \tag{12*}$$

the angular-momentum integrals

$$\sum_{i=1}^{n} (x_{ji}P_{hi} - x_{hi}P_{ji}) = \text{const} \tag{13*}$$

and the integrals of motion of the center of inertia

$$\sum_{i=1}^{n} P_{ji}t - \sum_{i=1}^{n} M_i x_{ji} = \text{const}. \tag{14*}$$

In these formulas, as usual,

$$P_{ji} = \partial L / \partial \dot{x}_{ji}.$$

In addition, by virtue of the homogeneity condition (8),

$$\sum_{i=1}^{n} E_i = \sum_{i=1}^{n} \sum_{j=1}^{3} \frac{\partial L}{\partial \dot{x}_{ji}} \dot{x}_{ji} - L,$$

as was to be expected.

From (12*) and (14*) it follows, in particular, that for every isolated system of bodies

$$\sum_{i=1}^{n} P_{ji} = \frac{d}{dt} \sum_{i=1}^{n} M_i x_{ji}, \tag{18}$$

so that for the relativistic coordinates of the center of inertia

$$X_j = \sum_{i=1}^{n} M_i x_{ji} \Big/ \sum_{i=1}^{n} M_i \quad (j = 1,2,3)$$

the Newtonian equations

$$d^2 X_j / dt^2 = 0.$$

remain valid.

2. SYSTEM OF INTERACTING ELECTRIC CHARGES

To formulate the four-dimensional equations of motion of a system of interacting electric charges, it is sufficient to find the four-dimensional Lagrangian function $\mathfrak{L}$ of the problem.

To terms of order $\dot{\mathbf{r}}_i^2/c^2$, the required four-dimensional Lagrangian function may be expressed in the form

$$\mathscr{L} = -\sum_i m_i c^2 \sqrt{\dot{x}_{0i}^2 - \frac{\dot{\mathbf{r}}_i^2}{c^2}} - \frac{1}{2} \sum_{i \neq k} \frac{e_i e_k}{S_{ik}} \mathscr{U}_{ik}, \tag{19}$$

$$S_{ik} = \sqrt{(\mathbf{r}_i - \mathbf{r}_k)^2 - c^2 (x_{0i} - x_{0k})^2}, \tag{20}$$

$$\mathscr{U}_{ik} = \sqrt{\dot{x}_{0i}\dot{x}_{0k} - c^{-2} (\dot{\mathbf{r}}_i \dot{\mathbf{r}}_k) - c^{-2} \mathscr{A}_{ik} / S_{ik}^2} \tag{21}$$

and

$$\mathscr{A}_{ik} = [(\dot{\mathbf{r}}_i (\mathbf{r}_i - \mathbf{r}_k)) - c^2 \dot{x}_{0i} (x_{0i} - x_{0k})]$$
$$\times [(\dot{\mathbf{r}}_k (\mathbf{r}_i - \mathbf{r}_k)) - c^2 \dot{x}_{0k} (x_{0i} - x_{0k})]. \tag{22}$$

In these formulas, m_i is the rest mass, e_i the charge, and $\mathbf{r}_i$ the radius vector of the ith particle. The summation over indices i and k is from 1 to n.

It is immediately evident that (19) is Lorentz-invariant, satisfies the homogeneity condition (8), and reduces for all $x_{0i} = t$ to the known (three-dimensional) Lagrangian function L (*cf.*, for example, Refs. 5 and 7) of a system of electric charges.

From the fact that for a system of interacting electric charges there exists a four-dimensional function $\mathfrak{L}$, satisfying all the requirements of Sec. 1, it follows that for such a system all the deductions of that section are valid.

The integrals of motion of a system of electric charges have been treated in complete detail in Refs. 5 and 7, and therefore we shall not concern ourselves with them here. We pause to consider only the total energy and the inertial mass of a particle.

By starting from the relation (16) and the explicit form of the function $\mathfrak{L}$ [*cf.* (19)], we arrive at the following expression for the total energy E_i of an electrically interacting particle, to terms of order $\dot{\mathbf{r}}_i^2/c^2$:

$$E_i = \frac{m_i c^2}{\sqrt{1-\dot{\mathbf{r}}_i^2/c^2}} + \frac{1}{2}\sum_k{}' \frac{e_i e_k}{|\mathbf{r}_i-\mathbf{r}_k|}\Big(1+\frac{1}{2c^2}(\dot{\mathbf{r}}_i\dot{\mathbf{r}}_k) + \frac{1}{2c^2}\frac{(\dot{\mathbf{r}}_i(\mathbf{r}_i-\mathbf{r}_k))(\dot{\mathbf{r}}_k(\mathbf{r}_i-\mathbf{r}_k))}{|\mathbf{r}_i-\mathbf{r}_k|^2}\Big). \quad (23)$$

The prime on the summation emphasizes that the summation index takes all values from 1 to n except one ($k \neq i$).

We get the magnitude of the inertial mass M_i of the same particle by dividing (23) by c^2. Thus to terms of order $\dot{\mathbf{r}}_i^2/c^2$, we have (*cf.* also Ref. 8)

$$M_i = \frac{m_i}{\sqrt{1-\dot{\mathbf{r}}_i^2/c^2}} + \frac{1}{2c^2}\sum_k{}' \frac{e_i e_k}{|\mathbf{r}_i-\mathbf{r}_k|}. \quad (24)$$

Formula (24) is the generalization of the well-known Einstein expression for inertial mass to the case of interacting electric charges.

3. SYSTEM OF BODIES INTERACTING GRAVITATIONALLY

To formulate the four-dimensional equations of motion of a system of bodies in the case of gravitational interaction, it is sufficient, as for a system of interacting electric charges, to find the four-dimensional Lagrangian function of the problem.

To terms of order $\dot{\mathbf{r}}_i^2/c^2$, the required four-dimensional Lagrangian function may be expressed in the form

$$\mathscr{L} = -\sum_i m_i c^2 \sqrt{\dot{x}_{0i}^2 - \frac{\dot{\mathbf{r}}_i^2}{c^2}} + \frac{1}{2}\sum_{i\neq k}\frac{\gamma m_i m_k}{S_{ik}}\Big(U_{ik} - \frac{1}{c^2}\sum_l{}' \frac{\gamma m_l}{S_{il}} U_{il}\Big). \quad (25)$$

Here

$$U_{ik} = [\dot{x}_{0i}\dot{x}_{0k} - c^{-2}(\dot{\mathbf{r}}_i\dot{\mathbf{r}}_k) - (\mathcal{A}_{ik}/c^2 S_{ik}^2) + 3c^{-2}\mathscr{B}_{ik}^2]^{\prime}$$
$$\mathscr{B}_{ik}^2 = (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_k)^2 - c^2(\dot{x}_{0i} - \dot{x}_{0k})^2,$$

S_{ik} and $\mathcal{A}_{ik}$ have the previous meanings (20) and (22).

It is evident that the function (25) is invariant with respect to Lorentz transformations, satisfies the homogeneity condition (8), and reduces for all $x_{0i} = t$ to the known (three-dimensional) Lagrangian function L (*cf.*, for example, Ref. 9) of our problem.

From the fact that for a system of bodies interacting gravitationally there exists a four-dimensional function $\mathscr{L}$, satisfying all requirements of Sec. 1, it follows that for such a system of bodies all the deductions of that section are valid. We consider some of them.

By starting from the relation (16) and the explicit form of the function $\mathscr{L}$ [*cf.* (25)], we get an expression for the total energy E_i of the ith particle in the case of gravitational interaction, to terms of order $\dot{\mathbf{r}}_i^2/c^2$. We get the magnitude of the inertial mass M_i of the same particle by dividing E_i by c^2. Thus to terms of order $\dot{\mathbf{r}}_i^4/c^4$ we get [*cf.* also Ref. 9]:

$$M_i = \frac{m_i}{\sqrt{1-\dot{\mathbf{r}}_i^2/c^2}} - \frac{1}{2c^2}\sum_k{}' \frac{\gamma m_i m_k}{|\mathbf{r}_i-\mathbf{r}_k|}\Big(1+\frac{1}{2c^2}(\dot{\mathbf{r}}_i\dot{\mathbf{r}}_k) + \frac{1}{2c^2}\frac{(\dot{\mathbf{r}}_i(\mathbf{r}_i-\mathbf{r}_k))(\dot{\mathbf{r}}_k(\mathbf{r}_i-\mathbf{r}_k))}{|\mathbf{r}_i-\mathbf{r}_k|^2} - \frac{3}{2c^2}(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_k)^2 - \frac{1}{c^2}\sum_l{}' \frac{\gamma m_l}{|\mathbf{r}_i-\mathbf{r}_l|}\Big). \quad (26)$$

Formula (26) is the generalization of the well-known Einstein expression for inertial mass to the case of gravitational interaction of the bodies.

Remembering that the relations of Sec. 1 hold whenever the four-dimensional function $\mathscr{L}$ satisfies the necessary general requirements, we easily get for the system of bodies under consideration, with the aid of the expression (26) for the inertial mass M_i, the momentum integrals [*cf.* (18) and (12*)] and the integrals of motion of the center of inertia [*cf.* (18) and (14*)] to terms of order $\dot{\mathbf{r}}_i^4/c^4$. (The second derivatives with respect to the variable t, which enter into the correction terms in M_i, are easily eliminated with the aid of the equations of motion, which are known to terms of order $\dot{\mathbf{r}}_i^2/c^2$.) Consequently, by using the general relations of Sec. 1, we can easily increase the accuracy of six of the integrals of motion as compared with the accuracy of the Lagrangian function L. As for the energy and angular-momentum integrals, increase of their accuracy requires a corresponding increase of the accuracy of the Lagrangian function itself. All these relations were established to terms of order $\dot{\mathbf{r}}_i^2/c^2$ in Ref. 9, already cited.

In closing, we pause to consider the relation between the inertial and the ponderable mass of a body. By virtue of (26), the inertial mass of a body depends on its rest mass, on the velocity of the body as a whole, and on the nature of the interaction of this body with all the other bodies. The inertial masses of interacting electric charges, according to (24), depend also on the values of the charges. From expressions (24) and (26) it follows that the inertial masses agree with the rest masses only in the first (Newtonian) approximation.

As for the ponderable mass ("gravitational charge") of a body, it follows from the Einstein gravitational equations that, except for the choice of the unit of measurement, the ponderable mass of a body always coincides with its rest mass. The experiments of Eötvös and the nondependence of the motion of a body (of small mass) in a fixed gravitational field upon the value of its mass lead to the same conclusion. But since the inertial mass of a body coincides with its rest mass only in the Newtonian approximation, therefore the equality between the ponderable and inertial masses of a body also holds only in the Newtonian approximation.

If we take account of the fact that (*cf.* Ref. 10) the motion of a body of finite (not small) mass in a gravitational field — which in this case is not fixed — depends on the value of the mass even in the first (Newtonian) approximation, we arrive at the conclusion that the problem of the dependence of the motion of a body on its mass does not reduce to the problem of the relation between the inertial mass of a body and its ponderable mass.

From the above discussion it follows, in particular, that the physical basis of the Einstein equivalence principle is not an equality between the ponderable and inertial masses, which holds only in the Newtonian approximation, but an equality of the ponderable mass ("gravitational charge") of a body to its rest mass.

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