trons was used. The differences between our data and the result of Flammersfeld are smaller. A number of lines which were detected in the photographic work were not seen in the present work since the sensitivity of the photographic method is higher.

The relative intensities of the conversion lines have been computed from the reading of the first counter. Since the cutoff energy of the film in the first counter was 4 kev, no corrections for absorption were introduced. According to Ref. 18, this effect is less than one percent for electron energies four or five times greater than the cutoff energy.

The accuracy in the relative intensity measurements for the conversion lines is 3–5 percent for the strong lines and 20–30 percent for the weak lines.

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[1] Latyshev, Sergeev, Krisiuk, Ostretsov, Egorov and and Shirshov, Izv. Akad. Nauk SSSR, Ser. Fiz. **20**, 354 (1956).

[2] Sergeev, Latyshev and Leonov, Izv. Akad. Nauk SSSR, Ser. Fiz. **20**, 369 (1956).

[3] K. Siegbahn and K. Edvardson, Nuc. 1, 1937 (1956).

[4] G. Lindstrom, Phys. Rev. **87**, 678 (1952).

[5] R. D. Hill in *Beta and Gamma-Ray Spectroscopy* (Interscience Pub., Amsterdam, 1955), p. 914.

[6] T. Gerholm, bid., p. 926

[7] Krisiuk, Latyshev and Sergeev, Izv. Akad. Nauk SSSR, Ser. Fiz. **20**, 367 (1956).

[8] Krisiuk, Vitman, Vorob'ev, Latyshev and Sergeev, Izv. Akad. Nauk SSSR, Ser. Fiz. **20**, 877 (1956).

[9] I. Bergstrom and R. D. Hill, Arkiv f Fysik **8**, 21, (1954).

[10] M. Mladjenovic and H. Slatis, Arkiv f Fysik **9**, 41 (1955).

[11] C. Ellis, Proc. Roy. Soc. A **139**, 336 (1933).

[12] C. Ellis, Proc. Roy. Soc. A **138**, 318 (1932).

[13] C. Ellis, Proc. Roy. Soc. A **143**, 350 (1934).

[14] J. Surugue, Ann. phys. **8**, 484 (1937).

[15] J. Surugue, J. phys. et radium **9**, 438 (1938).

[16] R. Arnoult, Ann. phys. **12**, 241 (1939).

[17] A. Flammersfeld, Z Physik **114**, 227 (1939).

[18] R. O. Lane and D. J. Zaffarano, Phys. Rev. **94**, 960 (1954).

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Generation of Slow π-Mesons by Cosmic Ray Particles

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Results of experiments on the transition effect of π-mesons are presented and discussed. The results presented allow one to conclude that low energy mesons are abundant in the spectrum of the generated mesons and that the plural mechanism of meson creation is correct. Most of the generating particles are probably cosmic ray neutrons.

1. SOURCES OF THE π-MESONS OBSERVED IN PHOTOGRAPHIC PLATES EXPOSED TO COSMIC RAYS

IN EXPERIMENTS, reported in Refs. 1-3, investigating the transition curve of π-mesons and the intensity of their generation as a function of the atomic weight of the target, no account was taken of the current of slow π-mesons from extraneous dense materials situated near the photographic plate or of the presence of π-mesons in the air. Both of these factors can influence the form of the transition curve.

Before embarking on an investigation of the transition effect of π-mesons and the intensity of their generation as a function of the atomic weight of the target, it is necessary to study the current of π-mesons coming from the air and from nearby extraneous dense absorbers. We set up special experiments for this purpose on a mountain top (altitude 4000 m). A pair of photographic plates were exposed during the course of two months on a mast 10 m high, with another pair in direct contact with the surface of the earth. After chemical treatment (developing and fixing), these photographic plates were scanned with a microscope at high magnification. Identification of the stopped π-mesons was made by means of π-μ decays and σ-stars.

The reduced results of this experiment are given in Table 1. It is clear from the table that the number of π-mesons observed in the photographic plates exposed at the surface of the earth is significantly greater (by 5 – 6 times) than in those on the mast. It follows from this that in studying the generation of slow π-mesons by cosmic rays all the photoemulsions used in the experiment should be placed as far as possible from extraneous dense absorbers.

TABLE I.

Number of the experiment	Position of the photographic plates	Number of π-mesons observed in 6 cm²	Number of π-mesons after exclusion of background
1	On the mast	8	5
	On the ground	30	27
2	On the mast	3	—
	On the ground	8	—

What fraction of the π-mesons recorded in a photographic emulsion are π-mesons formed in the air? An experimental study of this question by the method of photographic emulsions is extremely difficult, since these emulsions are always enveloped in wrapping material when they are exposed. Hence the question as to which part of the recorded mesons comes from the air and which part is formed in the wrapping material represents a definite experimental difficulty. By means of an elementary calculation, however, we can obtain the necessary information on the current of π-mesons coming from the air.

The number of π-mesons with energy less than E_0 originating in the atmosphere within a depth of x g/cm² from the boundary of the atmosphere is equal to

$$S_\pi = \int_0^{E_0} n(E)\left[\int_0^x \frac{S_0}{\lambda_0} e^{-t/l} e^{-(x-t)/\lambda} (t/x)^\gamma dt\right] dE, \quad (1)$$

where $n(E)$ is the differential energy spectrum of the generated π-mesons, $S_0 e^{-t/l}$ is the number of nucleons with energies greater than E_c at depth t, E_c is the threshold energy for the formation of mesons; $\exp\{-(x-t)/\lambda\}$ is the probability that a meson originating at depth t will reach depth x without undergoing a nuclear interaction; $(t/x)^2$ is the probability that a meson formed at depth t with momentum p_π will reach depth x without undergoing disintegration. Here.

$$\gamma = z_0 m_\pi / \tau_\pi p_\pi = z_0 m_\pi c^2 / \tau_\pi c\,(E^2 + 2 m_\pi c^2 E)^{1/2},$$

where $z = 6.4 \times 10^5$ cm in the stratosphere, λ_0 and λ are the respective ranges of the nucleon and meson for nuclear interaction. For air and also for the wrapping material and the glass backing of the photographic plates, we take $\lambda_0 = \lambda = 60$ g/cm² and $l = 2\lambda$. For a condensed medium of the type of carbon, formula (1) takes the form

$$S_{0\pi} = \int_0^{E_0} n(E)\left[\int_0^x \frac{S_0}{\lambda_0} e^{-t/l} e^{-(x-t)/\lambda} dt\right] dE. \quad (2)$$

Calculation shows that the π-mesons originating in the air and slowed down and stopped in the photographic emulsion amount to less than 1% of such mesons formed in the wrapping material and the glass backing of the photographic plate. Hence the basic sources of the π-mesons observed in photographic emulsions exposed to cosmic rays are the dense absorbers situated in direct proximity to the detector.

2. CHANGE IN THE NUMBER OF π-MESONS UPON PENETRATION FROM AIR INTO LEAD

Refs. 1–3 are devoted to the study of the penetration effect of π-mesons at mountain altitudes. However, it is not possible to make an unambiguous conclusion from these references as to the change in the number of slow π-mesons as a function of the thickness of the absorber. Moreover, the results of Refs. 1–4 and 5 do not agree among themselves, and there are no data on the penetration effect for π-mesons in the stratosphere given in the literature, unless we count the brief communication of Blau et al.[6], in which no experimental results are reported. The results given in Ref. 7 also seem quite doubtful.

We have carried out experiments studying the penetration effect of π-mesons. Electron sensitive photographic plates of type NIKFI were exposed in the stratosphere at an altitude of 25–27 km and at a mountain elevation (2.5 km). In one of the stratosphere experiments (Fig. 1a) some of the photographic plates were placed inside a hollow lead hemisphere with walls 2 cm thick and in an aperture made in the center of a solid hemisphere of lead of radius 8 cm, while the rest, kept far away from dense absorbers, were exposed without lead shielding and with thin shields. A schematic of the arrangement of the photographic plates in the other stratosphere experiment is given in Fig. 1b. Lead absorbers in the form of slabs of dimensions $9 \times 5 \times 2$ cm and $9 \times 5 \times 3$ cm and thin absorbers of thickness 3–4 g/cm^2 were used in this experiment.

In the mountain experiment the photographic plates were placed inside cylindrical absorbers made of lead (with walls of thickness 0.2, 2 and 6 cm) and aluminum (with walls of thickness 0.8 cm), and afterwards were exposed at a distance of 7 m from the earth. To reduce shrinkage, the photographic plates were placed inside a flat-sided glass vessel of thickness 1.5–2 mm, filled with nitrogen. In the stratosphere experiments the photographic plates were exposed without an absorber and with thin shields at a distance of 2.5 and 5 m above the container.

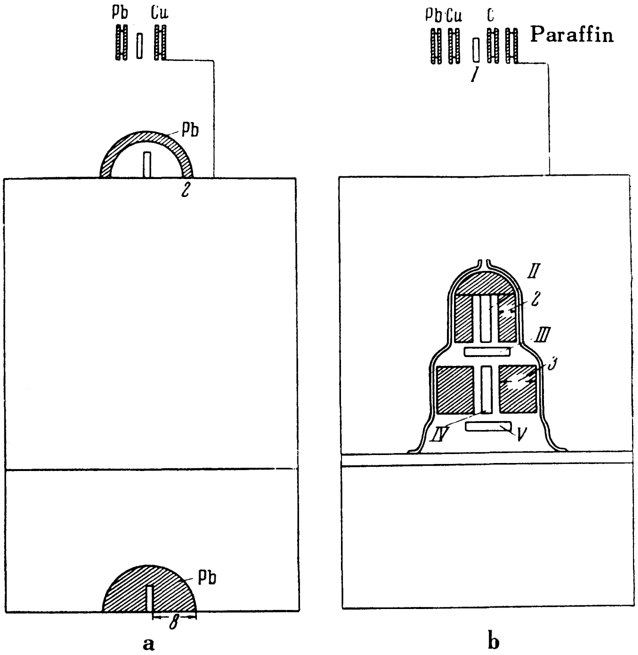

FIG. 1. Schematic of the stratosphere experiments.

The reduced results of the two stratosphere experiments and one of the experiments at a mountain elevation are given in Table 2. This table also includes ρ-mesons. It should be noted that not all of the ρ-mesons are μ-mesons. A certain part of the negative π-mesons (∼27%) does not give observable stars[8], and they are counted as ρ-mesons. Moreover, a certain part of the μ-mesons is connected with the decay of π^+-mesons generated in the absorber (local origin). Taking account of these factors leads to the estimate of the number of μ-mesons which is given in the last column of Table 2. This estimate corresponds approximately to an estimate of the number of μ-mesons by $\mu \to e$ decay.

It is clear from Table 2 that 1) the number of slow π-mesons increases strongly with increasing thickness of the lead absorber, while the number of μ-mesons is almost independent of the change in the thickness of the lead; 2) a very strong increase in the number of π-mesons occurs when the photographic emulsion is surrounded by a layer of lead of small thickness, while a further increase in thickness does not lead to a strong increase in the number of π-mesons; 3) no notable dependence of the penetration of π-mesons on height above sea level is observed.

It follows from these results that π-mesons are generated directly in the absorber surrounding the photographic emulsion, with low-energy mesons (having a range in lead less than 4 g/cm^2) being the most abundant among those generated. The fact that the form of the penetration curves for slow mesons in the stratosphere and at a mountain elevation are the same is probably due to the fact that

TABLE II.

No. of the experiment	Altitude above sea level, km	Thickness of the lead shielding	Area examined, cm^2	Number of mesons identified		
				π	ρ	μ
1	27	Air	22	7 ± 2.6	55	54
		1.7	22	25 ± 5	61	57
		23	22	40 ± 6.3	53	46
		90	22	57 ± 8.3	56	42
2	26	Air	20	6 ± 2.4	—	—
		3.4	20	26 ± 5.0	—	—
		23	20	28 ± 5.3	—	—
		III *	20	22 ± 4.7	—	—
		34	20	32 ± 5.6	—	—
		V *	20	30 ± 5.5	—	—
3	2.5	Air	17	2	102	102
		2.27	17	16	72	69
		23	17	22	98	94
		60	17	30	85	79

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*III and IV indicate the positions of the photographic plates relative to the lead absorbers (see Fig. 1).

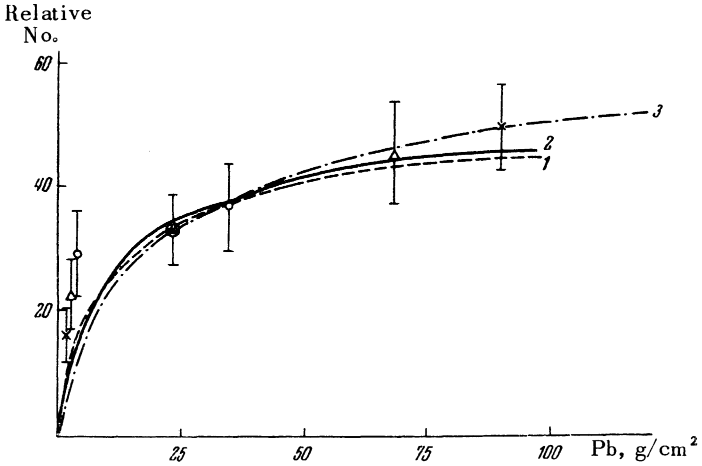

FIG. 2. Number of π-mesons generated: ×, experiment No. 1, o, experiment No. 2, Δ, mountain experiment.

they arise basically as secondary particles. The number of generated mesons is shown in Fig. 2 as a function of the thickness of the lead absorber.

We can obtain an idea of the form of the energy spectrum of the generated π-mesons from the penetration curve. The energy spectrum of the π-mesons formed in the nuclei of the emulsion has been investigated by Camerini et al.[9] and Yagoda[10]. According to the results of Ref. 9, the energy spectrum of the π-mesons in the energy region 200–1100 Mev may be represented by the expression $P(E)dE \sim E^{-1.4}\,dE$ (E is the total energy), and in the region of small energies by the expression[10] $P(E_k)dE_k \sim E_k^{0.6}\,dE_k$ (E_k is the kinetic energy). On the assumption of a spectrum of the form:

$$P(E)dE = AE^{-\gamma}dE \quad \text{for } E \geqslant 40 \text{ Mev} \qquad (3)$$

$$P(E)dE = BE^{0.6}\,dE \quad \text{for } E \leqslant 40 \text{ Mev} \qquad (4)$$

the expected penetration curves were calculated for values of γ of 1.5, 1.3 and 1 (the corresponding curves in Fig. 2 are indicated by the numbers 1, 2 and 3). It is clear from a comparison that in the region of small thickness the experimental points systematically go beyond the limits of the calculated curves. We are inclined to think that in the region of small energies the energy spectrum of the π-mesons is different from the spectrum (4) which we assumed. It should be pointed out, however, that the dependence of the angular distribution and the scattering of π-mesons on their energy may essentially influence the number of stopped π-mesons. Taking account of these factors in the calculation of the transition curve does not seem possible to us at the present time. But there seems no doubt that mesons of small energy, with ranges in lead of less than 4 g/cm^2, are abundantly represented among the mesons generated by cosmic rays.

3. DEPENDENCE OF THE CROSS SECTION FOR GENERATION OF π-MESONS ON THE ATOMIC WEIGHT OF THE MATERIAL OF THE TARGET

It is shown in Ref. 11 that the stratosphere data on the dependence of the cross section for formation of slow mesons on the atomic weight of the absorber can be explained by the essential role of the number of nucleons (in which number δ-nucleons are to be counted) which take part in the collisions.

It should be pointed out that taking account of the difference in stopping power of carbon, copper, and lead does not change the general conclusion just made, since according to the data on the penetration effect the number of stopped π-mesons is almost the same for lead thicknesses of 3–4 and 23 g/cm^2 (see Table 2). The results of our stratosphere experiments on the cross section for the formation of π-mesons in various nuclei confirm the results obtained by Abraham and Coldsack[12].

Thus, the conclusions made in Ref. 11 relative to the formation of slow π-mesons in heavy and light nuclei by cosmic rays in the stratosphere must be considered correct.

In this connection it is interesting to study the formation of slow π-mesons in various nuclei at mountain altitudes. The formation of π-mesons in various elements at a mountain altitude was studied in Ref. 13. According to the results of this reference, the cross section for the generation of π-mesons falls with increasing atomic weight of the target material. This fact cannot be accepted without a supplementary confirmation, since according to the results of Gregory and Tinlot[14], at an altitude of 3240 m the greater part ($\sim$90%) of the protons which produce nuclear interactions have energies less than 1 Bev, while only $\sim$ 10% of the nuclear interactions may be ascribed to protons with energies greater than 1 Bev. It must be presumed that the neutrons in the cosmic radiation at mountain altitudes possess approximately these same energies. It is clear from this that the average energy of the nucleons of cosmic rays at mountain altitudes cannot be much greater than the energy of the protons used in Refs. 15 and 16.

As is well known, the results given in Refs. 15 and 16 lead unambiguously to the conclusion that the cross section for the generation of π-mesons at a proton energy of approximately 400 Mev is proportional to the geometrical cross section of the nucleus. Hence it was expedient to set up a special experiment at a mountain altitude for an investigation of the formation of π-mesons in light and heavy nuclei. The experiment was set up at an altitude of 2500 m and the duration of the exposure was two months. Thin absorbers of lead and aluminum were used as generators, and photographic emulsions of type NIKFI of 400μ thickness were used as detectors. The absorbers were in the form of cylinders. The generator thicknesses were chosen to make the ionization ranges of the mesons in these substances equal. The use of thin absorbers, with thickness approximately equivalent with respect to ionization range, obviated the necessity of introducing corrections connected with nuclear and electromagnetic absorptions. Taking account of such corrections is difficult at present, since the energy spectrum of the low energy π-mesons generated in various substances at the given altitude has as yet been little studied. In order to guarantee the cleanness of the experiment, all the photographic emulsions used were removed a distance of 7 m from the surface of the earth. The average decay range of π-mesons with energies less than 50 Mev is about 6 m. Hence when the detector is taken a distance of 7 m from the surface of the earth, the contribution which π-mesons formed in the surface layer make to the number of recorded events is strongly dreceased.

The reduced results of the mountain experiment are given in Table 3. It is clear from the table that the number of π-mesons, relative to 1 g/cm^2 of material, decreases somewhat more weakly with increasing atomic weight than would be expected from the $A^{2/3}$ law. The relative cross section for generation $\sigma_{\rm Al}/\sigma_{\rm Pb}$, calculated from the data of Table 3, is equal to 0.16 ± 0.04. The corresponding relative cross section to be expected on the basis of the $A^{2/3}$ law is 0.25. It is clear from this that the cross section for the generation of mesons by cosmic ray particles at mountain altitudes increases with increasing A somewhat more rapidly than the geometrical cross section of the nucleus. If we attribute definite significance to this fact, then it shows that cosmic ray particles at mountain altitudes are capable of generating mesons not only by

TABLE III

Absorber	Absorber thickness, g/cm^2	Number of π-mesons per cm^3 day, g/cm^2	Number of π mesons per cm^3 day g/cm^2
Air	—	0.040 ± 0.03	—
Al	2.2	0.33 ± 0.026	0.132 ± 0.02
Pb	4.45	0.52 ± 0.03	0.108 ± 0.01

collision with surface nucleons, but also as a result of collisions with volume nucleons of a nucleus. The results of our mountain experiments are in agreement with the results of work carried out in a cloud chamber[17].

Using the results of Gregory and Tinlot[14], we can calculate the effective relative cross section for the generation of mesons, assuming that 90% of the cosmic ray nucleons at mountain altitudes generate mesons with cross section proportional to $A^{2/3} 2^{a}$, where $a = A^{1/3}/1.5$. Then the expected relative cross section calculated on the basis of these assumptions is given by $\sigma_{\mathrm{Al}}/\sigma_{\mathrm{Pb}} \approx 0.13$, which agrees with the experimentally observed magnitude of the relative cross section.

4. ON THE NATURE OF COSMIC RAY PARTICLES THAT GENERATE SLOW π-MESONS

In order to determine the nature of the meson-generating particles, we studied the altitude dependence of the meson production. For this purpose photographic plates were exposed at various altitudes under 2 cm of lead. Out of all the stratosphere flights made to determine the altitude dependence of the production of π-mesons, only those flights were used in which the drift time exceeded by 4 or more times the time of ascent and descent of the apparatus. Only in these cases can we ascribe the observed intensity to a definite height. The data from two stratosphere experiments and one mountain experiment have been reduced. The results concerning the altitude dependence of the number of π-mesons is the following:

Altitude above sea level, km	2.5	23.5	27
Number of π-mesons in 1 cm^2 in 1 min × 10^{-5}	1.96 ± 0.42	625 ± 96	625 ± 81

The absorption ranges for meson-generating particles have also been determined for two altitude intervals: 23.5–2.5 km and 27–2.5 km; they are equal respectively to $(127 \pm 11/7)$ g/cm^2 and $(130 \pm 11/7)$ g/cm^2. It should be noted that the absorption range of the meson-generating particles assumes a value intermediate between that for the range of particles generating penetrating showers (118 g/cm^2) and that for the range of particles which give rise to ordinary stars (140 g/cm^2). Although the statistical error does not allow us to make a choice as to which of these processes plays the main role, it seems quite probable that we should consider slow mesons to be generated both in showers and in ordinary stars.

In conclusion we give the results of an investigation of the ratio of positive and negative π-mesons, which may yield information on the nature of the generating particles (Table 4). Table 4 yields the following results:

1. An increase in the ratio π^+/π^- both in the stratosphere and at mountain altitude is observed with increasing energy interval of the π-mesons,

TABLE IV.

Absorber thickness g/cm^2	Altitude above sea level 26–27 km						Altitude above sea level 3–4 km		
	Pb target			C target			Pb target		
	π^+	π^-	π^+/π^-	π^+	π^-	π^+/π^-	π^+	π^-	π^+/π^-
3–5	29	48	0,60	—	—	—	6	18	0.33
23	13	22	0,82	—	—	—	18	39	0.46
20	—	—	—	16	34	0,50	—	—	—
26	—	—	—	26	30	0,87	—	—	—
70–80	37	37	1	—	—	—	13	21	0.62
114 (18)	—	—	—	—	—	—	30	41	0.73

which is (roughly) determined by the thickness of the absorber.

2. The negative "charge asymmetry" is significant for low energy π-mesons.

3. The ratio π^+/π^- changes only slightly in going from a heavy element (Pb) to a light element (C).

4. The magnitude of the ratio π^+/π^- increases in going from a mountain altitude to the stratosphere. All these regularities are easy to explain, if we take account of the fact that an overwhelming part of the slow π-mesons is generated by neutrons. The participation of protons in the formation of π-mesons increases with increasing energy of the π-mesons.

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[1] J. Harding and D. Perkins, Nature **164**, 285 (1949).

[2] J. Harding, Phil. Mag. **42**, 651 (1951).

[3] U. Camerini et al., Phil. Mag. **42**, 1241 (1951).

[4] V. Kamalian, Doklady Akad. Nauk SSSR **95**, 1169 (1954).

[5] A. Vaisenberg, Doklady Akad. Nauk SSSR **91**, 483 (1953).

[6] M. Blau, J. Nafe and H. Bramson, Phys. Rev. **78**, 320 (1950).

[7] M. Schein and J. Lord, Phys. Rev. **76**, 170 (1949).

[8] F. Adelman and S. Jones, Phys. Rev. **75**, 1468 (1949).

[9] U. Camerini et al., Phil. Mag. **41**, 413 (1950).

[10] H. Yagoda, Phys. Rev. **85**, 891 (1952).

[11] D. Kaipov and Zh. Takibaev, J. Exper. Theoret. Phys. (U.S.S.R.) **30**, 471 (1956), Soviet Phys. JETP **3**, 385 (1956).

[12] M. Abraham and S. Coldsack, Bull. Am. Phys. Soc. **30**, 15 (1955).

[13] Dallaporta, Merlin, Pierucci and Rostagni, Nuovo Cimento **9**, 202 (1952).

[14] B. Gregory and J. Tinlot, Phys. Rev. **81**, 667 (1951).

[15] R. Sagane and W. Dudziak Phys. Rev. **92**, 212 (1951).

[16] Block, Passman and Havens, Phys. Rev. **88**, 1239 (1952).

[17] B. Rossi, *High-Energy Particles*, (Prentice-Hall, Inc., New York, 1952).

[18] Brown, Camerini, Fowler, Muirhead, Powell and Ritson, Nautre **163**, 82 (1949).

Translated by M. J. Gibbons
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Construction of the Thermodynamic Potential of Rochelle Salt from the Results of the Optical Investigation of Domains

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From the experimental temperature dependence of the monoclinic parameter η and the specific heat c_p, the thermodynamic potential of Rochelle salt can be constructed with accuracy to terms of the order of η^4. The advantages of the optical method of determination of the monoclinic parameter are analyzed in comparison with the electrical and mechanical methods. Results of calculation of the thermodynamic potential surface from the temperature dependence of the angle of spontaneous rotation of the optical indicatrix are given. The possibilities of a rigorous construction (without interpolation) of this surface from data of the optical investigation of domains are pointed out.

As GINSBURG HAS SHOWN[1], the theory of piezoelectric phenomena can be developed on the basis of the general theory of phase transitions of the second kind[2]. In the general theory it is demonstrated that in the vicinity of the Curie point $T=\Theta$, the expansion of the thermodynamic potential in a power series of the parameter η which characterizes the degree of asymmetry of the system has the form

$$\Phi = \Phi_0 + A\eta^2 + C\eta^4 + \cdots \qquad (1)$$

Ordinarily it is assumed that in expansion (1) we can confine ourselves to only the terms written