Paramagnetic Resonance and Polarization of Nuclei in Metals*

M. IA. AZBEL', V. I. GERASIMENKO AND I. M. LIFSHITZ
Physico-Technical Institute, Academy of Sciences, Ukrainian SSR

(Submitted to JETP editor July 14, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1212-1225 (May, 1957)

A theory of paramagnetic resonances in metals is constructed, based on the simultaneous solution of Maxwell's equations and the kinetic equation for the density operator. The resultant nuclear polarization is determined. It is shown that this polarization varies very slowly with depth, decreasing exponentially up to depths of 10^{-3} to 1 cm, which is the mean distance traversed by an electron between collisions involving spin reversal. It is found that paramagnetic resonance brings about selective transparency of metallic films.

1. STATEMENT OF THE PROBLEM. A COMPLETE SET OF EQUATIONS

As is well known[1], paramagnetic resonance of of the conduction electrons takes place in metals placed in a constant magnetic field H_0 and a variable electromagnetic field of frequency $\omega = \Omega_0 \equiv 2\mu H_0/\hbar$ (μ is the magnetic moment of the electron).

As Overhauser has shown[2], this resonance is accompanied by polarization of the nuclei of the metal; in this case such polarization takes place as if the nuclei possessed an effective magnetic moment μ_{eff}, equal to

$$\mu_{\text{eff}} = \mu_{\text{nu}} + \mu\alpha T_{\text{ff}} / (1 + \alpha T_{\text{ff}}) \tag{1}$$

(μ_{nu} true magnetic moment of the nucleus; T_{ff} is the time of free flight of the electrons between collisions involving spin reversal; $\alpha = (4\mu^2 H_1^2/\hbar^2) T_{\text{ff}}$ is the probability of spin reversal of an electron per unit time in a variable magnetic field $2H_1 \cos\omega t$). However, it is easily seen that Eq. (1) is applicable only for very thin metallic samples, the thickness d of which is of the same order as, or small in comparison with, the thickness of the skin layer: 10^{-4} to 10^{-5} cm. In fact, the resonance probability of spin reversal per unit time can be introduced only when the electron is found in an almost homogenous field for a time interval significantly exceeding the period of the field. In the case of a large sample ($d \gg \delta$), this condition is satisfied for $\delta/v \gg 2\pi/\omega$, which corresponds to the frequencies

$$\omega \gg (2\pi v / c) \sqrt{2\pi\sigma / t_0} \sim 10^{13} \text{ sec}^{-1}$$

and in a magnetic field $H_0 = \hbar\omega/2\mu \gg 10^6$ Oe, which is practically unobtainable at the present time. Therefore, the polarization of nuclei by the Overhauser method takes place only in small particles of micron diameter, in which the electrons always move in a practically homogenous field[3].

At the same time, it can be shown that the Overhauser method permits polarization of the nuclei in layers whose thickness is tens and hundreds of times greater than that of the skin layer. In accord with Ref. 2, the degree of polarization P of the nuclei is determined only by the relative depolarization of the electrons along the direction x of the constant magnetic field:

$$P = \frac{1}{I}\left\{\left(I + \frac{1}{2}\right) \coth\left(I + \frac{1}{2}\right)\bar{\omega}_z - \frac{1}{2} \coth \frac{1}{2}\bar{\omega}_z\right\}; \quad \bar{\omega}_z = \frac{\chi H_0 - M_z}{\chi H_0}, \tag{1a}$$

where M is the spin magnetic moment of the electrons, I = nuclear spin. Polarization of the electron at a given point is determined by all the values of the magnetic field H_1 which it experiences along the path (up to the given point) from the previous collision involving spin reversal. Therefore, the magnetic moment at the given point is connected with the values of the magnetic field at all points within distances of the order of δ_{eff} passed by the electron between two successive collisions with spin reversal. Since the time T_{ff} between such collisions is much greater than the usual times of free flight t_0 of the electron, the diffusion length is $\delta_{\text{eff}} \approx v\sqrt{t_0 T_{\text{ff}}/3} \sim 10^{-3} - 1$ cm (v = velocity of the electron). Consequently, beginning with the low frequencies $\omega \gtrsim c^2/2\pi\sigma\delta_{\text{eff}}^2$, when $\delta_{\text{eff}} > \delta$ (for

*A preliminary note on this research has already been published[10].

$t_0 \sim 10^{-11}$ sec and $T_{ff} \sim 10^{-6}$ sec, $\omega \gtrsim 10^3$ sec^{-1}), a peculiar "anomalous skin effect" for the magnetic moment takes place; the coupling between the magnetic moment M and the variable magnetic field H_1 is an integral, in which the integration is carried out over a region with radius of the order δ_{eff}. This leads to a slow change in the magnetic moment with depth; the "depth of skin layer" for the magnetic moment is equal to δ_{eff}.

There then follow two important physical consequences:

1) Polarization of nuclei in the metal can take place in layers of thickness of the order $\delta_{eff} \sim 10^{-3}$ to 1 cm. This gives the possibility of obtaining rather thick polarized nuclear targets*.

2) The slow attenuation of the magnetic moment leads, in accord with Maxwell's equations, to the presence in E and H_1 of small, but slowly vanishing parts. In the case of a film of thickness $d \gg \delta$, just this part will determine the transmission coefficient for electromagnetic waves through the film in the vicinity of resonance. Consequently, for paramagnetic resonance, not only resonance absorption appears, but also resonance transmission of the film, in which the transmission coefficient can increase by many orders of magnitude. Thus, for low temperatures, the transmission coefficient of the wave through a film of about 0.1 mm thickness (at resonance) can have the order 10^{-9} to 10^{-13}, while away from resonance, it has the order of 10^{-40} to 10^{-50}. (We note that such a phenomenon occurs at all temperatures.)

The present research was also devoted to the determination of the degree of polarization of nuclei in metals and the transmission coefficient of metallic films with account of the spin diffusion†.

This problem is solved by use of the Maxwell equations

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}; \qquad \operatorname{curl} \mathbf{H} = \frac{4\pi}{c}\mathbf{j}; \quad \mathbf{B} = \mathbf{H}_1 + 4\pi\mathbf{M} \tag{2}$$

*As Rozentsveig and Fogel'[7] noted, nuclei of adsorbed hydrogen can be polarized in this manner.

†We note that determination of the power absorbed in paramagnetic resonance in a metal in the case of a constant magnetic field, perpendicular to the surface of the metal, and a weak electromagnetic field (when saturation of resonance is absent), was carried out by Dyson[4] on the basis of a study of the diffusion of electrons. The polarization of nuclei and selective transparency of a film was not considered at all by Dyson.

and the Boltzmann equation for the density operator of the electrons*:

$$\frac{\partial \hat f}{\partial t} + \mathbf{v}\frac{\partial \hat f}{\partial \mathbf{r}} + \frac{\partial \hat f}{\partial \mathbf{p}}\left\{e\mathbf{E} + \frac{e}{c}[\mathbf{vB}]\right\} + \frac{i}{\hbar}[\hat{\mathscr{H}}, \hat f] + \left(\frac{\partial \hat f}{\partial t}\right)_{\text{col}} = 0, \tag{3}$$

$$\hat{\mathscr{H}} = \mu\hat{\sigma}\mathbf{B}; \quad \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1(\mathbf{r}, t); \quad \mathbf{v} = \nabla_{\mathbf{p}}\varepsilon(\mathbf{p}).$$

Here ε, p and v are the energy, quasi-momentum and velocity of the electrons; $\hat\sigma$ is the spin operator:

$$\hat\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

the z-axis is chosen along the direction $\mathbf{H}_0$; $(\partial\hat f/\partial t)_{\text{col}}$ is the collision integral for the electrons.

It remains to write down the boundary condition for $\hat f$. Describing the reflection of the electrons from the surface $\zeta = 0$ semi-phenomenologically[6], and considering that the electron spin does not change in collisions with the surface, we have (for $\zeta = 0$†):

$$\hat f(v_\zeta) = (1 - q)\bar{\hat f} + q\hat f(-v_\zeta), \quad v_\zeta > 0 \tag{4}$$

(the bar denotes averaging over the momenta).

Solution of the Boltzmann equation permits us to determine the relation between the current density j and the direction of the electric field E, and between the spin magnetic moment M and the direction of the variable magnetic field $\mathbf{H}_1$:

$$\mathbf{j} = \frac{e}{h^3}\int \mathbf{v}\,\mathrm{Sp}\hat f\, d\tau_{\mathbf{p}}; \qquad \mathbf{M} = \frac{\mu}{h^3}\int \mathrm{Sp}(\hat\sigma\hat f)d\tau_{\mathbf{p}}; \quad d\tau_{\mathbf{p}} = dp_x\,dp_y\,dp_z. \tag{5}$$

Equations (2), (3) and (5) form the complete system of equations for the problem under consideration.

2. REDUCTION OF THE EQUATION TO CANONICAL FORM

We set

$$\hat f = \hat f^0 + \hat f', \tag{6}$$

where $\hat f^0$ is a function which at each given moment corresponds to the equilibrium state for E = 0.

*After completion of this research, a paper appeared[5], in which the same equation was used.

†In Ref. 5, this condition was written for $q = 1$. Evidently, $q \approx 0$ almost always.

Evidently, in a system of coordinates in which the direction of the magnetic induction B coincides with the axis ξ, $\hat{f}^0$ has the form

$$\hat{f}^0 = \begin{pmatrix} f_0(\varepsilon_0 - \mu B) & 0 \\ 0 & f_0(\varepsilon_0 + \mu B) \end{pmatrix} = \frac{1}{2}\left[f_0(\varepsilon_0 - \mu B) + f_0(\varepsilon_0 + \mu B)\right]\hat{I} + $$
$$+ \frac{1}{2}\hat{\sigma}_\xi\left[f_0(\varepsilon_0 - \mu B) - f_0(\varepsilon_0 + \mu B)\right] = \frac{1}{2}\left[f_0(\varepsilon_0 + \mu B) + f_0(\varepsilon_0 - \mu B)\right]\hat{I} \tag{7}$$
$$- \frac{1}{2}\left[f_0(\varepsilon_0 + \mu B) - f_0(\varepsilon_0 - \mu B)\right]\mu\mathbf{H}\hat{\boldsymbol{\sigma}} = f_1^0\hat{I} + \mathbf{f}^0\hat{\boldsymbol{\sigma}}.$$

For $\mu H \ll kT$,

$$\hat{f}^0 = f_0(\varepsilon)\hat{I} - \mu\hat{\boldsymbol{\sigma}}\mathbf{B}f_0'(\varepsilon), \quad f_0(x) = \{e^{(x-\varepsilon_0)/kT} + 1\}^{-1}. \tag{8}$$

Here f_0 is the equilibrium Fermi function, $\hat{I}$ is a unit operator.

Taking as variables ζ – the direction of the normal to the surface of the metal (which does not coincide, generally speaking, with the direction z of the constant magnetic field), the energy ε, the projection of the momentum p_z, and the dimensionless time $\tau = (\frac{1}{2}\pi m_0)\partial S_T/\partial\varepsilon$ of rotation of the electron about the orbit (S_T is the area of the sector in the intersection of the plane $\varepsilon(\mathbf{p}) = \varepsilon$ with the plane p_z = const – see Ref. 8), we get for the zeroth approximation in E,

$$\frac{\partial\hat{f}'}{\partial t} + \frac{\partial\hat{f}'}{\partial\zeta}v_\zeta + \frac{1}{T_0}\frac{\partial\hat{f}'}{\partial\tau} + \frac{i}{\hbar}\left[\mu\hat{\boldsymbol{\sigma}}\mathbf{B}, \hat{f}'\right]$$
$$+ \left(\frac{\partial\hat{f}'}{\partial t}\right)_{\text{col}} = -\frac{\partial\hat{f}^0}{\partial t} - \frac{\partial\hat{f}^0}{\partial\varepsilon}e\mathbf{v}\mathbf{E}. \tag{9}$$

(Here we have assumed that $(\partial\hat{f}^0/\partial t)_{\text{col}} = 0$ and that $[\hat{f}, \hat{\mathscr{H}}] = 0$, where $\hat{\mathscr{H}}$ is the Hamiltonian operator for E = 0). The term $(i/\hbar)[\mu\hat{\boldsymbol{\sigma}}\mathbf{B}, \hat{f}']$ in this equation describes the change in the operator of the electron density in the magnetic field, connected with the presence of electron spins. This change is brought about for two reasons: first, the variable magnetic field leads to the equalization of the electron densities in states with spins oriented parallel and antiparallel to the constant magnetic field $\mathbf{H}_0$ (see, for example, Ref. 2), and second, in an inhomogeneous magnetic field, forces act on the spin which are proportional to $\partial H_1/\partial\zeta$. The first reason leads to resonance reversal of spins and it itself determines the degree of depolarization of the electrons at resonance. The second reason leads only to "fine tuning" of the electrons according to the direction of the total magnetic induction B (the latter is accounted for mainly by the form of the operator $\hat{f}^0$). It is natural that in the determination of the depolarization of the electrons and the polarization of the nuclei, it does not have to be considered. (The same is also done in Ref. 10.) In order to demonstrate this fact, we note that

$$\frac{i}{\hbar}\left[\hat{\boldsymbol{\sigma}}\mathbf{B}, \hat{f}'\right] = \frac{i}{\hbar}\left[\hat{\boldsymbol{\sigma}}, \hat{f}'\right]\mathbf{B} + \hat{\boldsymbol{\sigma}}\frac{i}{\hbar}\left[\mathbf{B}, \hat{f}'\right],$$

or, in the quasi-classical approximation,

$$\frac{i}{\hbar}\left[\mu\hat{\boldsymbol{\sigma}}\mathbf{B}, \hat{f}'\right] = \frac{i}{\hbar}\left[\hat{\boldsymbol{\sigma}}, \hat{f}'\right]\mathbf{B} - \hat{\boldsymbol{\sigma}}\frac{\partial\mathbf{B}}{\partial\zeta}\frac{\partial\hat{f}'}{\partial p_\zeta}. \tag{10}$$

The first term on the right leads to spin reversal, the second corresponds to the classical force acting on the spin.

We now set

$$\hat{f}' = f_1'\hat{I} + \mathbf{f}'\hat{\boldsymbol{\sigma}}, \quad \hat{f}^0 = f_1^0\hat{I} + \mathbf{f}^0\boldsymbol{\sigma}. \tag{11}$$

From (9) and (10) we get, taking into account the commutation law for $\hat{\boldsymbol{\sigma}}$:

$$\frac{\partial f_1'}{\partial t} + \frac{\partial f_1'}{\partial\zeta}v_\zeta + \frac{1}{T_0}\frac{\partial f_1'}{\partial\tau} + \left(\frac{\partial f_1'}{\partial t}\right)_{\text{col}}$$
$$- \mu\frac{\partial\mathbf{B}}{\partial\zeta}\frac{\partial\mathbf{f}'}{\partial p_\zeta} = -\frac{\partial f_1^0}{\partial t} - \frac{\partial f_1^0}{\partial\varepsilon}e\mathbf{v}\mathbf{E},$$
$$\frac{\partial\mathbf{f}'}{\partial t} + \frac{\partial\mathbf{f}'}{\partial\zeta}v_\zeta + \frac{1}{T_0}\frac{\partial\mathbf{f}'}{\partial\tau} + [\mathbf{f}'\boldsymbol{\Omega}] + \left(\frac{\partial\mathbf{f}'}{\partial t}\right)_{\text{col}}$$
$$- \mu\frac{\partial\mathbf{B}}{\partial\zeta}\frac{\partial f_1'}{\partial p_\zeta} + i\mu\left[\frac{\partial\mathbf{B}}{\partial\zeta}, \frac{\partial\mathbf{f}'}{\partial p_\zeta}\right]$$
$$= -\frac{\partial\mathbf{f}^0}{\partial t} - \frac{\partial\mathbf{f}^0}{\partial\varepsilon}e\mathbf{v}\mathbf{E}, \quad \boldsymbol{\Omega} = 2\mu\mathbf{B}/\hbar.$$

We can show, using direct estimates, that we can neglect all terms pertaining to the second component in (10). Here the equations take the form

$$\frac{\partial f_1'}{\partial t} + \frac{\partial f_1'}{\partial \zeta} v_\zeta + \frac{1}{T_0}\frac{\partial f_1'}{\partial \tau} + \left(\frac{\partial f_1'}{\partial t}\right)_{\text{col}} = -\frac{\partial f_0}{\partial \varepsilon} e\mathbf{vE}, \tag{12}$$

$$\frac{\partial \mathbf{f}'}{\partial t} + \frac{\partial \mathbf{f}'}{\partial \zeta} v_\zeta + \frac{1}{T_0}\frac{\partial \mathbf{f}'}{\partial \tau} + [\mathbf{f}'\Omega] + \left(\frac{\partial \mathbf{f}'}{\partial t}\right)_{\text{col}} = -\frac{\partial \mathbf{f}^0}{\partial t}.$$

The first expression permits us to determine, in accord with (5), the relation between the current density j and the intensity of the electric field E:

$$\mathbf{j} = 2eh^{-3}\int \mathbf{v} f_1' d\tau_p.$$

This connection was found in Refs. 7 and 9. We are interested only in the function f' which makes possible the determination of the spin moment M:

$$\mathbf{M} = \chi\mathbf{B} + 2\mu h^{-3}\int \mathbf{f}' d\tau_p. \tag{13}$$

For the solution of Eq. (12) we must write out the concrete form of the collision integral. The vector f' is changed in collisions both as a consequence of the redistribution of electrons in energy and momentum (with relaxation times t_0^ϵ and t_0^p), and as a consequence of the redistribution of their spins (with relaxation time T_{ff}). In this case, as has already been pointed out $T_{ff} \gg t_0$, so that the two types of collisions can be considered separately:

$$\left(\frac{\partial \mathbf{f}'}{\partial t}\right)_{\text{col}} = \left(\frac{\partial \mathbf{f}'}{\partial t}\right)_{t_0} + \left(\frac{\partial \mathbf{f}'}{\partial t}\right)_{T_{ff}}.$$

Without taking a specific form for these operators, let us write them out as is usually done, with the aid of the corresponding relaxation times. It is obvious that in a wholly equilibrium state, $f' = 0$; therefore,

$$(\partial \mathbf{f}'/\partial t)_{T_{ff}} = \mathbf{f}'/T_{ff}.$$

Let us determine to what equilibrium value f'_{eq} the collisions without spin reversal. Since in such collisions the probability density of finding a given projection of the spin, independent of the values of the energy and momentum, does not change,

$$\int \mathbf{f}' d\tau_p = \int \mathbf{f}'_{eq}(\varepsilon)\, d\tau_p.$$

For simplicity, we shall consider that any change in energy in the collisions can be neglected. Then

$$\mathbf{f}_{eq} = \int \hat{\mathbf{f}}' \frac{ds}{v} \Big/ \int \frac{ds}{v} = \overline{\mathbf{f}}'; \qquad \left(\frac{\partial \mathbf{f}'}{\partial t}\right)_{t_0} = \frac{\mathbf{f}' - \overline{\mathbf{f}}'}{t_0}.$$

(This is clearly valid either for sufficiently low temperatures, when $t_0^\epsilon \gg t_0^p$, and the collisions can be regarded as elastic, or in sufficiently weak magnetic fields, in which $\mu H_0 \ll kT$.) In the general case,

$$\left(\frac{\partial \mathbf{f}}{\partial t}\right)_{\text{col}} = \frac{1}{t_0^p}(\mathbf{f} - \overline{\mathbf{f}}) + \frac{1}{t_0^\varepsilon}(\mathbf{f} - \widetilde{\mathbf{f}}\chi^0/\widetilde{\chi}^0), \quad \widetilde{\varphi} = \int \varphi\, d\tau_p,$$

$$\chi^0 = {}^1/{}_2[f_0(\varepsilon - \Delta) - f_0(\varepsilon + \Delta)], \quad |\widetilde{\mathbf{f}}| = \widetilde{\chi}^0.)$$

Thus the problem reduces to the solution of the Maxwell equations (2) with the current density j(E) determined in Refs. 7 and 9, and the magnetic moment

$$\mathbf{M} = \chi\mathbf{B} + 2\mu h^{-3}\int \mathbf{f}' d\tau_p,$$

where f' satisfies the equation

$$\frac{\partial \mathbf{f}'}{\partial t} + \frac{\partial \mathbf{f}'}{\partial \zeta} v_\zeta + \frac{1}{T_0}\frac{\partial \mathbf{f}'}{\partial \tau} + [\mathbf{f}'\Omega] + \frac{\mathbf{f}'}{t_0^*} = \frac{\overline{\mathbf{f}}'}{t_0} - \frac{\partial \mathbf{f}^0}{\partial t}, \quad \frac{1}{t_0^*} = \frac{1}{t_0} + \frac{1}{T_{eq}}$$

with boundary conditions

$$\mathbf{f}'(v_\zeta)|_{\zeta=0} = (1 - q)\overline{\mathbf{f}}'|_{\zeta=0} + q\mathbf{f}'_1(-v_\zeta)|_{\zeta=0}.$$

In the case of a half space, evidently $\mathbf{f}'(-v_\zeta < 0) = 0$ for $\zeta = \infty$; the function f' must be periodic in τ with period $\theta = (\frac{1}{2}\pi m_0)\,\partial S/\partial\varepsilon$.

For simplicity, we shall consider that $\mu H/kT \ll 1$, so that $\partial \mathbf{f}^0/\partial t = -\mu f_0'(\varepsilon)\,\partial \mathbf{H}/\partial t$. Let us set $\mathbf{f}' = \mu f_0'(\varepsilon)\mathbf{w}'$. Then we get

$$\mathbf{M} = \chi(\mathbf{B} - \overline{\mathbf{w}}'), \tag{14}$$

$$\frac{\partial \mathbf{w}'}{\partial t} + \frac{\partial \mathbf{w}'}{\partial \zeta} v_\zeta + \frac{1}{T_0}\frac{\partial \mathbf{w}'}{\partial \tau} + [\mathbf{w}'\Omega] + \frac{\mathbf{w}'}{t_0^*} = \frac{\overline{\mathbf{w}}'}{t^0} + \frac{\partial \mathbf{B}}{\partial t},$$

$$\frac{1}{t_0^*} = \frac{1}{t_0} + \frac{1}{T_{ff}}. \tag{15}$$

We now introduce the cyclic variables w and w_z:

$$iwH_0 = w'_x + iw'_y, \quad w'_z B_0 = w_z,$$
$$B_1 = B_{1x} + iB_{1y}, \quad M_1 = M_{1x} + iM_{1y}. \tag{16}$$

Then Eqs. (14) and (15) take the form

$$M_z = \chi B_0 (1 - \bar{w}_z), \quad M = -i\chi B_0 \bar{w},$$
$$\left(\frac{\partial}{\partial t} + \hat{D}\right) w_z = \bar{w}_z / t_0 + \operatorname{Re}(w \Omega_1^*),$$
$$\left(\frac{\partial}{\partial t} + \hat{D} - i\Omega_0\right) w = \frac{\bar{w}}{t_0} - i \frac{1}{B_0} \frac{\partial B_1}{\partial t} - \frac{2\mu}{h} B_1 w_z, \tag{17}$$
$$\hat{D} = \frac{\partial}{\partial \zeta} v_\zeta + \frac{1}{T_0} \frac{\partial}{\partial \tau} + \frac{1}{t_0^*}, \quad \Omega = \frac{2\mu \mathbf{H}}{h}.$$

We consider only that component Ω_1 which yields a resonance, *i.e.*, $\Omega_1 = \Omega_1 e^{i\omega t}$ (we shall denote the amplitude by the same letter as the function). Then the solution has the form: $w = we^{i\omega t}$; w_z does not depend on the time. The equations for the determination of $w(\zeta)$ and $w_z(\zeta)$ close to resonance, at $\omega = \Omega_0$, are written

$$\{\hat{D} + i(\omega - \Omega_0)\} w = \bar{w}/t_0 + \Omega_1 (1 - w_z),$$
$$\hat{D} w_z = \bar{w}_z / t_0 + \operatorname{Re}(\Omega_1^* w). \tag{18}$$

Thus the problem reduces to the solution of the system of Eq. (18) and the Maxwell equations. Evidently the system of these equations is non-linear in general, because of the nonlinearity of the coupling of the magnetic moment M with the field $B_1 = \hbar \Omega_1 / 2\mu$ [this coupling is also determined by the equations (18)].

Chief interest is presented by the case of sufficiently large fields B_1, in which the electron gas close to the metallic surface is almost completely depolarized: $w_z \approx 1$, but at sufficiently great depths, the depolarization is naturally small: $w_z \ll 1$. In this case the usual linearization is not possible. It would appear that an essential nonlinearity can take place only in the region close to resonance. Let us investigate this region in somewhat more detail. First of all, we note that the solution of the first of Eqs. (18):

$$\frac{\partial w}{\partial \zeta} v_\zeta + \frac{1}{T_0} \frac{\partial w}{\partial \tau} + \left\{\frac{1}{t_0} + \frac{1}{T_{ff}} + i(\omega - \Omega_0)\right\} w = \frac{\bar{w}}{t_0} + \Omega_1 (1 - w_z) \tag{19}$$

has a sharply resonant character at $\omega T_{ff} \gg 1$, ωt_0, *independently* of the relation between ω and $1/t_0$ (in the particular case when $\omega t_0 \ll 1$). This is connected with the fact that, at $\omega = \Omega_0$, $1/T_{ff} = 0$ is an *eigenvalue* of Eq. (19), since in this case, the homogeneous equation

$$\frac{\partial w}{\partial \zeta} v_\zeta + \frac{1}{T_0} \frac{\partial w}{\partial \tau} + \frac{w}{t_0} - \frac{\bar{w}}{t_0} = 0$$

has the nontrivial solution $w = w(\varepsilon)$, which is independent of the coordinates and of τ.

In correspondence with this, the solutions of Eqs. (18) near resonance (for $|\omega - \Omega_0| \lessdot 1/T_{ff} \ll 1/t_0$) will, in the first place, have a significantly larger $\Omega_1 t_0$ [since for $\omega = \Omega_0$ and $T_{ff} \to \infty$, w generally diverges as $(t_0/T_{ff})^{-1/2}$], meaning that it will differ only slightly from $\bar{w}$ and $\bar{w}_z$ (since $w - \bar{w} \sim \Omega_1 t_0$); in the second place, they are more slowly varying with distance than Ω_1 (the reason for this is discussed in Sec. 1), and in the third place they depend on the behavior of Ω_1 only at small distances [since Eq. (19) has a smoothly varying solution even for Ω_1 which is a δ-function in the coordinates]. Therefore,

$$\Omega_1 (1 - w_z(\zeta)) \approx \Omega_1 (1 - \bar{w}_z(\zeta)) \approx \Omega_1 (1 - \bar{w}_z(0));$$
$$\Omega_1^* w(\zeta) \approx \Omega_1^* \bar{w}(\zeta) \approx \Omega_1^* \bar{w}(0).$$

At large distances, Ω_1 changes as slowly as $\bar{w}$ and $\bar{w}_z$; however, as was pointed out, the value of the right side at such distances has no effect on the form of $\bar{w}$ and $\bar{w}_z$. Of course, all these assertions can be verified.

Thus, Eq. (18) near resonance can be written in the form

$$\{\hat{D} + i(\omega - \Omega_0)\} w = \bar{w}/t_0 + \Omega_1 [1 - \bar{w}_z(0)], \tag{20}$$
$$\hat{D} w_z = \bar{w}_z / t_0 + \operatorname{Re}[\Omega_1^* \bar{w}(0)]. \tag{21}$$

We set

$$w = [1 - \bar{w}_z(0)] u, \quad w_z = \operatorname{Re}[\bar{w}(0) u_z]. \tag{22}$$

Then Eqs. (20) take the form

$$\{\hat{D} + i(w - \Omega_0)\} u = \bar{u}/t_0 + \Omega_1,$$
$$u_z = u^*|_{\omega = \Omega_0} \tag{23}$$

$$u(0, v_\zeta) = (1 - q)\bar{u}(0) + qu(0, -v_\zeta), \quad v_\zeta > 0, \tag{24}$$

where

$$w_z(\zeta) = \frac{\operatorname{Re}[\bar{u}(0) u_z(\zeta)]}{1 + \operatorname{Re}[\bar{u}(0) \bar{u}_z(0)]},$$
$$w(\zeta) = \frac{u(\zeta)}{1 + \operatorname{Re}[\bar{u}(0) \bar{u}_z(0)]}. \tag{25}$$

The magnetization M_z and M and the polarization of the nuclei P are determined by Eqs. (17) and (1a), as before.

3. SOLUTION OF THE EQUATIONS FOR THE BULK METAL

a) *Special case*

Let us consider the simplest case (in mathematical behavior) of a quadratic dispersion $\varepsilon = p^2/2m^*$ (m^* = effective mass), a field H_0 perpendicular to the surface of the metal (here the z and ζ axes coincide) and specular reflection of the electrons from the surface, *i.e.*, $q = 1$. (We note that the quantity q does not depend on the qualitative results.) Then Eq. (23) takes the form (u independent of τ):

$$v_z\frac{\partial u}{\partial z}+\frac{u}{t}=\frac{\bar{u}}{t_0}+\Omega_1,\quad \frac{1}{t}=\frac{1}{t_0}+\frac{1}{T_{ff}}+i(\omega-\Omega_0). \tag{26}$$

The boundary conditions are

$$u(0, v_z)=u(0,-v_z),\quad u(\infty,-v_z)=0,\ v_z>0.$$

Finding the solution of Eq. (26), and averaging it over the Fermi surface, we get an integral equation for $\bar{u}$:

$$\bar{u}(z)=\int_{-\infty}^{\infty}R(|z-\zeta|)\left\{\frac{t}{t_0}\bar{u}(\zeta)+t\,\Omega_1(\zeta)\right\}d\zeta,$$

$$R(|z-\zeta|)=\frac{1}{2l}\int_1^{\infty}\exp\left\{-\frac{|z-\zeta|}{l}x\right\}\frac{dx}{x}, \tag{27}$$

$$l=v_0t,\ \Omega_1(-\zeta)=\Omega_1(\zeta).$$

[We note that for $\Omega_1 = 0$ and $t/t_0 = 1$, $\bar{u}$ = const is a solution of Eq. (27)]. From (27) we find

$$\bar{u}(z)=\frac{1}{\pi}\int_0^{\infty}\frac{R_k t_0\,\Omega_{1k}\cos kzdk}{t_0/t-R_k}, \tag{28}$$

$$R_k=\tan^{-1}kl/kl,\ \Omega_{1k}=2\int_0^{\infty}\Omega_1(z)\cos kzdz.$$

by making use of a Fourier transformation. We note that close to resonance ($|\omega-\Omega_0|\sim 1/T_{ff}$) the only essential k are those for which $kl\lesssim(t_0/T_{ff})^{1/2}$. For such kl, $R_k\approx 1-k^2l^2/3$ and

$$\bar{u}(z)\approx\frac{1}{\pi}\int_0^{\infty}\frac{t_0\,\Omega_{10}\cos kzdk}{t_0/T_{ff}+it_0(\omega-\Omega_0)+k^2l^2/3}$$

$$=\frac{t_0\,\Omega_{10}}{2l}\sqrt{\frac{3}{\varkappa}}\exp\left\{-\frac{z}{l}\sqrt{3\varkappa}\right\},$$

$$\varkappa=t_0/T_{ff}+it_0(\omega-\Omega_0).$$

The Maxwell equations (2) for

$$H_1=H_{1x}+iH_{1y}=H_1e^{i\omega t};$$

$$E=E_x+iE_y=Ee^{i\omega t};\ j=j_x+ij_y=je^{i\omega t}$$

are written in the form

$$dE/dz=-\omega B_1/c;\quad dH_1/dz=-4\pi ij/c.$$

Therefore,

$$\Omega_{10}=2\int_0^{\infty}\Omega_1(z)\,dz=\frac{4\mu c}{\hbar\omega}E(0),$$

where $E(0)$ is the field at the surface of the metal. Thus, close to resonance,

$$\bar{u}(z)=\frac{3cE(0)\,\delta_{\mathrm{eff}}}{vB_0l}e^{-z/\delta_{\mathrm{eff}}};$$

$$\delta_{\mathrm{eff}}=v\sqrt{\frac{t_0T_{ff}}{3(1+i(\omega-\Omega_0)T_{ff})}}. \tag{29}$$

It is seen from this equation that the width of the resonance line is determined only by the quantity T_{ff}: $|\omega-\Omega_0|\sim 1/T_{ff}$ which was first shown by Dyson[4].

For the magnetization, substituting the value of $\bar{u}(z)$ in Eq. (17), we get, at resonance,

$$M_z=\chi B_0\left\{1-\frac{|\alpha|^2}{1+|\alpha|^2}e^{-z/\delta_{\mathrm{eff}}}\right\},$$

$$M=M_x+iM_y=-i\chi B_0\frac{\alpha}{1+|\alpha|^2}e^{-z/\delta_{\mathrm{eff}}},$$

where

$$\alpha=\frac{3c\{E_x(0)+iE_y(0)\}\,\delta_{\mathrm{eff}}}{B_0v_0l}.$$

We note that for sufficiently weak field B_1, when saturation is absent, *i.e.*, $|\alpha|\ll 1$, the equations for M_x and M_y undergo (with accuracy up to an exponential factor) a transition to the Dyson formula[4], where we must set $\nu=\omega$.

Thus, we have shown that $\bar{u}(z)$ actually vanishes at the depth $\delta_{\mathrm{eff}}\gg\delta$. Moreover, u and $\bar{u}\gg\Omega_1t_0$, but $u-\bar{u}\sim\Omega_1t_0$, *i.e.*, $u-\bar{u}\ll\bar{u}$. Thus the assumptions of the preceding section are valid.

We note that $H_1(z)$ can be represented qualitatively in the form of two parts: a large, rapidly attenuating part, and a small, slowly attenuating part:

$$H_1(z)\sim H_1(0)\,e^{-z/\delta}-4\pi i\chi H_0\,\alpha e^{-z/\delta_{\mathrm{eff}}}.$$

b) *General case.*

Let us find the quantities of interest to use for arbitrary assumptions on the dispersion law $\varepsilon(\mathbf{p})$ and for arbitrary magnitude and direction of the constant field $\mathbf{H}_0$. For simplicity, we consider only the case of resonance. The reflection of the electrons from the surface we shall consider diffuse ($q = 0$) which is practically always the case.

As was shown, the problem reduced to finding a solution of the equation

$$v_\zeta \frac{\partial u}{\partial \zeta} + \frac{1}{T_0}\frac{\partial u}{\partial \tau} + \frac{u}{t_0^*} = \frac{\bar{u}}{t_0} + \Omega_1 \tag{30}$$

which is periodic in τ with the boundary conditions:

$$u(0, \mathbf{vn}) = \bar{u}(0); \quad u(\infty, -\mathbf{vn}) = 0, \quad \mathbf{vn} > 0. \tag{31}$$

We introduce

$$\xi = \zeta/r_0, \quad \gamma^* = T_0/t_0^*, \quad \gamma = T_0/t_0,$$

$$v_\zeta/v_0, = V_\zeta, \quad r_0 = v_0 T_0,$$

v_0 = characteristic velocity on the Fermi surface. Then Eq. (30) is written

$$\frac{\partial u}{\partial \xi} + \hat{L}u = \frac{\gamma}{V_\zeta}(\bar{u} + t_0\Omega_1) \equiv \frac{\gamma}{V_\zeta}\psi;$$

$$\hat{L} = \frac{1}{V_\zeta}\left(\frac{\partial}{\partial \tau} + \gamma^*\right). \tag{32}$$

For solution of this equation, we apply the method developed in Ref. 9. We replace $\mathbf{p}$ in Eq. (32) by $-\mathbf{p}$; then we obtain for the function $u(-\mathbf{v})$ the equation

$$\partial u(-\mathbf{v})/\partial \xi - \hat{L}u(-\mathbf{v}) = -\gamma\psi/V_\zeta. \tag{33}$$

Here use is made of the fact that $\varepsilon(-\mathbf{p}) = \varepsilon(\mathbf{p})$, $\mathbf{v}(-\mathbf{p}) = -\mathbf{v}(\mathbf{p})$,

$$\frac{1}{T_0}\frac{\partial}{\partial \tau}\Big|_{\mathbf{p}} \equiv \left[\mathbf{v}\frac{\partial}{\partial \mathbf{p}}\right] = \frac{1}{T_0}\frac{\partial}{\partial \tau}\Big|_{-\mathbf{p}}.$$

Acting on Eq. (32) with the operator $\partial/\partial\xi - \hat{L}$, and on (33) with the operator $\partial/\partial\xi + \hat{L}$, and reducing them, we obtain an equation for the function $f = \frac{1}{2}[u(\mathbf{v}) + u(-\mathbf{v})]$:

$$(\partial^2/\partial\xi^2 - \hat{L}^2) f = -\gamma\psi\hat{L}/V_\zeta, \tag{34}$$

where it is taken into account that $\bar{f}(\xi) = \bar{u}(\xi)$.

In this equation, we continue the functions $f(\xi)$ and $\Omega_1(\xi)$ as even functions into the region $\xi < 0$:

$$f(-\xi) = f(\xi), \quad \Omega_1(-\xi) = \Omega_1(\xi)$$

and go over to the Fourier transforms

$$\varphi(k) = \int_{-\infty}^{\infty} f(\xi) e^{ik\xi} d\xi, \quad \Omega_1(k) = \int_{-\infty}^{\infty} \Omega_1(\xi) e^{ik\xi} d\xi,$$

(obviously, $\bar{\varphi}(k) = \bar{u}(k)$):

$$(\hat{L}^2 + k^2)\varphi(k) = \gamma\psi(k)\hat{L}/V_\zeta - 2f'(0). \tag{35}$$

For determination of $f'(0)$, we note that, from Eqs. (32) and (33),

$$f'(0) = -\tfrac{1}{2}\hat{L}\{u(\mathbf{v}) - u(-\mathbf{v})\}.$$

We now make use of the boundary condition (31). Since, for $V_\zeta > 0$ on the surface of the metal, independently of the other projections of the velocity $u = \bar{u} = \bar{f}$, then, as is easy to see,

$$u(\mathbf{v}) - u(-\mathbf{v}) = \operatorname{sign} V_\zeta \cdot (\bar{f}(0) - f(0)),$$
$$f'(0) = -\hat{L}\operatorname{sign} V_\zeta \cdot (\bar{f}(0) - f(0)).$$

Consequently, Eq. (35) takes the form

$$(\hat{L}^2 + k^2)\varphi(k)$$
$$= (2\hat{L}/V_\zeta)\{\tfrac{1}{2}\gamma\psi(k) + V_\zeta|[\bar{f}(0) - \bar{f}(0)]\},$$
$$\varphi(k) = \{(\hat{L} + ik)^{-1} + (\hat{L} - ik)^{-1}\}$$
$$\times\{\tfrac{1}{2}\gamma\psi(k) + |V_\zeta|[\bar{f}(0) - f(0)]\}\, 1/V_\zeta \equiv g_+ + g_-.$$

Computation of the right side of this expression reduces to finding the periodic solution of the linear equation

$$(\partial/\partial\tau + \gamma^* \pm ikV_\zeta)g_\pm$$
$$= \tfrac{1}{2}\gamma\psi(k) + |V_\zeta|[\bar{f}(0) - f(0)].$$

This solution has the form

$$g_\pm = [\exp(\gamma^*\theta \pm ik\tilde{V}_\zeta) - 1]^{-1}$$
$$\times \int_\tau^{\tau+\theta} \exp\{\gamma^*(\tau' - \tau) \pm ik\int_\tau^{\tau'} V_\zeta\, d\tau''\}\{\tfrac{1}{2}\gamma\psi(k)$$
$$+ |V_\zeta|[\bar{f}(0) - f(0)]\}\, d\tau';$$
$$\tilde{V}_\zeta = \int_0^\theta V_\zeta\, d\tau.$$

Therefore, remembering that $\psi(k) = \bar{u}(k) + t_0\Omega_1(k)$, we get

$$\varphi(k,\tau)=R(k,\tau)\,\bar{u}(k)+R(k,\tau)\,t_0\,\Omega_1(k)+\int\limits_{\tau}^{\tau+\theta}N(k,\tau,\tau')\,[\bar{f}(0)-f(0)]\,d\tau', \tag{36}$$

where

$$R(k,\tau)=\operatorname{Re}\gamma\,[\exp(\gamma^*\theta+ik\widetilde{V}_\zeta)-1]^{-1}\int\limits_{\tau}^{\tau+\theta}\exp\{\gamma^*(\tau'-\tau)+ik\int\limits_{\tau}^{\tau'}V_\zeta\,d\tau''\}\,d\tau',$$

$$N(k,\tau,\tau')=2\operatorname{Re}|V_\zeta|\,[\exp(\gamma^*\theta+ik\widetilde{V}_\zeta)-1]^{-1}\exp\{\gamma^*(\tau'-\tau)+ik\int\limits_{\tau}^{\tau'}V_\zeta d\tau''\}.$$

(We note that although $\bar{f}=f\ll\bar{f}$, $\bar{f}-f\sim t_0\,\Omega_1$. Therefore we cannot neglect this difference.)

For convenience, we introduce

$$\Sigma(k,\tau)=\bar{\varphi}(k,\tau)-\varphi(k,\tau),\qquad \overline{\Sigma(k,\tau)}=0,$$

$$S(\tau)=\overline{f(0,\tau)}-f(0,\tau)=\lim_{\xi\to0}\frac{1}{\pi}\int\limits_0^\infty\Sigma(k,\tau)\cdot\cos\xi k\cdot dk=\int\limits_0^\infty\frac{1}{\pi}\Sigma(k,\tau)\,dk, \bar{S}=0.$$

Then Eq. (36) can be written in the form

$$\Sigma(k,\tau)=[1-R(k,\tau)]\,\bar{u}(k)-R(k,\tau)\,t_0\,\Omega_1(k)-\int\limits_{\tau}^{\tau+\theta}N(k,\tau,\tau')\,S(\tau')\,d\tau'. \tag{37}$$

Averaging this equation over the Fermi surface, we find the function $\bar{u}(k)$:

$$\bar{u}(k)=\frac{\overline{R(k,\tau)\,t_0\Omega_1(k)}}{1-\overline{R(k,\tau)}}+\frac{1}{1-\overline{R(k,\tau)}}\overline{\int\limits_{\tau}^{\tau+\theta}N(k,\tau,\tau')\,S(\tau')\,d\tau'}. \tag{38}$$

For the determination of $S(\tau)$, we substitute $u(k)$ in Eq. (37) and integrate the latter over k from 0 to ∞. We then obtain the integral equation

$$S(\tau)=\frac{1}{\pi}\int\limits_0^\infty\frac{\overline{R(k,\tau)}-R(k,\tau)}{1-\overline{R(k,\tau)}}\,t_0\Omega_1(k)\,dk$$

$$-\frac{1}{\pi}\int\limits_0^\infty dk\int\limits_{\tau}^{\tau+\theta}N(k,\tau,\tau')\,S(\tau')\,d\tau'$$

$$+\frac{1}{\pi}\int\limits_0^\infty\frac{1-R(k,\tau)}{1-\overline{R(k,\tau)}}\,dk\,\overline{\int\limits_{\tau}^{\tau+\theta}N(k,\tau,\tau')\,S(\tau')\,d\tau'},$$

wherein we must set $\gamma^*=\gamma$. In a fashion similar to that of Ref. 9, we can solve the resultant equation by the method of successive approximations, and show that $S(\tau)\sim t_0\Omega_1(k)$ and that it has no singularities for any values of τ.

Returning to Eq. (38), we note that for sufficiently small k (such that $kl\ll1$)

$$\overline{R(k,\tau)}=t_0^*/t_0-l^2k^2,$$

where

$$l^2=(\gamma/\theta\int dp_z)\int\frac{dp_z}{(e^{\gamma^*\theta}-1)^3}\int\limits_0^\theta d\tau\int\limits_0^\theta e^{\gamma^*\tau'}\varphi(\tau,\tau')\,d\tau',$$

$$\varphi(\tau,\tau')=e^{\gamma^*\theta}(e^{\gamma^*\theta}+1)\,\widetilde{V}_\zeta^2$$

$$-2\int\limits_{\tau}^{\tau+\tau'}V_\zeta d\tau''e^{\gamma^*\theta}(e^{\gamma^*\theta}-1)\,\widetilde{V}_\zeta$$

$$+\Big(\int\limits_{\tau}^{\tau+\tau'}V_\zeta d\tau''\Big)^2(e^{\gamma^*\theta}-1)^2.$$

It is easy to see that in weak fields, $H_0\,(\gamma\gg1)\,l\sim1/\gamma$. For strong fields $H_0\,(\gamma\ll1)$ two cases are possible:

1. If the field H_0 forms an angle with the surface $\varphi\gtrsim\gamma$ (in this case $\widetilde{V}_\zeta$ is not small), then $\varphi(\tau,\tau')\approx2\widetilde{V}_\zeta^2$ and, as before, $l\sim1/\gamma$.

2. If the field H_0 forms and angle with the surface $\varphi\lesssim\gamma$ (in this case $\widetilde{V}_\zeta\approx0$), then $l\sim1$.

As in case (a), we have

$$\bar{u}(\xi) = \frac{1}{\pi}\left\{t_0\Omega_1(0) + \overline{\int_\tau^{\tau+0} N(0,\tau,\tau')\,S(\tau')\,d\tau'}\right\} \int_0^\infty \frac{\cos k\xi\cdot dk}{t_0/T_{ff} + l^2k^2}.$$

It follows from the Maxwell equations that $\Omega_1(0) = (4\mu c/\hbar\omega)E(0)$.

Taking it into account that $S(\tau) \sim t_0\Omega_1(0)$, we get

$$\bar{u}(\zeta) = A\frac{ct_0E(0)}{B_0r_0l}\sqrt{\frac{T_{ff}}{t_0}}\,e^{-\zeta/\delta_{\text{eff}}},\quad \delta_{\text{eff}} = lr_0\sqrt{T_{ff}/t_0},\quad A\sim 1,\quad E(0) = (cZ/4\pi)\,H_1(0), \tag{39}$$

Z is the surface impedance which was found in Ref. 7.

The exact value of l depends on the dispersion law and the direction of the field H_0. Thus, for quadratic dispersion and a constant field perpendicular to the surface of the metal, $l = 1/\gamma\sqrt{3}$, and we again obtain Eq. (29).

Making use of Eqs. (25) and (1a), we obtain the polarization of the nuclei in the bulk metal at resonance:

$$P = \frac{1}{I}\left\{\left(I+\frac{1}{2}\right)\coth\left(I+\frac{1}{2}\right)\frac{|\alpha|^2e^{-\zeta/\delta_{\text{eff}}}}{1+|\alpha|^2}\frac{\mu H_0}{kT} - \frac{1}{2}\coth\frac{1}{2}\frac{|\alpha|^2e^{-\zeta/\delta_{\text{eff}}}}{1+|\alpha|^2}\frac{\mu H_0}{kT}\right\},$$

$$\alpha = A\,(ct_0E(0)/H_0r_0l)\sqrt{T_{ff}/t_0}.$$

Thus, in the bulk metal, a substantial polarization of the nuclei takes place to a depth of $\delta_{\text{eff}} \sim v_0T_0(T_{ff}/t_0)^{1/2}$ in the case of a strong field H_0 parallel to the surface, and to a depth $\delta_{\text{eff}} \sim v_0t_0(T_{ff}/t_0)^{1/2}$ for all other cases.

4. SELECTIVE TRANSPARENCY OF A FILM

In order to find the transmission coefficient of an electromagnetic wave through a film of thickness d, it is necessary to solve Eq. (23) with the two boundary conditions (24).

For simplicity we consider the case of a square law of dispersion $\varepsilon = p^2/2m^*$, a field H_0 perpendicular to the surface, and mirror reflection of electrons from the surface: $q = 1$ (inasmuch as the character of the dispersion law and the boundary conditions affect the results only quantitatively, as we have already seen).

In this case, Eq. (23) takes the form (since u obviously does not depend on τ)

$$v_z\partial u/\partial z + u/t_0^* = \bar{u}/t_0 + \Omega_1. \tag{40}$$

The boundary conditions are written

$$u(0, v_z) = u(0, -v_z),\quad u(d, -v_z) = u(d, v_z),\quad v_z > 0.$$

Finding $u(z)$ and averaging it over the Fermi surface, we obtain the integral equation for $\bar{u}(z)$:

$$\bar{u}(z) = \frac{1}{2v_0}\int_{-k}^{k} R(|t-\zeta|)\,[\bar{u}(\zeta) + t_0\Omega_1(\zeta)]\,d\zeta, \tag{41}$$

$$k = \frac{d}{v_0t_0^*},\qquad R(t) = \frac{t_0^*}{t_0}\int_1^\infty \frac{\cosh(k-t)x}{\sinh kx}\frac{dx}{x}.$$

The function $R(t)$ is even and periodic in the interval $(-2k, 2k)$ with period $2k$. Thanks to this, we can solve Eq. (41) by the expansion of all functions in Fourier series (cosines) with period π/k. The solution has the form

$$\bar{u}(z) = \frac{\bar{u}_0}{2} + \sum_{n=1}^\infty \bar{u}_n\cos\frac{\pi nz}{k},$$

$$\bar{u}_n = \frac{kt_0R_n\Omega_{1n}}{1-kR_n},\qquad \Omega_{1n} = \frac{2}{k}\int_0^k \Omega_1(z)\cos\frac{\pi nz}{k}\,dz,$$

$$R_n = \int_0^\infty R(t)\cos\frac{\pi nt}{k}\,dt = \frac{t_0^*}{kt_0}\frac{\tan^{-1}(\pi n/k)}{\pi n/k}. \tag{42}$$

In resonance, the chief contributions in (42) are clearly made by the $\bar{u}_n$ with small n (so that $\pi n/k \ll 1$). In this case,

$$u_n \approx t_0\Omega_{1n}\Big/\left[\frac{t_0}{T_{ff}} + \frac{1}{3}\left(\frac{\pi n}{k}\right)^2\right].$$

But in $\Omega_1(z)$ the essential $z \lessapprox \delta/v_0t_0$, where $\pi nz/k \lessapprox \delta/\delta_{\text{eff}} \ll 1$; therefore,

$$\Omega_{1n} \approx \frac{2}{k}\int_0^k \Omega_1(z)\cos\frac{\pi nz}{k}\,dz \approx \frac{2}{k}\int_0^k \Omega_1(z)\,dz = \Omega_{10}.$$

Thus,

$$u_n \approx t_0\Omega_{10} \Big/ \left[\frac{t_0}{T_{ff}} + {}^1/_3\left(\frac{\pi n}{k}\right)^2\right]$$
$$\bar{u}(z) = \frac{u_0}{2} + \sum_{n=1}^{\infty} u_n \cos\frac{\pi n z}{k} = {}^1/_2\,\Omega_{10} T_{ff} \frac{d}{\delta_{eff}} \frac{\cosh[(d-z)/\delta_{eff}]}{\sinh(d/\delta_{eff})}. \quad (43)$$

From Maxwell's equations we get

$$\Omega_{10} = (4\mu c/\hbar\omega)\,[E(0) - E(d)]/d.$$

(We note that for $d \to \infty$ Eq. (43) goes over into Eq. (29).

Of fundamental interest (see below) is the consideration of selective transparency of films for which $\delta \ll d \ll \delta_{eff}$. Here (at resonance),

$$\bar{u}(z) \approx cT_{ff} E(0)/H_0 d.$$

Hence

$$M = -i\chi B_0 \frac{\bar{u}}{1+|\bar{u}|^2} e^{i\omega t} \equiv M_0 e^{i\omega t}.$$

From Maxwell's equations,

$$E' = -\omega B_1/c, \quad H_1' = -4\pi i j/c;$$

since $H_1 = B_1 - 4\pi M$, then

$$E' = -\omega B_1/c, \quad B_1' = -4\pi i j/c,$$

i.e., B_1 in this approximation does not depend on the magnetization and falls off rapidly (at depths of order δ).

Consequently, at a depth $\delta_{eff} \gg z$, $d - z \gg \delta$, we have a homogeneous magnetic field $-4\pi M_0 e^{i\omega t}$ (obviously, this field always has circular polarization). Hence, taking into account the boundary conditions on the surface of the film, we easily obtain the transmission coefficient for electromagnetic waves through the film:

$$K = \left|\frac{H_{1trans}}{H_{1inc}}\right|^2 \approx \left|\frac{\chi T_{ff} c^3 Z^2}{2\pi d\,\{1 + |c^2 Z T_{ff} H_{1inc}/2\pi d H_0|^2\}}\right|^2,$$

where Z is the surface impedance without account of spin polarization.

The unusual form of the equation for K is connected with the specific change of the field in the film as a result of the diffusion of the spin [see Eq. (29a)].

The power of the previous wave W_{trans} will be maximum in that case in which

$$H_{inc}^{opt} = 2\pi d H_0/c^2 T_{ff}|Z|.$$

Here

$$W_{trans}^{max} = \frac{\pi}{16}\left(\frac{\chi\hbar c^2|Z|}{\mu\lambda}\right); \qquad H_{trans}^{max} = \frac{\pi\chi\hbar c^2|Z|}{2\mu\lambda},$$

i.e., both these quantities are independent of the thickness of the film (but $H_{inc}^{opt} \sim d$).

In the general case,

$$W_{trans} = 4W^{max}\{H_{1inc}/H_{1inc}^{opt}\}^2/\{1 + (H_{1inc}/H_{1inc}^{opt})^2\}^{-2};$$

$$H_{1trans} = 2H_{trans}^{max}\,|H_{1inc}/H_{1inc}^{opt}|/[1 + |H_{1inc}/H_{1inc}^{opt}|^2].$$

5. POLARIZATION OF NUCLEI IN FILMS

We write out the formula for the polarization of nuclei in films* (at resonance):

$$P = \frac{1}{I}\{(I + {}^1/_2)\coth(I + {}^1/_2)A - {}^1/_2\coth{}^1/_2 A\};$$

$$A = \frac{|\alpha|^2}{1+|\alpha|^2}\frac{\mu H_0}{kT}\frac{\cosh[(d-z)/\delta_{eff}]}{\cosh(d/\delta_{eff})},$$

where

$$\alpha = \frac{cT_{ff}[E(0) - E(d)]}{H_0\delta_{eff}} \coth\frac{d}{\delta_{eff}}$$

for arbitrary d. In the case $d < \delta$,

$$a = 4\mu T_{ff} H_1/\hbar,$$

i.e., we get the Overhauser formula. In the case $\delta \ll d \ll \delta_{eff}$:

$$a = cT_{ff}E(0)/H_0 d = c^2 Z T_{ff} H_{inc}/2\pi d H_0.$$

Finally, for $d \gg \delta_{eff}$, we get the formula for the bulk metal:

$$a = cT_{ff}E(0)/H_0\delta_{eff} = c^2 T_{ff} Z H_{inc}/2\pi d H_0.$$

*This formula is correct for $d - z \ll \delta$ since at such a distance from the second surface the polarization is significantly less.

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Translated by R. T. Beyer
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SOVIET PHYSICS JETP VOLUME 5, NUMBER 5 DECEMBER, 1957

Polarization Correlation in Nucleon-Nucleon Scattering

A. G. ZIMIN

(Submitted to JETP editor December 2, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1226-1232 (May, 1957)

Equations are obtained for the polarization correlation in proton-proton scattering, taking into account four phases: ${}^1S_0$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$ and Coulomb interaction. A computation using phases for the isotropic states as obtained from scattering data shows that the Coulomb interaction plays an essential role for energies of 10–30 Mev. Polarization correlation can thus be used to give a more precise determination of the isotropic phases (which do not give rise to polarization), and to estimate other phases in the energy region in which they begin to appear. We also consider the scheme of experiments for measuring the polarization correlation and obtain the combinations of components of the polarization tensor which are measured in the experiments.

1. INTRODUCTION

THE SCATTERING OF PARTICLES with spin is described by the average values of spin operators over the scattered wave. For two particles with spins $\boldsymbol{\sigma}^{(1)}$ and $\boldsymbol{\sigma}^{(2)}$, these operators are:

$$\hat{1},\ \sigma_i^{(1)},\ \sigma_i^{(2)},\ \hat{P}_{ik} = \hat{\sigma}_i^{(1)}\hat{\sigma}_k^{(2)}. \qquad (1)$$

The corresponding average values are: the scattering cross section, the polarization of the first (1) and second (2) particle, and the polarization correlation. This last quantity has a tensor character ($i, k = x, y, z$) and may be called the polarization tensor. If we represent the asymptotic form of the scattered wave as a sum of partial waves (with given j, l, s), these average quantities will be expressed in terms of the corresponding phases. The analysis of scattering of nucleons requires the inclusion of phases with $l > 0$. To determine them unambiguously we must measure all the characteristics of the scattering which relate the phases (cross section, polarization, and polarization correlation). As we shall show in detail later, measurement of the polarization correlation is especially important for determining the phases in the region of isotropic scattering of the protons. It is known that the scattering of protons is isotropic over a wide range of energy (up to 400–450 Mev), and is therefore described by the phases of the isotropic states ${}^1S_0$ and ${}^3P_0$. To separate them one might measure polarization in addition to the cross section. However, the isotropic phases give no nuclear polarization, while its Coulomb part is sizeable only at very small an-