Magnetic Interaction of Electrons and Anomalous Diamagnetism

B. T. GEILIKMAN
Moscow State Pedagogical Institute
(Submitted to JETP editor July 13, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1206-1211 (May, 1957)

It is shown that the taking into account of magnetic interaction, within the framework of perturbation theory for an ideal electron gas, does not lead to anomalous diamagnetism.

IN THE PRESENT WORK the diamagnetism of electrons in a metal is studied with the magnetic interaction of the electrons taken into account. Some considerations regarding the importance of magnetic interaction of electrons for superconductivity were presented by Welker[1]. However, in view of the complete absence there of quantitative calculations, the question remained open.

It is possible to point out more convincing arguments than those in Ref. 1 in favor of the possible importance of magnetic interaction for anomalous diamagnetism. It is not difficult to see that the energy of a system in a magnetic field, for arbitrary specification of the vector potential A, must have the following form:

$$E = \sum_{\mathbf{q}} [(\mathbf{A_q}, \mathbf{A_{-q}}) - q^{-2} (\mathbf{q}, \mathbf{A_q}) (\mathbf{q}, \mathbf{A_{-q}})] \varphi (q^2)$$
$$\mathbf{A_q} = \frac{1}{V} \int e^{-i(\mathbf{qr})/\hbar} \mathbf{A} (\mathbf{r})\, d\mathbf{r}. \quad (1)$$

Relation (1) follows from the equation of continuity; one can easily check this by using the expression for the current density $\mathbf{j}_s = -c\, \delta E/\delta \mathbf{A}$. In the case of a slowly changing field, φ can be expanded in a series,

$$\varphi (q^2) = \varphi_0 + \varphi_1 q^2 + \ldots$$

Anomalous diamagnetism is possible only when the term of zero order, φ_0, differs from zero. In fact, in this case we get for the current London's equation

$$\mathbf{j_q} \approx -c\varphi_0 (\mathbf{A_q} - q^{-2} (\mathbf{q}, \mathbf{A_q})\, \mathbf{q}),$$

$$i.e., \ \mathrm{curl}\ \mathbf{j} = -c\varphi_0 \mathbf{H}.$$

Thus in order that London's equation may be obtained (for div $\mathbf{A} \neq 0$), there must be a pole of type q^{-2} in the expression for E or $\mathbf{j_q}$. Schafroth has shown that interaction of electrons with the lattice vibrations, within the framework of perturbation theory, gives no poles of this type[2]. But Coulomb interaction of the electrons, at first glance, might give such a pole. In fact, the correction to the energy in the third approximation of perturbation theory (the second with respect to the interaction of electrons with a magnetic field, is

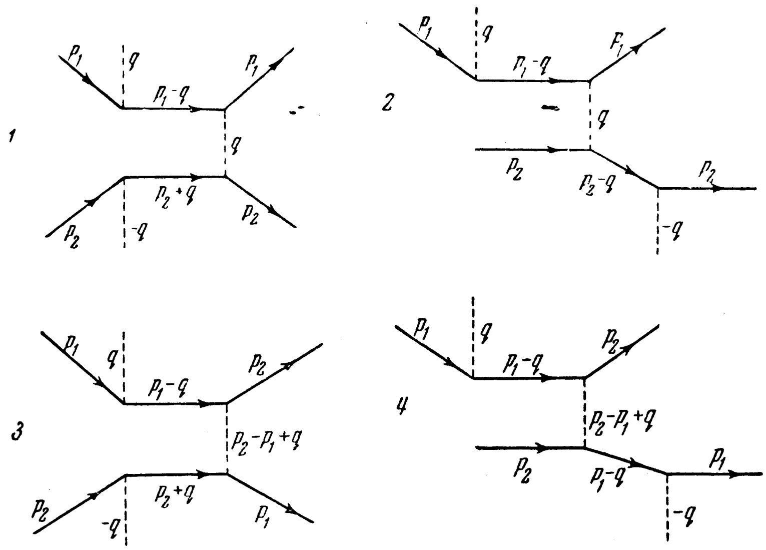
$$-(e/2mc) \sum_i [\mathbf{p}_i \mathbf{A}_i + \mathbf{A}_i \mathbf{p}_i]$$

and the first with respect to the Coulomb interaction is $\frac{1}{2} \Sigma e^2/r_{ij}$) has the form (the factor q^{-2} is characteristic of an interaction of the type $1/r$):

$$\left(\frac{e}{mc}\right)^2 \sum_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}} \frac{(\mathbf{p}_1 + \mathbf{q}/2, \mathbf{A_q})(\mathbf{p}_2 - \mathbf{q}/2, \mathbf{A_{-q}})}{(E_0 - E_1)(E_0 - E_2)} \frac{4\pi\hbar^2 e^2}{q^2 V},$$
$$\mathbf{q} = \mathbf{p}_1' - \mathbf{p}_1 = \mathbf{p}_2 - \mathbf{p}_2'. \quad (2)$$

However, as was shown by Schafroth, Coulomb interaction in perturbation theory does not lead to anomalous diamagnetism, because the summation over $\mathbf{p}_1$ and $\mathbf{p}_2$, which enters to the first power in the numerator, gives zero. In the case of magnetic interaction, there is an additional factor $(\mathbf{p}_1\mathbf{p}_2 - \frac{1}{2}\mathbf{p}_1\mathbf{q} + \frac{1}{2}\mathbf{p}_2\mathbf{q})$ in the numerator, thanks to which expression (2) will not reduce to zero. But in this case the Hamiltonian contains other terms that are likewise capable of giving an energy correction quadratic in A. It will be shown below that magnetic interaction of the electrons similarly does not lead to anomalous diamagnetism, within the framework of perturbation theory.

In the nonrelativistic approximation, the Hamiltonian contains a rather large number of terms that depend on A; it therefore proves more convenient to use the relativistic Hamiltonian for electrons interacting with a quantized electromagnetic field. In this formulation the interaction of the electrons, either Coulomb or magnetic, is not introduced ex-



plicitly but is a consequence of the exchange of virtual photons (longitudinal and transverse). Then the first nonvanishing energy correction quadratic in A is obtained in the fourth approximation of perturbation theory, *viz.*, in the second with respect to the external potential and in the second with respect to the radiation field. To this correction correspond eight diagrams: the four shown in the figure and four obtained from them by reversal of the directions of the electron paths. The contributions to the energy from two diagrams that differ only in the directions of the electron paths are exactly equal; it is therefore possible to consider only the four diagrams shown in the figure, by doubling the energy correction that corresponds to these diagrams.

We write the matrix elements for the first and second diagrams (the notation is the same as in Ref. 4):

$$S_1 = \frac{e^4 i}{(2\pi)^4}\int \bar{u}(p_2)\,\gamma_\nu \frac{i(\hat{p}_2+\hat{q})-m}{(p_2+q)^2+m^2}\,\hat{a}(-q)\,u(p_2)\,\frac{1}{q^2}\,\bar{u}(p_1)\,\gamma_\nu \frac{i(\hat{p}_1-\hat{q})-m}{(p_1-q)^2+m^2}\,\hat{a}(q)\,u(p_1)\,d^4q;$$

$$S_2 = \frac{e^4 i}{(2\pi)^4}\int \bar{u}(p_2)\,\hat{a}(-q)\,\frac{i(\hat{p}_2-\hat{q})-m}{(p_2-q)^2+m^2}\,\gamma_\nu u(p_2)\,\frac{1}{q^2}\,\bar{u}(p_1)\,\gamma_\nu \frac{i(\hat{p}_1-\hat{q})-m}{(p_1-q)^2+m^2}\,\hat{a}(q)\,u(p_1)\,d^4q.$$

It is evident that in the case of the third and fourth diagrams, the factor q^{-2} is replaced by $(p_2 - p_1 + q)^{-2}$. Because of the absence of a pole of type q^{-2}, these diagrams cannot give anomalous diamagnetism; we shall therefore not consider them.

We calculate S_1 and S_2 according to the general rules:

$$S_1 = \frac{ie^4}{(2\pi)^4}\int\frac{4/m^2}{q^2(-2(p_1q)+q^2)(2(p_2q)+q^2)}[2(p_1a(q))\,p_{1\nu} - (p_1a(q))\,q_\nu - (qa(q))\,p_{1\nu} + (p_1q)\,a_\nu(q)]\,[2(p_2a(-q))\,p_{2\nu} + (p_2a(-q))\,q_\nu + (qa(-q))\,p_{2\nu} - (p_2q)\,a_\nu(-q)]\,d^4q; \quad (3)$$

$$S_2 = \frac{ie^4}{(2\pi)^4}\int\frac{4/m^2}{q^2(-2(p_1q)+q^2)(-2(p_2q)+q^2)}[2(p_1a(q))\,p_{1\nu} - (p_1a(q))\,q_\nu - (qa(q))\,p_{1\nu} + (p_1q)\,a_\nu(q)]\,[2(p_2a(-q))\,p_{2\nu} - (p_2a(-q))\,q_\nu - (qa(-q))\,p_{2\nu} + (p_2q)\,a_\nu(-q)]\,d^4q. \quad (4)$$

Henceforth we shall suppose that $a_4 = 0$ and that A is independent of time. Then

$$\mathbf{a}(q) = \int \mathbf{A}(\mathbf{r}) e^{-i(qx)} d^4x = \mathbf{A_q} \int e^{iq_4x_4} dx_4.$$

Since all the terms in (3) and (4) contain $a(q)\,a(-q)$, it is appropriate to replace $a_\mu(q)\,a_\nu(-q)$ everywhere by the product

$$A_\mu(\mathbf{q})\,A_\nu(-\mathbf{q})\,2\pi\delta(q_4)\,\Delta t$$

(Δt is a normalizing interval). As is well known, the energy correction $\Delta\varepsilon$ is connected with the matrix element by the relation[4] $S = -i\Delta t\,\Delta\varepsilon$.

Thus for $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ we get the same expressions (3) and (4), except that the integration must be carried out only over the three-dimensional volume $d\mathbf{q}$, not over d^4q, and the four-dimensional vectors q and $a(q)$ in all the terms must be replaced by the three-dimensional vectors $\mathbf{q}$ and $\mathbf{A_q}$.

Upon adding S_1 and S_2 and taking account of the fact that $(p_1p_2) = \mathbf{p}_1\mathbf{p}_2 - \varepsilon_1\varepsilon_2$, we get ($\mathbf{p}_2$ has been replaced by $-\mathbf{p}_2$ in S_1):

$$\begin{aligned}\Delta\varepsilon = \frac{i}{\Delta t}(S_1 + S_2) = \frac{-2e^4}{(2\pi)^3 m^2}\int \frac{1}{q^2(-\mathbf{p}_1\mathbf{q} + q^2/2)(-\mathbf{p}_2\mathbf{q} + q^2/2)}&[4(\mathbf{p}_1\mathbf{a})(\mathbf{p}_2\mathbf{a}_-)(\mathbf{p}_1\mathbf{p}_2) \\ - 2(\mathbf{p}_1\mathbf{a})(\mathbf{p}_2\mathbf{a}_-)(\mathbf{p}_1\mathbf{q}) - 2(\mathbf{p}_1\mathbf{a})(\mathbf{q}\mathbf{a}_-)(\mathbf{p}_1\mathbf{p}_2) + 2(\mathbf{p}_1\mathbf{a})(\mathbf{p}_2\mathbf{q})(\mathbf{p}_1\mathbf{a}_-) \\ - 2(\mathbf{p}_1\mathbf{a})(\mathbf{p}_2\mathbf{a}_-)(\mathbf{p}_2\mathbf{q}) + 2(\mathbf{p}_1\mathbf{q})(\mathbf{p}_2\mathbf{a}_-)(\mathbf{p}_2\mathbf{a}) - 2(\mathbf{q}\mathbf{a})(\mathbf{p}_2\mathbf{a}_-)(\mathbf{p}_1\mathbf{p}_2) \\ + (\mathbf{p}_1\mathbf{a})(\mathbf{p}_2\mathbf{a}_-)q^2 + (\mathbf{q}\mathbf{a})(\mathbf{q}\mathbf{a}_-)(\mathbf{p}_1\mathbf{p}_2) - (\mathbf{q}\mathbf{a})(\mathbf{p}_2\mathbf{q})(\mathbf{p}_1\mathbf{a}_-) \\ - (\mathbf{p}_1\mathbf{q})(\mathbf{q}\mathbf{a}_-)(\mathbf{p}_2\mathbf{a}) + (\mathbf{p}_1\mathbf{q})(\mathbf{p}_2\mathbf{q})(\mathbf{a}\mathbf{a}_-)]\,d\mathbf{q};\end{aligned} \tag{5}$$

here $\mathbf{a} = \mathbf{A_q}$, $\mathbf{a}_- = \mathbf{A_{-q}}$.

To get the correction ΔE to the energy of the whole system, $\Delta\varepsilon$ must be summed over $\mathbf{p}_1$ and $\mathbf{p}_2$. We integrate first over angles in $\mathbf{p}_1$- and $\mathbf{p}_2$-space, *i.e.*, we find

$$\Delta\varepsilon' = \int \Delta\varepsilon \sin\vartheta_1\,d\vartheta_1 \sin\vartheta_2\,d\vartheta_2\,d\varphi_1\,d\varphi_2 = \int \Delta\varepsilon\,d\Omega_1\,d\Omega_2.$$

We point the z axis for $\mathbf{p}_1$ and $\mathbf{p}_2$ along $\mathbf{q}$. The integration of all the twelve terms that enter in (5) reduces to the calculation of the following three integrals, which are to be understood as principal values:

$$J_1 = \int_0^\pi \frac{\sin\vartheta\,d\vartheta}{-pq\cos\vartheta + q^2/2} = \frac{1}{pq}\int_{-1}^{1}\frac{d\mu}{\mu + q/2p} = \frac{1}{pq}\ln\left|\frac{1+q/2p}{1-q/2p}\right| \approx \frac{1}{p^2}\left(1 + \frac{q^2}{12p^2} + \ldots\right);$$

$$\begin{aligned}J_2 &= \int_0^\pi \frac{\cos\vartheta\sin\vartheta\,d\vartheta}{-pq\cos\vartheta + q^2/2} = -\frac{2}{pq} + \frac{qJ_1}{2p} \approx -\frac{2}{pq}\left(1 - \frac{q^2}{4p^2} + \ldots\right); \\ J_3 &= \int_0^\pi \frac{\cos^2\vartheta\sin\vartheta\,d\vartheta}{-pq\cos\vartheta + q^2/2} = -\frac{1}{p^2} + \left(\frac{q}{2p}\right)^2 J_1 \approx -\frac{1}{p^2}\left(1 - \frac{q^2}{4p^2} + \ldots\right);\end{aligned} \tag{6}$$

here $p^2 = \mathbf{p}^2$.

To clarify the problem of anomalous diamagnetism, it is sufficient to consider merely the term of zero order, φ_0, in the expression (1). In this case only terms quadratic in q need to be retained in the expansions of J_1, J_2, and J_3.

We calculate the first term $\Delta\varepsilon_1'$ in the expression $\Delta\varepsilon'$:

$$\begin{aligned}\Delta\varepsilon_1' &= 4b\int\frac{d\mathbf{q}}{q^2}\int(\mathbf{p}_2\mathbf{a}_-)\,p_1^2 a p_2 \frac{[\cos\vartheta_1\cos\theta + \sin\vartheta_1\sin\theta\cos(\varphi_1 - \Phi)]}{(-p_1q\cos\vartheta_1 + q^2/2)(-p_2q\cos\vartheta_2 + q^2/2)} \times \\ &\times[\cos\vartheta_1\cos\vartheta_2 + \sin\vartheta_1\sin\vartheta_2\cos(\varphi_1 - \varphi_2)]\sin\vartheta_1\,d\vartheta_1\,d\varphi_1\,d\Omega_2 = \\ &= 4b(2\pi)\,p_1^2 a p_2(\mathbf{p}_2, \mathbf{a}_-)\int\frac{d\mathbf{q}}{q^2}\int\frac{[\cos^2\vartheta_1\cos\theta\cos\vartheta_2 + {}^1/_2\sin^2\vartheta_1\sin\theta\sin\vartheta_2\cos(\varphi_2 - \Phi)]}{(-p_1q\cos\vartheta_1 + q^2/2)(-p_2q\cos\vartheta_2 + q^2/2)} \\ &\times\sin\vartheta_1\,d\vartheta_1\,d\Omega_2 \approx 8\pi b p_2^2 a a_-\int\frac{d\mathbf{q}}{q^2}\int\Big[-\left(1 - \frac{q^2}{4p_1^2}\right)\cos\theta\cos\vartheta_2 + \\ &+\left(1 - \frac{q^2}{12p_1^2}\right)\sin\theta\sin\vartheta_2\cos(\varphi_2 - \Phi)\Big]\frac{[\cos\vartheta_2\cos\theta_- + \sin\vartheta_2\sin\theta_-\cos(\varphi_2 - \Phi_-)]}{(-p_2q\cos\vartheta_2 + q^2/2)}\sin\vartheta_2\,d\vartheta_2\,d\varphi_2 \\ &= 4b(2\pi)^2\int\frac{d\mathbf{q}}{q^2}\left[(\mathbf{a}\mathbf{a}_-)\left(1 - \frac{q^2}{12p_1^2} - \frac{q^2}{12p_2^2}\right) - \frac{(\mathbf{a}\mathbf{q})(\mathbf{a}_-\mathbf{q})}{6}\left(\frac{1}{p_1^2} + \frac{1}{p_2^2}\right)\right];\end{aligned}$$

$$b = -2e^4/(2\pi)^2 m^2; \quad \mathbf{a} = \{a, \theta, \Phi\}; \quad \mathbf{a}_- = \{a_-, \theta_-, \Phi_-\}.$$

In the same way we calculate the remaining terms:

$$\frac{1}{4b\pi^2}\Delta\varepsilon_2' \approx -4\int(\mathbf{aq})(\mathbf{a}_-\mathbf{q})\frac{d\mathbf{q}}{q^4}\left(1-\frac{q^2}{4p_1^2}-\frac{q^2}{4p_2^2}\right); \quad \Delta\varepsilon_3' \approx \Delta\varepsilon_2';$$

$$\frac{1}{4b\pi^2}\Delta\varepsilon_4' = -4\int\frac{d\mathbf{q}}{q^2}\left[\left(1-\frac{q^2}{12p_1^2}\right)(\mathbf{aa}_-) - \frac{(\mathbf{aq})(\mathbf{a}_-\mathbf{q})}{q^2}\left(2-\frac{q^2}{3p_1^2}\right)\right]\left(1-\frac{q^2}{4p_2^2}\right);$$

$$\Delta\varepsilon_5' \approx \Delta\varepsilon_2'; \quad \frac{1}{4b\pi^2}\Delta\varepsilon_6' \approx -4\int\frac{d\mathbf{q}}{q^2}\left[\left(1-\frac{q^2}{12p_2^2}\right)(\mathbf{aa}_-) - \right.$$

$$\left. -\frac{(\mathbf{aq})(\mathbf{a}_-\mathbf{q})}{q^2}\left(2-\frac{q^2}{3p_2^2}\right)\right]\left(1-\frac{q^2}{4p_1^2}\right); \quad \Delta\varepsilon_7' \approx \Delta\varepsilon_2';$$

$$\Delta\varepsilon_8' \approx \Delta\varepsilon_9' \approx -\Delta\varepsilon_2'; \quad \Delta\varepsilon_{10}' \approx \Delta\varepsilon_{11}' \approx \Delta\varepsilon_2';$$

$$\frac{1}{4b\pi^2}\Delta\varepsilon_{12}' \approx 4\int\frac{d\mathbf{q}}{q^2}(\mathbf{a}_-\mathbf{a})\left(1-\frac{q^2}{4p_1^2}-\frac{q^2}{4p_2^2}\right).$$

Adding, we find

$$\Delta\varepsilon' = \sum_{i=1}^{12}\Delta\varepsilon_i' = 0.$$

Thus the magnetic interaction (likewise the Coulomb interaction, since we have also taken account of exchange via longitudinal photons) gives no anomalous diamagnetism. The neglected terms of higher order in q^2 give a correction to the ordinary diamagnetism. There is no point in considering higher approximations of perturbation theory in the case of magnetic interaction, *i.e.*, the sixth (of fourth order in the radiation field), *etc.*, since the fourth approximation, already considered, would give for the London constant in the case of anomalous diamagnetism an extra factor of order 1/137, the sixth would give one of order $(1/137)^2$ and so on.

We point out that the application of perturbation theory to magnetic interaction is apparently justified. In fact, the mean interaction energy of a single pair of electron is

$$U_{\mathrm{Av}} \approx (e/mc)^2 p_{\mathrm{Av}}^2/r_{\mathrm{Av}} \approx (e/mc)^2 p_0^3/\hbar;$$

p_0 is the maximum momentum.

The mean kinetic energy of the electrons is $T_{\mathrm{Av}} \approx p_0^2/m$;

$$U_{\mathrm{Av}}/T_{\mathrm{Av}} \approx (e^2/\hbar c)\,p_0/mc = p_0/137\,mc \ll 1.$$

In conclusion, I express my gratitude to I. M. Polievktov-Nikoladze for an interesting discussion.

APPENDIX

In the appendix we present a simple proof of the absence of anomalous diamagnetism in the case of a system of free electrons within the framework of perturbation theory [2].

The Hamiltonian of a single free electron in a magnetic field has the form

$$H = H_0 + H'; \; H_0 = \frac{p^2}{2m}; \, .$$

$$H' = -\frac{e}{2mc}[\mathbf{Ap}+\mathbf{pA}] + \frac{e^2}{2mc^2}A^2.$$

The correction to the energy of the electron, quadratic in A, in the second approximation of perturbation theory, is evidently equal to

$$\Delta\varepsilon = \Delta\varepsilon^{(1)} + \Delta\varepsilon^{(2)}; \quad \Delta\varepsilon^{(1)} = \frac{e^2}{2mc^2V}\int A^2 d\mathbf{r} = \frac{e^2}{2mc^2}\sum_{\mathbf{q}}(\mathbf{A_q A_{-q}});$$

$$\Delta\varepsilon^{(2)} = \left(\frac{e}{mc}\right)^2\sum_{\mathbf{q}}\frac{(\mathbf{p}+\mathbf{q}/2, \mathbf{A_q})(\mathbf{p}+\mathbf{q}/2, \mathbf{A_{-q}})}{[p^2-(\mathbf{p}+\mathbf{q})^2]/2m} = -\frac{e^2}{mc^2}\sum_{\mathbf{q}}\frac{1}{\mathbf{pq}+q^2/2}\Big[(\mathbf{pA_q})(\mathbf{pA_{-q}}) \tag{A.1}$$

$$+\frac{1}{2}(\mathbf{pA_q})(\mathbf{qA_{-q}}) + \frac{1}{2}(\mathbf{pA_{-q}})(\mathbf{qA_q}) + \frac{1}{4}(\mathbf{qA_q})(\mathbf{qA_{-q}})\Big] \equiv \sum_{i=1}^{4}\Delta\varepsilon_i^{(2)}.$$

In the summation over all electrons, *i.e.*, over p, we first carry out the integration over angles in p-space. In this step the calculation of $\Delta\varepsilon' = \int \Delta\varepsilon d\Omega$ reduces again to the calculation of the integrals J_1, J_2, and J_3, in which we limit ourselves to terms quadratic in q. According to (6),

$$\Delta\varepsilon_1^{(2)\prime} = -(e^2/mc^2)\,2\pi \sum [(\mathbf{A_q}\,\mathbf{A_{-q}}) - 2q^{-2}(\mathbf{A_q q})(\mathbf{A_{-q} q})];$$

$$\Delta\varepsilon_2^{(2)\prime} = \Delta\varepsilon_3^{(2)\prime} = -(e^2/mc^2)\,2\pi \sum_{\mathbf{q}} (\mathbf{A_q q})(\mathbf{A_{-q} q})/q^2,$$

and $\Delta\varepsilon_4^{(2)\prime}$, it is easily seen, has the form $\sim \Sigma(\mathbf{q}\mathbf{A_q})(\mathbf{q}\mathbf{A_{-q}})$, *i.e.*, in general gives no anomalous diamagnetism.

Thus the sum becomes

$$\sum_{\mathbf{p}} \sum_{i=1}^{4} \Delta\varepsilon_i^{(2)} = \sum_{\mathbf{p}} \Delta\varepsilon^{(2)} = -\frac{Ne^2}{2mc^2} \sum_{\mathbf{q}} (\mathbf{A_q A_{-q}}),$$

i.e., it is equal to the sum $\Sigma\Delta\varepsilon^{(1)}$ with the sign reversed. Consequently, anomalous diamagnetism is absent. This proof relates to an arbitrary system of free electrons with an isotropic distribution in p-space and with momenta **p** different from zero, *i.e.*, to a Fermi gas and to a Bose gas not in a state of condensation. In the case of a charged Bose gas in a state of condensation, this result retains its validity for electrons with momenta $\mathbf{p} \neq 0$. For electrons with $\mathbf{p} = 0$ (let their number be N_s), the energy in a magnetic field may be found directly from formula (A.1) by setting $\mathbf{p} = 0$. Then

$$\Delta E = \frac{N_s e^2}{2mc^2} \Big\{ \sum_{\mathbf{q}} \mathbf{A_q A_{-q}} - \sum_{\mathbf{q}} (\mathbf{A_q q})(\mathbf{A_{-q} q})/q^2 \Big\}. \quad (A.2)$$

We see that (A.2) agrees with expression (1) for $\varphi_0 = N_s e^2/2mc^2$; that is, a Bose gas in a state of condensation possesses anomalous diamagnetism[5-7]

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Translated by W. F. Brown Jr.

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