$$e = P_y^{(a)} \langle \sigma_{y'}^{(a)} \rangle_0 / (d\sigma / do)'_0 = P_y \overline{\sigma_{y'}^{(a)}}, \qquad (6)$$

where $\overline{\sigma_y^{(a)}}$ is the polarization of particle a in the reaction $b + Y \to a + X$, when b and Y are unpolarized. In the case of elastic scattering $(a + X \to a + X)$ equation (6) becomes the well known formula of Wolfenstein.[2] Note that Wolfenstein only proved his theorem for the case where inelastic scattering is absent (he assumed the scattering operator M was unitary). Our demonstration is free of this limitation. The third formula in (5) relates the polarization of particle Y in the reaction $a + X \to b + Y$ to the correlated polarization in the reverse reaction. The remaining formulas in (5) need no special elucidation except for the formula of line four. The asterisk attached to the brackets < >, such as in $\langle \sigma_x^{(b)} \rangle_x^*$, denotes the fact that it describes the polarization of particle b for the reaction $a + X \to b + Y$ wherein particle X was polarized in the initial state while particle a was completely unpolarized. The index x attached to the bracket < >, refers here to the x-component of polarization of particle X. The rest of the formulas of line four may be interpreted in this way.

In conclusion the author wishes to thank Ia. A. Smorodinskii for a discussion of the results.

[1] J. M. Blatt and V. Weisskopf, *Theoretical Nuclear Physics*, John Wiley and Sons, Inc., New York (1952).

[2] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).

Translated by M. A. Melkanoff
155

Polarization of Cerenkov Radiation

A. A. SOKOLOV AND IU. M. LOSKUTOV
Moscow State University

(Submitted to JETP editor January 3, 1957)
J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 630-632 (March, 1957)

In order to analyze the dependence of Cerenkov radiation upon the spin of the charged particles, we have utilized the method developed in Ref. 1 (see also Ref. 2)., which allows one to solve for the intensities of both linearly and circularly polarized radiation.

When we consider linear polarization, we must resolve the amplitude of the vector potential of the quantized photon field into two mutually perpendicular components in the following fashion:

$$\mathbf{a} = \mathbf{a}_2 + \mathbf{a}_3 = \beta_2 q_2 + \beta_3 q_3, \quad \beta_2 = [\varkappa^0 \mathbf{k}^0] / \sqrt{1 - (\varkappa^0 \mathbf{k}^0)}, \quad \beta_3 = [\varkappa^0 \beta_2]. \qquad (1)$$

$\boldsymbol{\varkappa}^0 = \boldsymbol{\varkappa}/\varkappa$ is a unit vector which characterizes the motion of the photon and the unit vector $\mathbf{k}^0$ must be assigned some definite direction (in our problem, we shall assume that the vector $\mathbf{k}^0$ is in the direction of the electron motion, *i.e.*, along the z-axis).

In the case of circular polarization the vector potential is resolved into two different components:

$$\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_{-1} = \beta_1 q_1 + \beta_{-1} q_{-1}, \quad \sqrt{2}\beta_\lambda = \beta_2 + i\lambda\beta_3, \quad \lambda = 1, -1. \qquad (2)$$

The quantized part of the vector potential appearing in Eq. (1) and (2) must satisfy the relations

$$q_{j'}^+ q_j = 0, \quad q_j q_{j'}^+ = \delta_{jj'}, \quad j, j' = 2, 3, 1, -1.$$

In contructing the quantized transverse electromagnetic field in a medium of refractive index $n (n = \sqrt{\varepsilon}, \mu = 1)$ (*cf.* Refs. 3 and 4, where the quantum theory of the Cerenkov effect is developed), we find that the vector potential $\mathbf{A}$ is related to the quantized amplitudes $\mathbf{a}$ (*cf.* Ref. 1 or 2) through the following expression

$$\mathbf{A} = L^{-3/2} \sum_{\varkappa} \sqrt{\frac{2\pi c\hbar}{n\varkappa}} (\mathbf{a} \exp\{-ic(\varkappa t / n) + i\boldsymbol{\varkappa}\mathbf{r}\} + \mathbf{a}^+ \exp\{ic(\varkappa t / n - i\boldsymbol{\varkappa}\mathbf{r}\}). \qquad (3)$$

We shall choose to write the wave function for a free electron in the form

$$\psi = L^{-3/2} \sum_{\mathbf{k}'} C' b' \exp\{-icK't + i\mathbf{k}'\mathbf{r}\}, \qquad (4)$$

where $\hbar\mathbf{k}$ is the momentum of the electron, $c\hbar K = c\hbar\sqrt{k^2 + k_0^2}$ the electron energy and $\hbar k_0/c$ its mass. We shall denote the initial state of the electron by an unprimed symbol and its final state by a primed symbol

Introducing the perturbation energy $U = (e/c)(\alpha\mathbf{A})$, we must consider the coefficients C' as time-dependent, satisfying the initial condition: $C' = C(k', 0) = \delta_{\mathbf{k}'\mathbf{k}}$.

When solving the Dirac equation (*i.e.*, including electron spin), the ψ-function (4) is given by a four-row matrix. When solving the Klein-Gordon Equation (*i.e.*, for spinless particles), we must limit ourselves to two wave functions assuming

$$b_1 = (2c\hbar K)^{-1/2}, \quad b_2 = (c\hbar K / 2)^{1/2}. \qquad (5)$$

In that case the expression for the density becomes $\rho = \psi_1^+ \psi_2 + \psi_2^+ \psi_1$. One then finds the following expression for the radiation intensity per unit legth of electron motion:

$$W_{(j)}^{(s)} = \frac{\hbar\omega}{v}\frac{\partial}{\partial t}\sum_{\mathbf{k}'} C'^{+}C' = \frac{e^2}{2\pi}\int \frac{c}{nv}\,\delta\left(K - \frac{\varkappa}{n} - K'\right)$$

$$R_{(j)}^{(s)+} R_{(j)}^{(s)} \sin\theta d\theta d\varphi \varkappa^2 d\varkappa, \quad R_{(j)}^{(s)} = b'^{+} (\alpha^{(s)} \alpha_{(j)}^{+})\, b, \qquad (6)$$

where v is the electron velocity. The index $s(s = \frac{1}{2}$; $s\,(s = \frac{1}{2}, 0)$ denotes here the spin of the particle, $j\,(j = 2, 3, +1, -1)$ denotes the polarization of the radiated photons, and θ is the angle between the initial momentum of the electron and that of the radiated photon.

The expression for $\cos\theta$ may be obtained, as usual, from conservation of energy and momentum (*cf.* Ref. 3 and 4):

$$\cos\theta = (1 / \beta n) + (n\omega\hbar / 2cp)\,(1 - n^{-2}). \qquad (7)$$

Taking into account the dependence of n upon the frequency ω and the fact that n tends to unity for large values of ω, radiation is possible if $\beta n(\omega) > 1$, and is cut off at a frequency $\omega = \omega_{max}$ for which $\cos\theta = 1$.

When solving the Dirac equation, we must replace the $a^{(s)}$ by the well-known Dirac matrices ($a^{(1/2)} = a$), while in the case of zero-spin particles we must set $b\,a^{(0)} = b\mathbf{k}/K$ and $b'a^0 = b'\mathbf{k}/K$. The radiation due to zero-spin particles ($s = 0$), and that due to particles possessing spin, is thus found to be:

$$W_{(j)}^{(0)} = \frac{e^2}{v}\int_0^{\omega_{max}} \omega\, \frac{(\mathbf{k}\mathbf{a}_{(j)})(\mathbf{k}'\mathbf{a}_{(j)}^{+})}{nKK'}\,\delta\left(K - \frac{\varkappa}{n} - K'\right)\sin\theta\, d\theta\, \varkappa d\varkappa,$$

$$W_{(j)}^{(1/2)} = \frac{e^2}{v}\int_0^{\omega_{max}} \omega \left[\frac{(\mathbf{k}\mathbf{a}_{(j)})\,(\mathbf{k}'\mathbf{a}_{(j)}^{+})}{nKK'} + \frac{1}{2}\left(1 - \frac{k_0^2 + (\mathbf{k}\mathbf{k}')}{KK'}\right)(\mathbf{a}_{(j)}\,\mathbf{a}_{(j)}^{+})\right] \qquad (8)$$

$$\times\,\delta\left(K - \frac{\varkappa}{n} - K'\right)\sin\theta d\theta \varkappa d\varkappa.$$

In order to take polarization into account, we must apply formulas (10)–(12) of Ref. 1 to eliminate the quantized amplitudes of the vector potential. The intensity of radiation emitted per unit length of a zero-spin particle is then found to be

$$W_{(2)}^{(0)} = 0, \quad W_{(3)}^{(0)} = \frac{e^2}{c^2}\int_0^{\omega_{max}} \omega\,(1 - \cos^2\theta)\, d\omega, \qquad (9)$$

i.e., the radiation will be strongly linearly polarized over the whole range of frequency ω, and the polarization vector (the electric intensity vector for the radiated photons) must be in the plane ($\varkappa\mathbf{k}$).

In the case of radiation due to particles of spin one half, we find

$$W_{(2)}^{(1/2)} = \frac{e^2}{c^2}\int_0^{\omega_{max}} \omega\,\frac{n^2\omega^2\hbar^2}{4c^2p^2}\left(1 - \frac{1}{n^2}\right) d\omega, \qquad (10)$$

$$W_{(3)}^{(1/2)} = W_{(3)}^{(0)} + W_{(2)}^{(1/2)}; \quad W_{(-1)}^{(1/2)} = W_{(1)}^{(1/2)}. \qquad (11)$$

Thus it may be seen that in the classical approximation ($\hbar \to 0$), the radiation will be completely linearly polarized just as in the case of zero-spin particles.

The presence of spin leads to an additional unpolarized radiation which does not disappear at the radiation threshold ($E = E_0$). The linearly polarized part of the radiation is missing at threshold, but it grows in proportion to $(E - E_0)$ as the electron energy E ($(E - E_0)/E_0 \ll 1$) grows:

$$W_{(3)}^{(0)} = 2\,\frac{e^2}{c^2}\,\frac{E - E_0}{mc^2}\int_0^{\omega_{max}} \omega n^2 d\omega. \qquad (12)$$

Harding and Henderson[5] state in a brief note that they have observed non-polarized Cerenkov radiation near threshold. However, in view of the small intensity of the unpolarized part of the radiation, proportional to $\hbar^2$, it is difficult to believe that these observations are somehow tied up with spin effects. Still, it should be noted that, due to the wide use use of light counters, experimental technique has reached such a degree of accuracy that it has become possible to observe Cerenkov effects due to single particles.[6]

---

[1] A. A. Sokolov and I. M. Termov, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 473 (1956); Soviet Phys. JETP **4**, 396 (1957).

[2] A. A. Sokolov and A. I. Mukhtarov, Vestn. Moscow State Univ. **8**, 63 (1948).

[3] A. A. Sokolov, Dokl. Akad. Nauk SSSR **28**, 415 (1950); see also A. A. Sokolov and D. D. Ivanenko, *The Quantum Theory of Fields*, Moscow-Lenningrad (1952) p. 222.

[4] V. L. Ginzburg. J. Exptl. Theoret. Phys. (U.S.S.R.) **10**, 589 (1940).

[5] J. M. Harding and J. E. Henderson, Phys. Rev. **74**, 1560 (1948).

[6] I. M. Frank, Izv. Fyz. Nauk **58**, 111 (1956).

Translated by M. A. Melkanoff
156