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Polarization Correlation in Nucleon-Nucleon Scattering

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Equations are obtained for the polarization correlation in proton-proton scattering, taking into account four phases: ${}^1S_0$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$ and Coulomb interaction. A computation using phases for the isotropic states as obtained from scattering data shows that the Coulomb interaction plays an essential role for energies of 10–30 Mev. Polarization correlation can thus be used to give a more precise determination of the isotropic phases (which do not give rise to polarization), and to estimate other phases in the energy region in which they begin to appear. We also consider the scheme of experiments for measuring the polarization correlation and obtain the combinations of components of the polarization tensor which are measured in the experiments.

1. INTRODUCTION

THE SCATTERING OF PARTICLES with spin is described by the average values of spin operators over the scattered wave. For two particles with spins $\boldsymbol{\sigma}^{(1)}$ and $\boldsymbol{\sigma}^{(2)}$, these operators are:

$$\hat{1},\ \sigma_i^{(1)},\ \sigma_i^{(2)},\ \hat{P}_{ik} = \hat{\sigma}_i^{(1)}\hat{\sigma}_k^{(2)}. \qquad (1)$$

The corresponding average values are: the scattering cross section, the polarization of the first (1) and second (2) particle, and the polarization correlation. This last quantity has a tensor character ($i, k = x, y, z$) and may be called the polarization tensor. If we represent the asymptotic form of the scattered wave as a sum of partial waves (with given j, l, s), these average quantities will be expressed in terms of the corresponding phases. The analysis of scattering of nucleons requires the inclusion of phases with $l > 0$. To determine them unambiguously we must measure all the characteristics of the scattering which relate the phases (cross section, polarization, and polarization correlation). As we shall show in detail later, measurement of the polarization correlation is especially important for determining the phases in the region of isotropic scattering of the protons. It is known that the scattering of protons is isotropic over a wide range of energy (up to 400–450 Mev), and is therefore described by the phases of the isotropic states ${}^1S_0$ and ${}^3P_0$. To separate them one might measure polarization in addition to the cross section. However, the isotropic phases give no nuclear polarization, while its Coulomb part is sizeable only at very small angles of scattering [$\leqslant$ 5–10° in the center-of-mass system (c.m.s.) at medium energies], and is therefore difficult to measure. On the other hand, the polarization correlation has a measurable value even when only the singlet phase is included. Consequently its measurement (along with the cross section) makes it possible to determine these phases in the high energy region (> 100 Mev), where the Coulomb interference term, which enables us to separate these phases at low energies, becomes negligibly small.

In the present paper we derive equations for the polarization and polarization correlation in terms of the scattering phases for orbital angular momenta $l = 0$ and 1 (J = 0, 1, 2), with allowance for the Coulomb interaction which plays an important part in the scattering of charged particles (protons) for energies < 100 Mev.

2. DERIVATION OF EQUATIONS

The polarization correlation in the scattering of two particles is defined as the average of the operator $\hat{P}_{ik}$ over the scattered wave. It gives the probability that, after scattering, one of the particles (1) is in the spin state σ_i, while the other is in the state σ_k.

In order to express P_{ik} in terms of the scattering phases by means of the method of phase analysis, we consider the asymptotic expression for the wave function in the scattering of a plane wave with spin. We consider the scattering of initially unpolarized particles. In the c.m.s. the system of two particles with spins s_1 and s_2 is described by the plane wave $\chi_m^s \exp(i\mathbf{k}\cdot\mathbf{r})$, which is the product of the spin wave function χ_m^s with total spin $s_1 + s_2, \ldots |s_1 - s_2|$ and projection m on the axis of quantization (taken along the direction of relative motion of the particles), and the plane wave $\exp(i\mathbf{k}\cdot\mathbf{r})$ of the relative motion. Since the particles are not polarized prior to scattering, the scattering of waves with given s and $m = m_s$ proceeds independently, *i.e.*, $\chi_m^s \to \chi_{m'}^{s'}$ where the prime denotes quantities after the scattering. The asymptotic form of the scattered wave is:

$$\psi_{\text{sc}}(s'm') = \frac{e^{ikr}}{r}\sum_{sm} M_{s'm'}^{sm}(\theta, \varphi)\,\chi_m^s. \tag{2}$$

After the scattering, the original spin s and its projection m are in general changed and take on some new values s', m'. The quantity which is conserved is the total angular momentum J, which we write schematically as a sum $\mathbf{J} = \mathbf{l} + \mathbf{s}$ of the spin and the orbital angular momentum of the relative motion.

The quantities $M_{s'm'}^{sm}$, whose angular dependence is given by generalized Legendre polynomials, are the amplitudes for transitions between spin states $(sm) \to (s'm')$. There quantities are the generalization of the single amplitude which appears in the scattering of spinless particles and which is given by the Legendre polynomial $P_l(\cos\theta)$. In the scattering of nucleons ($s_1 = s_2 = \frac{1}{2}$) the total spin takes on two values: $s = 0$ (singlet) and $s = 1$ (triplet), so that the scattering matrix M is made up from the 16 amplitudes $M_{s'm'}^{sm}$. It is easy to express all the quantities characterizing the scattering in terms of this matrix:

$$\text{cross section:}\quad \sigma = \mathrm{Sp}\,(M^+M), \tag{3}$$

$$\text{polarization:}\; P_i = \mathrm{Sp}\,(M^+\sigma_i M)\,/\,\mathrm{Sp}\,(M^+M) \tag{4}$$

polarization correlation:

$$P_{ik} = \mathrm{Sp}\,(M^+\hat{\sigma}_i^{(1)}\hat{\sigma}_k^{(2)}M)/\mathrm{Sp}\,(M^+M). \tag{5}$$

As usual, Sp denotes the sum of the diagonal matrix elements and M^+ is the Hermitean conjugate.

A knowledge of the phases is necessary for a complete determination of the scattering matrix.

We first treat the scattering of two protons (n–n scattering is obtainable from the general formulas by setting the charge equal to zero). When the Coulomb interaction is included, the scattering matrix can be written as a sum of nuclear and Coulomb terms. The nuclear part is obtained from the asymptotic form of the scattered wave function, where the expansion of the plane wave must be carried out using radial Coulomb functions F and G in place of the free radial functions R_{kl}.

For the case of identical particles, the matrix must be symmetrized appropriately: the coordinate part must be symmetric for singlet states and antisymmetric for triplet states. Taking account of the symmetry, the Coulomb part of the matrix can be written in the form:

$$M_{\text{coul}} = K_{ss'}\,\delta_{ss'}\,\delta_{mm'}, \tag{6}$$

where

$$K_s = -\,{}^1/_4\eta\omega\,(s^{-2}e^{-i\alpha} + (-)^s c^{-2}e^{-i\beta})\exp(2i\sigma_0) \tag{7}$$

is the Coulomb scattering amplitude. We have introduced the symbols

$$\eta = e^2/\hbar v, \quad c = \cos\theta/2, \quad s = \sin\theta/2,$$
$$\alpha = \eta \ln s^2, \quad \beta = \eta \ln c^2,$$

$\omega = \sqrt{(\gamma + 1)/2\gamma^2}$; ω is a relativistic correction for small angles of scattering. In this part of the matrix, the spin variables appear in delta functions since the spin is conserved in a Coulomb field.

For identical particles there are no transitions with change of spin (singlet $\rightleftarrows$ triplet) in the nuclear part of the matrix.

The matrix elements of M are expressed in terms of generalized Legendre polynomials and the phase matrix

$$\overset{J}{S}{}^{sl}_{s'l'} = \exp(2i\overset{J}{\delta}{}^{sl}_{s'l'}),$$

which gives the connection between the incoming and outgoing spin amplitudes $(sl) \to (s'l')$.[1] The matrix S is defined for a given J and parity $P_+ = (-)^J$, $P_- = (-)^{J+1}$. In the case of no spin, the S matrix has one element $S_l = \exp(2i\,\delta_l)$ for each l. We shall calculate only that part of the matrix which is important for practical purposes, *i.e.*, the part with orbital angular momentum $l = 0$ and 1. Formulae for higher l's can be gotten without any fundamental difficulties (it is sufficient to include transitions between states with $l > 1$), but are very cumbersome.

Using the selection rules, we obtain from the asymptotic form of the scattered wave the following expressions for the nonzero elements of the scattering matrix (the Coulomb factor $\exp(2i\,\delta_l)$ is omitted):

$$\begin{gathered}
M^{00}_{00} = \frac{1}{2}(1 - \overset{0}{S}{}^{00}_{00}), \quad M^{11}_{11} = M^{1-1}_{1-1} = \frac{3}{4}\cos\theta\,(2 - \overset{1}{S}{}^{11}_{11} - \overset{2}{S}{}^{11}_{11}), \\
M^{10}_{10} = \frac{1}{2}\cos\theta\left(3 - 2\overset{2}{S}{}^{11}_{11} - \overset{0}{S}{}^{11}_{11}\right), \quad M^{10}_{1-1} = -\frac{1}{2\sqrt{2}}e^{i\varphi}\sin\theta\left(\overset{0}{S}{}^{11}_{11} - \overset{2}{S}{}^{11}_{11}\right), \\
M^{11}_{10} = -\frac{3}{4\sqrt{2}}e^{i\varphi}\sin\theta\left(\overset{1}{S}{}^{11}_{11} - \overset{2}{S}{}^{11}_{11}\right), \quad M^{1-1}_{10} = \frac{3}{4\sqrt{2}}e^{-i\varphi}\sin\theta\left(\overset{1}{S}{}^{11}_{11} - \overset{2}{S}{}^{11}_{11}\right), \\
M^{10}_{11} = \frac{1}{2\sqrt{2}}e^{-i\varphi}\sin\theta\left(\overset{0}{S}{}^{11}_{11} - \overset{2}{S}{}^{11}_{11}\right).
\end{gathered} \tag{8}$$

The polarization tensor for identical particles is obviously symmetric in the particle and coordinate indices (since the particles are indistinguishable), *i.e.*, it must be written in the form:

$$\hat{P}_{ik} = \frac{1}{2}(\hat{\sigma}^{(1)}_i \hat{\sigma}^{(2)}_k + \hat{\sigma}^{(1)}_k \hat{\sigma}^{(2)}_i).$$

The action of the operator $\hat{P}_{ik}$ on the spin indices $(s'm')$ of the matrix $M^{sm}_{s'm'}$ is completely analogous to its action on the spin indices of the wave functions $\chi^s_{m'}$: for example

$$\hat{P}_{xx}\chi^0_0 = -\chi^0_0, \quad \hat{P}_{xx}\chi^1_0 = \chi^1_0, \tag{9}$$
$$\hat{P}_{yz}\chi^1_0 = -\frac{i}{\sqrt{2}}(\chi^1_{-1} + \chi^1_1)$$

etc.

To simplify the writing of the polarization tensor, we define the quantity

$$\sigma_{ik} = P_{ik}\,\mathrm{Sp}\,(M^+M). \tag{10}$$

Unlike the normalized P_{ik}, this quantity has the dimensions of a differential cross section (barns/sterad.). We choose our coordinate system as follows (*cf.* Fig. 1): the z axis is along the direction of the incident flux, the y axis is perpendicular to the plane of scattering.

Let us consider the scattering the c.m.s. in the (xz) plane, which corresponds to azimuthal angle $\varphi = 0$. From formula (5), by using (6) – (9), we get the following expressions for the components σ_{ik} in terms of the four phases: $\delta_0({}^1S_0)$, $\delta^0_1({}^3P_0)$, $\delta^1_1({}^3P_1)$, $\delta^2_1({}^3P_2)$, which for simplicity we write in terms of elements of the scattering matrix:

$$\begin{gathered}
k^2\sigma_{xx} = 4\mathrm{Re}\,[(M^{11}_{11} + K_1)\,M^{11^*}_{1-1}] + 2\,(|M^{10}_{11}|^2 - |M^{11}_{10}|^2) + |M^{10}_{10} + K_1|^2 - |M^{00}_{00} + K_0|^2, \\
k^2\sigma_{xz} = 2\sqrt{2}\,\mathrm{Re}\,[(M^{11}_{11} + K_1)\,M^{10^*}_{11} + (M^{10}_{10} + K_1)\,M^{11^*}_{10} - M^{11}_{1-1}M^{10^*}_{11}], \\
k^2\sigma_{zz} = 2\,(|M^{11}_{11} + K_1|^2 + |M^{11}_{1-1}|^2 + |M^{11}_{10}|^2 - |M^{10}_{11}|^2) - |M^{10}_{10} + K_1|^2 - |M^{00}_{00} + K_0|^2; \\
k^2\sigma_{yy} = -4\mathrm{Re}\,[(M^{11}_{11} + K_1)\,M^{11^*}_{11}] + 2\,(|M^{10}_{11}|^2 + |M^{11}_{10}|^2) + |M^{10}_{10} + K_1|^2 - |M^{00}_{00} + K_0|^2.
\end{gathered} \tag{11}$$

Thus only four components of the tensor are different from zero.

Similarly we get the expression for the polarization from formula (4):

$$\sigma_0 P_y = \frac{\sqrt{2}}{4} \operatorname{Re} [i (M_{10}^{11} - M_{11}^{10}) \times (M_{11}^{11} + M_{10}^{10} - M_{1-1}^{11} + 2K_1)^*]. \quad (12)$$

Only the y-component of the polarization is different from zero. The polarization is perpendicular to the plane of scattering because the polarization pseudovector is constructed from the two available vectors: $\mathbf{k}$ and $\mathbf{k}'$ (the wave vectors before and after scattering), so that $\mathbf{P} \sim \mathbf{k} \times \mathbf{k}'$.

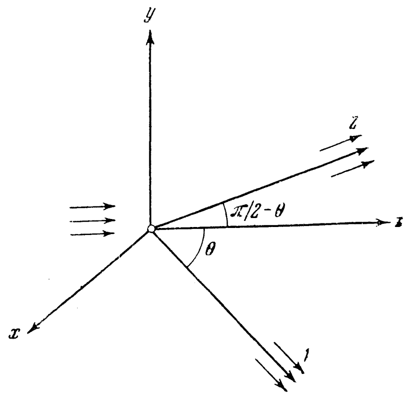

FIG. 1. Orientation of coordinate axes.

We first consider the formula for the polarization. It consists of two terms: a nuclear term and an interference term (proportional to $\eta = e^2/\hbar v$). Pure Coulomb terms naturally do not occur, since the Coulomb field does not change the polarization. In the nuclear part only the triplet phases occur, and they appear in linear combinations which vanish if they are equal or if one of them is equal to zero. It is clear that for isotropic phases only the Coulomb interference term is left, and this term is important only for very small angles of scattering ($\lesssim 5$–$10°$ in the c,m.s. at medium energies). The measurement of polarization at small, scattering angles presents well-known difficulties.

Next we treat the expressions for σ_{ik}. They consist of a nuclear, a Coulomb ($\sim \eta^2$), and an interference ($\sim \eta$) part. Unlike the polarization, the correlation contains pure Coulomb terms, so that it even occurs in the scattering in a pure Coulomb field. It is not hard to see that the Coulomb term is itself an interference between Coulomb scattering amplitudes and is consequently closely related to the identify of the particles. To make this clearer, we consider in particular the Coulomb term in the expression for P_{xx}.

The spin state is not changed during scattering in a Coulomb field, so that the scattering matrix is diagonal. The diagonal terms have the form $K_s = f(\theta) + (-)^s f(\pi-\theta) \equiv f_+ + (-)^s f_-$, while the scattered wave from a given spin state is $K_s \chi_m^s$.

We find the average value of $\hat{P}_{xx} = \hat{\sigma}_x^{(1)} \hat{\sigma}_x^{(2)}$ for the individual states, *i.e.*,

$$[(K_s \chi_m^s)^+, \quad \hat{\sigma}_x^{(1)} \hat{\sigma}_x^{(2)} (K_s \chi_m^s)] = K_s^* K_s (\chi_m^{s+} \hat{\sigma}_x^{(1)} \hat{\sigma}_x^{(2)} \chi_m^s).$$

For the singlet we get $- K_0^* K_0$; for the triplet with $m = 0$, $K_1^* K_1$; while for the triplets with $m = \pm 1$ we get zero, since, for example, $(\chi_1^1 \hat{P}_{xx} \chi_1^1) = (\chi_1^{1+} \chi_{-1}^1) = 0$. To determine P_{xx} we need only add the results obtained for singlet and triplet waves, multiplying them by the statistical weights which are equal to ¼, since there was no initial polarization. We get $\sigma_{xx}^c = K_1^* K_1 - K_0^* K_0 = -2(f_+^* f_- + f_+ f_-^*)$. In other words, this term occurs because singlet and triplet are scattered differently.

3. MEASURABLE CONBINATIONS OF CORRELATIONS

As was shown above, in the plane of scattering only four of the tensor components are different from zero: P_{yy}, P_{xx}, P_{xz}, P_{zz}. However, in an experiment we can only perform two independent measurements of the correlation: one in which the planes of the first and second scatterings are parallel, the other in which they are mutually perpendicular[2] (*cf.* Fig. 2). By the second scattering we mean the scattering by the nuclei of the analyzers (de-

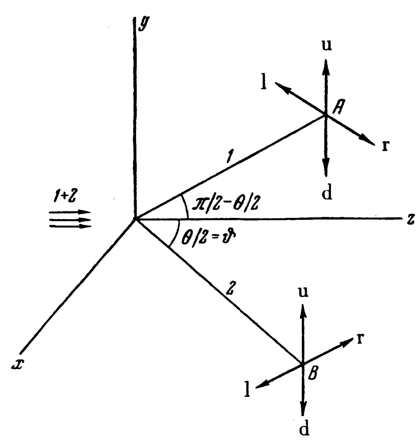

FIG. 2. Scheme of experiment for measurement of polarization correlation.

noted by A and B). The following two correlation measurements are independent:

1) By measuring the intensity when both particles are scattered upward (I_{uu}), both downward (I_{dd}), one scattered up, the other down (I_{ud}), and vice versa (I_{du}), we measure the correlation along the y axis:

$$P_{yy} = P_{\rm I} = \frac{\sigma_{\rm I}}{\sigma} = \frac{(I_{uu} - I_{du}) - (I_{ud} - I_{dd})}{I_{uu} + I_{du} + I_{ud} + I_{dd}}; \tag{13}$$

2) The second measurement is a measurement of the intensity when both particles are scattered to the left (I_{ll}), both scattered to the right (I_{rr}), the first to the right and the second to the left (I_{rl}) and vice versa (I_{lr}). By means of a formula analogous to (13) with the substitutions $u \to l$, $d \to r$, we determine a linear conbination of the remaining components: P_{xx}, P_{xz}, P_{zz}. These three components form a tensor in the xz plane. We bring it to axes $x'z'$ by a rotation through the angle ϑ ($\vartheta = \theta/2$ is the angle of scattering in the laboratory system). In this case we obviously are measuring the component of the rotated tensor with mixed indices, or more precisely the sum $\frac{1}{2}(P'_{x'z'} + P'_{z'x'})$ (in view of the identity of the particles). So the second combination is the following:

$$P_{\rm II} = {}^1/_2 (P'_{x'z'} + P'_{z'x'}) = P_{xz} \cos 2\vartheta + {}^1/_2 (P_{xx} - P_{zz}) \sin 2\vartheta.$$

The formulas for $P_{\rm I}$ and $P_{\rm II}$ must be expressed in terms of the scattering angle in the lab system. Neglecting relativistic effects (for protons of a few hundred Mev), we must set $\theta = 2\vartheta$. The measurable combinations are

$$k^2\sigma_0 P_{\rm I} = k^2\sigma_{yy},$$

$$k^2\sigma_0 P_{\rm II} = \frac{1}{2\sin\theta}\left(|M_{11}^{10}|^2 - |M_{10}^{11}|^2\right).$$

4. PHASE ANALYSIS

Starting from the isotropy of the $p-p$ scattering, an attempt was made to describe the scattering in terms of the two phases of the isotropic states $\delta_0(^1S_0)$ and $\delta_1(^3P_0)$.[3] Inclusion of the Coulomb interaction makes it possible, by analyzing experimental data on scattering cross sections, to determine magnitudes and signs of these phases in the energy region where the interference between Coulomb and nuclear scattering is not very small compared to the nuclear scattering. Protons with energies of 70–80 Mev in the laboratory system belong in this energy region.

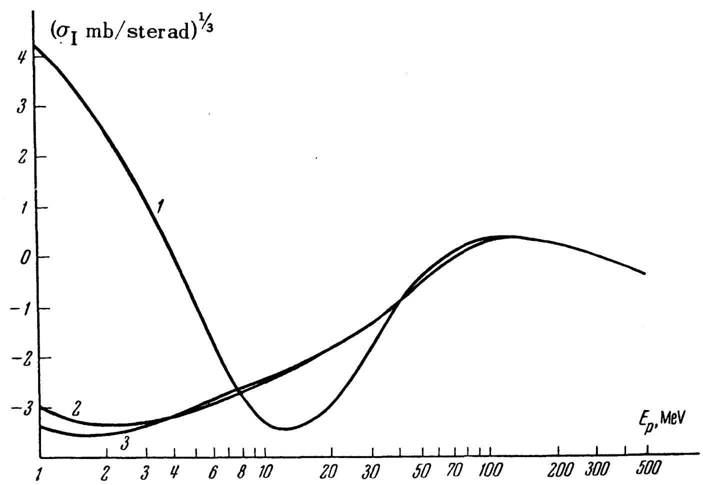

FIG. 3. Energy dependence of the polarization correlation, of measured with scattering planes perpendicular. *1*. Correlation for $\theta = 20°$; *2*. Correlation for $\theta = 90°$; *3*. Nuclear part for $\theta = 20°$.

For higher energies the interference term (which falls off essentially like $E^{-3/2}$) becomes so small that separation of the phases which are combined in the nuclear scattering cross section $\sigma_{\rm nuc} = f(\delta_0, \delta_1)$ is no longer possible. By using the energy dependence of the cross section, the energy variation of the phases δ_0 and δ_1 over the whole region of isotropy (up to 400 – 450 Mev) was predicted[3].

We give a table of the values of the phases which we shall use in estimating the correlation:

E (Mev)	δ_0 (deg)	δ_1 (deg)	E (Mev)	δ_0 (deg)	δ_1 (deg)
0.5	18	~0	46	32	32
1	33	~0	100	10	52
1.5	42	~0	150	—7	61
2	46	~0	200	—23	70
10	56	2	300	—56	84
18	54	3	450	—90	90
30	48	11			

More accurate values for the phases can be obtained only from measurement of the polarization correlation (since, as already noted, measurement of the polarization gives nothing).

In order to estimate the importance of the polarization correlation, we give graphs of the energy dependence of σ_{ik} (for both σ_I and σ_{II}) for two typical angles: $\theta = 20°$ (where the Coulomb part is important), and $\theta = 90°$ (where the Coulomb part is neglibly small except at the very lowest energies). Figure 3 gives the quantity σ_I, which is measured when the planes of scattering are perpendicular, while Fig. 4 shows $\sigma_{II} = \sigma P_{II}$ while is measured with parallel planes of scattering. In order to make the drawings compact the energy is measured on a logarithmic scale, while the vertical axis gives the cube root of the correlation (in mb/sterad). Such a scale enables us to visualize the behavior of the curves over the whole energy range.

From the figures we see that the Coulomb terms play an important part not just for intermediate energies (*cf.* σ_I), which makes possible a more detailed phase analysis over the region of isotropy of the p–p scattering.

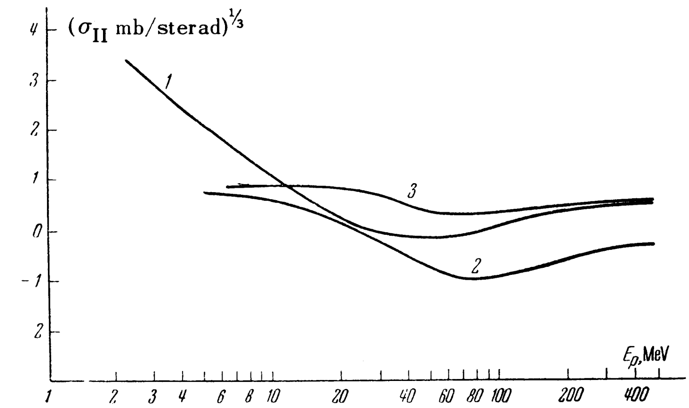

FIG. 4. Energy dependence of the polarization correlation, as measured with scattering planes parallel. *1*. Correlation for $\theta = 20°$; *2*. Total and nuclear correlation for $\theta = 90°$; *3*. Nuclear part for $\theta = 20°$.

We get the important result that the Coulomb terms cannot be neglected in an analysis which includes the polarization correlation. Measurement of the correlation at intermediate energies (20–100 Mev) will make it possible to get the isotropic phases more accurately and to determine the contribution of the anisotropic phases.

In conclusion, I express my thanks to Prof. Ia. A. Smorodinskii for suggesting this topic.

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