For sufficiently large p_0, Eq. (5) can be calculated in the same way as was done by Ekstein[4], obtaining

$$\{\cdots\} = (4\pi)^2 \chi_a(\mathbf{r}) \left[\frac{1}{1+b^2} - \frac{m}{2\mu}\right] \int_{p_0}^{\infty} dp. \tag{6}$$

From the definition of b and μ it follows that the expression in square brackets in Eq. (6) vanishes. It is not difficult to show that nonzero neutron energies also give a finite result.

Thus when using wave functions for real molecules in the calculation according to Eq. (1), we obtain a finite result. This conclusion contradicts that of Ekstein, since he mistakenly omitted the factor a in the exponent of Eq. (1) and stated that $k_\gamma = ip$, rather than the correct expression $k_\gamma = ip\sqrt{\mu/\mu_p}$.

The results of Breit and Zilsel are correct because an artificial (mathematically nonrigorous) method was used to eliminate the divergent part of the integral, which in turn, is due to the idealization of the problem being considered.

[1] E. Fermi, Ricerca sci. 7, 2, 13 (1936).

[2] G. Breit, P. Zilsel, Phys. Rev. 71, 232 (1947).

[3] B. Lippmann, J. Schwinger, Phys. Rev. 79, 481 (1950).

[4] H. Ekstein, Phys. Rev. 87, 31 (1952).

Translated by E. J. Saletan
203

Fluctuations in Gases

B. B. KADOMTSEV

(Submitted to JETP editor January 24, 1957)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 943-944 (April, 1957)

THE STATE OF A GAS is completely described by the particle-distribution function in the phase space $f(\mathbf{r}, \mathbf{v}, t)$; therefore the problem of fluctuations in gases leads to the study of the correlation characteristics of the distribution function. Ordinarily one understands by f the statistically-average density of the particles in the phase space which, naturally, cannot fluctuate. Speaking of the fluctuations of the distribution function we must, however, bear in mind the "true" density

$$F(\mathbf{r}, \mathbf{v}, t) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i)\,\delta(\mathbf{v} - \mathbf{v}_i),$$

where the summation is carried out with respect to all particles. We shall consider the density F to be a random quantity which only on the average coincides with f.

The function F satisfies the equation

$$\frac{\partial F}{\partial t} + (\mathbf{v}\nabla)F = \frac{1}{m}\frac{\partial F(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{v}} \int \frac{\partial U(|\mathbf{r} - \mathbf{r}'|)}{\partial \mathbf{r}} F(\mathbf{r}', \mathbf{v}', t)\, d\mathbf{r}'\, d\mathbf{v}', \tag{1}$$

where m is the mass of a molecule, and U is the potential energy of the interaction of the molecules among themselves. In an ideal gas we may neglect the interaction of the particles and obtain then from (1) $F(\mathbf{r}, \mathbf{v}, t) = F(\mathbf{r} - \mathbf{v}(t - t_0), \mathbf{v}, t_0)$ whence

$$\langle \varphi(\mathbf{r}, \mathbf{v}, t)\,\varphi(\mathbf{r}_0, \mathbf{v}_0, t_0)\rangle = f(\mathbf{r}, \mathbf{v}, t)\,\delta(\mathbf{r} - \mathbf{v}(t - t_0) - \mathbf{r}_0)\,\delta(\mathbf{v} - \mathbf{v}_0), \tag{2}$$

where $\varphi = F - f$ and the angular brackets denote the averaging.

Our problem consists in finding the correlation of (2) with allowance for the collisions. If the gas is not very dense, we may confine ourselves only to the accounting of paired collisions, and then the right half of Eq. (1) may be approximately represented in the form of the collision term $S(F, F)$:

$$S(F, F) = \int \{F(\mathbf{r}, \mathbf{v}', t)\,F(\mathbf{r}_1, \mathbf{v}_1', t) - F(\mathbf{r}, \mathbf{v}, t)\,F(\mathbf{r}_1, \mathbf{v}_1, t)\}\,|\mathbf{v} - \mathbf{v}_1|\,\rho\, d\rho\, d\chi\, d\mathbf{v}_1, \tag{3}$$

where ρ is the collision parameter, $\rho\, d\rho\, d\chi$ is an element of the surface which is perpendicular to the relative velocity $\mathbf{v} - \mathbf{v}_1$ and passes through the point $\mathbf{r}$, $\mathbf{r}_1$ is the coordinate of this element, $\mathbf{v}'$ and $\mathbf{v}_1'$ are the velocities of the particles before the impact which are transformed after the collision into $\mathbf{v}$, $\mathbf{v}_1$. It is approximately assumed here that the collision occurs at that instant when both particles intersect the surface which is perpendicular to their relative velocity.

If we average (1) using the collision term in the form of (3) and neglect the correlation of the particles before the collision and the difference between $\mathbf{r}$ and $\mathbf{r}_1$, we shall obtain the ordinary Boltzmann equation

$$\frac{\partial f}{\partial t} + (\mathbf{v}\nabla)f = \int \{f(\mathbf{r}, \mathbf{v}', t)\,f(\mathbf{r}, \mathbf{v}_1', t) - f(\mathbf{r}, \mathbf{v}, t)\,f(\mathbf{r}, \mathbf{v}_1, t)\}\,|\mathbf{v} - \mathbf{v}_1|\,\rho\, d\rho\, d\chi\, d\mathbf{v}_1. \tag{4}$$

Taking (4) into consideration, we transform (1) into the form

$$(\partial\varphi/\partial t) + (\mathbf{v}\nabla)\varphi + A\varphi = q, \quad (5)$$

where A is an operator determined by the expression $A\varphi = -S(f, \varphi) - S(\varphi, f)$ and $q = S(\varphi, \varphi)$. Since use of a distribution function makes sense only where one deals with sufficiently large volumes containing many particles, we must assume the function φ to be a small quantity. Therefore, we may approximately use in q, which is a quantity of the second order of smallness with respect to φ, the correlation (2) whence we obtain

$$\langle q(\mathbf{r}, \mathbf{v}, t)\, q(\mathbf{r}_0, \mathbf{v}_0, t_0)\rangle = (A + A^*) f(\mathbf{r}, \mathbf{v}, t)\, \delta(\mathbf{r} - \mathbf{r}_0)\, \delta(\mathbf{v} - \mathbf{v}_0)\, \delta(t - t_0), \quad (6)$$

where A^* is the operator acting upon the velocity $\mathbf{v}_0$.

Let us denote by G the Green function for Eq. (5), *i.e.*, the operator by whose action on the source we can obtain the solution of this equation. Then we obtain from (5) and (6):

$$\langle \varphi(\mathbf{r}, \mathbf{v}, t)\, \varphi(\mathbf{r}_0, \mathbf{v}_0, t_0)\rangle = GG^*(A + A^*) f(\mathbf{r}, \mathbf{v}, t)\, \delta(\mathbf{r} - \mathbf{r}_0)\, \delta(\mathbf{v} - \mathbf{v}_0)\, \delta(t - t_0), \quad (7)$$

where G^* is the operator acting upon $\mathbf{r}_0$, $\mathbf{v}_0$, t_0.

Eq. (7) also furnishes a solution for our problem. If we put, approximately, $A = 1/\tau$, where τ is the average time between collisions and assume that f is not a function of $\mathbf{r}$ and t, then (7) takes the form

$$\langle \varphi(\mathbf{r}, \mathbf{v}, t)\, \varphi(\mathbf{r}_0, \mathbf{v}_0, t_0)\rangle = e^{-|t-t_0|/\tau} f(\mathbf{v})\, \delta(\mathbf{r} - \mathbf{r}_0 - \mathbf{v}(t - t_0))\, \delta(\mathbf{v} - \mathbf{v}_0). \quad (8)$$

However, for a more exact consideration of the problem and also in the non-stationary case, we must solve Eq. (5), that is, a linearized kinetic equation with a random source.

The physical sense of this equation is evident. Actually, every act of collision leads to two particles being withdrawn from the initial density and to two particles with different velocities appearing in their place at that same point in the space. Eq. (5) also describes the further development of such a random disturbance of the distribution function.

Eq. (5) is, with respect to form, entirely analogous to the Maxwell equations with random sources[1] used in the theory of electric fluctuations. This is not surprising, since in the case of thermodynamic equilibrium, equations of this type can be obtained by starting from the general theory of fluctuations[2,3]. The kinetic derivation of the formulae (5) and (6) considered here possesses, in addition to greater clarity, the advantage that it is correct also in the non-stationary case.

It must be noted that according to (7) the particles are found to be only slightly correlated before collision. The correlation arises from such chains of collisions where two impinging particles collide with two others and these latter collide with one another. Since four particles participate in this chain and we have even neglected triple collisions, we may neglect the correlation of the particles before the collision resulting from (7).

I should like to express my deep gratitude to Academician M. A. Leontovich fordiscussing this report with me.

[1] S. M. Rytov, *Theory of Electric Fluctuations*, AN SSSR, 1953.

[2] Callen, Barasch, and Jackson, Phys. Rev. **88**, 1382 (1952).

[3] S. M. Rytov, Dokl. Akad. Nauk SSSR **110**, 371 (1956).

Translated by L. Mahler
204

The Effect of Neutron Irradiation on the Compressibility of Metals

A. I. LIKHTER AND A. I. KIKOIN

Institute of the Physics of Metals of the Ural Branch of the Academy of Sciences of the USSR

Ultra-High-Pressure Physics Laboratory of the Academy of Sciences of the USSR

(Received by JETP editor January 25, 1957)

J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 945 (April, 1957)

THE FEW STUDIES that have been made so far on the effect of fast neutron irradiation on the elastic properties of metals and alloys show either that the effect is entirely absent or that it is exceedingly small. The modulus of elasticity, as far as we know, has been studied only in austenite steel and in copper. Neither case showed any change in the modulus of elasticity for a total flux of 10^{19} neutrons/cm^2 [1]. The shear modulus was studied in neutron-irradiated copper, and the residual change at room temperature was not greater than 1%[1].