process for which the matrix element is given by

$$S_{i\to f} = -\frac{e}{\sqrt{2\omega}}\int \overline{\psi}_p^{(-)}(\gamma e)\, e^{ikx}\, \psi_{-\tilde{p}}^{(+)}\, d^4x; \qquad (1)$$

Here $\psi_p^{(-)}$ is the wave function of a proton in the field of the nucleus, behaving asymptotically like a plane wave of momentum p together with an ingoing spherical wave. $\psi_{-\tilde{p}}^{(+)}$ is the wave-function of a proton with negative energy $-\tilde{E}$, behaving asymptotically like a plane wave of momentum $-\tilde{p}$ with an outgoing spherical wave. **e** is the polarization vector of the photon. The proton-antiproton interaction is neglected; this is allowable when the photon energy ω is not too close to the threshold. The wave-functions $\psi_p^{(-)}$ and $\psi_{-\tilde{p}}^{(+)}$ are constructed by means of the optical model, in which the interactions of the proton and antiproton with the nucleus are described phenomenologically.

The problem becomes very simple in the limit $\omega >> 2M$, when the important part of the matrix element comes from a region far from the nucleus, where the wave functions behave like a superposition of a plane wave and a wave diffracted by the nucleus. Akhiezer[1] has worked out the theory of diffraction for spinor waves, and he finds for the wave function of a spin $-1/2$ particle scattered by a completely black absorbing nucleus of radius R

$$\psi_{-\tilde{p}}^{(+)} = -\frac{1}{4\pi}\int\left(\gamma\frac{\partial}{\partial \mathbf{x}} + \gamma_4\tilde{E} - M\right) \qquad (2)$$

$$\times \frac{e^{i\tilde{p}\,|\mathbf{x}-\rho|}}{|\mathbf{x}-\rho|}(\gamma\tilde{\mathbf{n}})\, v_{-\tilde{p}}\, d\rho\cdot e^{i\tilde{E}t},$$

$$(\rho \geqslant R,\ \rho \perp \mathbf{n}).$$

Here the integration extends over a plane through the center of the nucleus and perpendicular to the unit vector $\tilde{\mathbf{n}} = (\tilde{\mathbf{p}}/|\tilde{\mathbf{p}}|)$. $v_{-\tilde{p}}$ is the spinor amplitude of the plane wave with momentum $-\tilde{p}$ and negative energy $-\tilde{E}$. Similarly,

$$\overline{\psi}_p^{(-)} = \frac{1}{4\pi} \qquad (3)$$

$$\times\int \overline{u}_p(\gamma\mathbf{n})\left(\gamma\frac{\partial}{\partial \mathbf{x}} + \gamma_4 E + M\right)\frac{e^{ip\,|\mathbf{x}-\rho|}}{|\mathbf{x}-\rho|}\, d\rho\cdot e^{iEt},$$

$$(\rho \geqslant R,\ \rho \perp \mathbf{n}),$$

with $\mathbf{n} = (\mathbf{p}/|\mathbf{p}|)$. The Coulomb interaction between the nucleons and the nucleus is neglected.

The idea of diffractive scattering only makes sense at small angles. Supposing the angles between the pair momenta **p**, $\tilde{\mathbf{p}}$ and the phonon momentum **k** to be small, and the energies E, $\tilde{E}$ large compared with M, the cross-section for proton-antiproton pair production by a black uncharged nucleus becomes

$$d\sigma = \frac{e^2}{32\pi^3}\frac{E\tilde{E}}{\omega^3} \qquad (4)$$

$$\times \frac{R^2 J_1^2(MR\,|\xi+\eta|)}{|\xi+\eta|^2}\left[\frac{2(\xi+\eta)^2}{(1+\xi^2)(1+\eta^2)}\right.$$

$$+\frac{\omega^2}{E\tilde{E}}\left(\frac{\xi}{1+\xi^2} - \frac{\eta}{1+\eta^2}\right)^2$$

$$\left.+\left(2+\frac{\omega^2}{E\tilde{E}}\right)\left(\frac{1}{1+\xi^2}+\frac{1}{1+\eta^2}\right)^2\right] d\tilde{E}\, d\xi\, d\eta;$$

with the vectors ξ and η defined by

$$\mathbf{p} = (\mathbf{p}\varkappa)\varkappa + M\xi, \quad \tilde{\mathbf{p}} = (\tilde{\mathbf{p}}\varkappa)\varkappa + M\eta.$$
$$\varkappa = \mathbf{k}/\omega, \quad \xi\varkappa = \eta\varkappa = 0; \qquad (5)$$
$$d\xi = (E/M)^2\, d\Omega, \quad d\eta = (\tilde{E}/M)^2\, d\tilde{\Omega}.$$

Unlike the Born approximation formulas, the expression (4) cannot be obtained from the corresponding expression[1] for the bremsstrahlung of a proton diffractively scattered by a black uncharged nucleus by substituting $-\tilde{p}$, p, $-k$ for p, p', k. The reason is that in the extreme relativistic limit the diffracted waves hardly overlap at all in bremsstrahlung but overlap strongly in pair-production.

If we assume a finite size for the nucleon,[2] a form-factor will appear in Eq.(4). We have not included any effect of the anomalous magnetic moment of the nucleon.

1 A. I. Akhiezer, Dokl. Akad.Nauk SSSR 94, 651 (1954).
2 I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR 96, 265 and 481 (1954).

Translated by F. J. Dyson
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Internal Compton Effect in Pair Conversion

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(Submitted to JETP editor October 30, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 384–385 (February, 1957)

IF the energy difference between excited and ground state is greater than $2mc^2$ (m is the electron mass), then a nucleus in the excited state may decay by emission of an electron-positron pair togetherwith a photon. We call this process the internal Compton effect in pair conversion. We exhibit below the formulas for the relative differential probability of this process (i.e., the ratio of the absolute probability of internal Compton effect in pair conversion to the probability of a simple radiative transition), calculated in Born approximation.

The Feynman diagrams for this process are analogous to those appearing in the calculation[1] of internal Compton effect involving the atomic electrons. We maintain the notations of the earlier calculation[1], except that ϵ_+, ϵ_-, p_+, p_- now denote the energy and momentum of the positron and electron.

The relative differential cross-section for a magnetic 2^j-pole transition is

$$
\begin{aligned}
d\gamma_j^{(\mathrm{M})} = {} & \frac{4(2j+1)\alpha^2 k p_+ p_-}{(2\pi)^3 P}\left(\frac{P}{\Delta E}\right)^{2j+1}\frac{1}{(P^2-\Delta E^2)^2}\Bigl\{kp_+\,[\cos(\mathbf{p}_+\mathbf{k})-\cos(\mathbf{p}_+\mathbf{P})\cos(\mathbf{kP})] \\
& \times\Bigl[\frac{m^2+(p_-k)}{(p_-k)^2}+\frac{(p_+p_-)}{(p_-k)(p_+k)}\Bigr]+kp_-\,[\cos(\mathbf{p}_-\mathbf{k})-\cos(\mathbf{p}_-\mathbf{P})\cos(\mathbf{kP})]\Bigl[\frac{m^2+(p_+k)}{(p_+k)^2} \\
& +\frac{(p_+p_-)}{(p_+k)(p_-k)}\Bigr]+p_+p_-\,[\cos(\mathbf{p}_+\mathbf{p}_-)-\cos(\mathbf{p}_+\mathbf{P})\cos(\mathbf{p}_-\mathbf{P})]\Bigl[\frac{m^2}{(p_-k)^2}+\frac{m^2}{(p_+k)^2} \\
& +\frac{(p_+p_-)+(p_++p_-,k)}{(p_-k)(p_+k)}\Bigr]+\frac{m^2k^2}{(p_-k)(p_+k)}\sin^2(\mathbf{kP})-\frac{p_-^2}{(p_-k)}\sin^2(\mathbf{p}_-\mathbf{P})-\frac{p_+^2}{(p_+k)}\sin^2(\mathbf{p}_+\mathbf{P}) \\
& -\frac{m^2(p_+P)}{(p_-k)^2}-\frac{m^2(p_-P)}{(p_+k)^2}-\frac{m^2+(p_+k)}{(p_-k)}-\frac{m^2+(p_-k)}{(p_+k)} \\
& -2\,\frac{(p_+p_-)[(p_+p_-)+(p_++p_-,k)-m^2]}{(p_+k)(p_-k)}\Bigr\}\sin\vartheta_-d\vartheta_-do_+dk\,d\varepsilon_+,
\end{aligned}
$$

Here $(p_+p_-) = \mathbf{p}_+\mathbf{p}_- - \epsilon_+\epsilon_-$ is the scalar product of the 4-vectors p_+ and p_-. The z-axis is taken along the direction of $\mathbf{k}$, $\vartheta_\pm$ are the polar angles of the vectors $\mathbf{p}_\pm$, and

$$do_+ = \sin\vartheta_+ d\vartheta_+\,d\varphi_+;\ \mathbf{P}=\mathbf{p}_++\mathbf{p}_-+\mathbf{k}.$$

The relative differential cross-section for an electric $2j$-pole transition is

$$
\begin{aligned}
d\gamma_j^{(\text{э})} = {} & \frac{4(2j+1)\alpha^2 p_+p_-k}{(2\pi)^3(j+1)P}\left(\frac{P}{\Delta E}\right)^{2j+1}\frac{1}{(P^2-\Delta E^2)^2}\Bigl\{2\Bigl[\frac{m^2+(p_-k)}{(p_-k)^2}+\frac{(p_+p_-)}{(p_-k)(p_+k)}\Bigr] \\
& \times\Bigl[jk\varepsilon_+-jk\varepsilon_+\frac{\Delta E}{P}\cos(\mathbf{kP})-jkp_+\frac{\Delta E}{P}\cos(\mathbf{p}_+\mathbf{P})+\frac{k\Delta E^2}{2P^2}p_+((j+1)\cos(\mathbf{kp}_+) \\
& +(j-1)\cos(\mathbf{kP})\cos(\mathbf{p}_+\mathbf{P}))\Bigr]+2\Bigl[\frac{m^2+(p_+k)}{(p_+k)^2}+\frac{(p_+p_-)}{(p_+k)(p_-k)}\Bigr] \\
& \times\Bigl[jk\varepsilon_--jk\varepsilon_-\frac{\Delta E}{P}\cos(\mathbf{kP})-jkp_-\frac{\Delta E}{P}\cos(\mathbf{p}_-\mathbf{P})+p_-\frac{k\Delta E^2}{2P^2}((j+1)\cos(\mathbf{p}_-\mathbf{k}) \\
& +(j-1)\cos(\mathbf{kP})\cos(\mathbf{p}_-\mathbf{P}))\Bigr]+2\Bigl[\frac{m^2}{(p_-k)^2}+\frac{m^2}{(p_+k)^2}+\frac{(p_+p_-)+(p_++p_-,k)}{(p_-k)(p_+k)}\Bigr] \\
& \times\Bigl[j\varepsilon_+\varepsilon_--j\varepsilon_+p_-\frac{\Delta E}{P}\cos(\mathbf{p}_-\mathbf{P})-j\varepsilon_-p_+\frac{\Delta E}{P}\cos(\mathbf{p}_+\mathbf{P})+p_+p_-\frac{\Delta E^2}{P^2}((j+1)\cos(\mathbf{p}_+\mathbf{p}_-) \\
& +(j-1)\cos(\mathbf{p}_+\mathbf{P})\cos(\mathbf{p}_-\mathbf{P}))\Bigr]-\frac{2}{(p_-k)}\Bigl[j\varepsilon_-^2-2j\varepsilon_-p_-\frac{\Delta E}{P}\cos(\mathbf{p}_-\mathbf{P}) \\
& +\frac{p_-^2\Delta E^2}{2P^2}((j-1)\cos^2(\mathbf{p}_-\mathbf{P})+2)\Bigr]-\frac{2}{(p_+k)}\Bigl[j\varepsilon_+^2-2j\varepsilon_+p_+\frac{\Delta E}{P}\cos(\mathbf{p}_+\mathbf{P}) \\
& +\frac{p_+^2\Delta E^2}{2P^2}((j-1)\cos^2(\mathbf{p}_+\mathbf{P})+2)\Bigr]+\frac{2m^2k^2}{(p_+k)(p_-k)}\Bigl[j-2j\frac{\Delta E}{P}\cos(\mathbf{kP}) \\
& +\frac{\Delta E^2}{2P^2}((j-1)\cos^2(\mathbf{kP})+2)\Bigr]+\Bigl[j-(2j+1)\frac{\Delta E^2}{P^2}\Bigr]\Bigl[\frac{m^2}{(p_-k)^2}(p_+P) \\
& +\frac{m^2}{(p_+k)^2}(p_-P)+\frac{m^2+(p_+k)}{(p_-k)}+\frac{m^2+(p_-k)}{(p_+k)}+2\,\frac{(p_+p_-)[(p_+p_-)+(p_++p_-,k)-m^2]}{(p_+k)(p_-k)}\Bigr]\Bigr\} \\
& \times\sin\vartheta_-d\vartheta_-do_+dk\,d\varepsilon_+.
\end{aligned}
$$

These formulas become much simpler in the extreme relativistic limit. After integrating over angles, the relative differential probability becomes identical for magnetic and electric transitions, namely

$$d\gamma_j = \frac{(2j+1)\,\alpha^2}{2\,(2\pi)^2}$$

$$\times\left(1-\frac{m^2\,(\Delta E-k)}{\Delta E\varepsilon_+\varepsilon_-}\right)^j \frac{(\varepsilon_+^2+\varepsilon_-^2)\,k)}{(\Delta E-k)^2\Delta E^3}\,dk\,d\varepsilon_+.$$

The ratio of the differential probability for internal Compton effect to the probability for ordinary internal pair-conversion is roughly given by

$$d\gamma_j/d\beta_j = (\alpha/2\pi)\,kdk/(\Delta E-k)^2,$$

provided that ΔE-$K >> m$.

In conclusion the author thanks I. S. Shapiro for valuable advice and assistance.

1. E. G. Melikian, J. Exptl, Theoret, Phys. (U.S.S.R.) 31, 1088 (1956); Soviet Phys. JETP

Translated by F. J. Dyson
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The Momentum Distribution of Interacting Fermi Particles

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(Submitted to JETP editor November 22, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R) 32, 399–400 (February, 1957)

WE consider a system composed of a large number of interacting Fermi-particles. It is to be expected that among the excited states of the system there will exist states whose energy can be expressed as a sum of energies of quasi-particles. The energy of a quasi-particle of momentum p is

$$\varepsilon_p = v_0\,(p-p_0),$$

where p_0 is the momentum at the top of the Fermi sea of the quasi-particles, $v_0 = v\,(p_0)$ is the velocity of a quasi-particle at the Fermi surface, $p > p_0$ for quasi-particles and $p < p_0$ for holes. The momentum p_0 need not coincide with the limiting momentum p_0^0 determined by the density,

$$p_0^0 = (3\pi^2 n)^{1/3}, \qquad (\hbar = 1).$$

It is easy to see that the quasi-particles have an attenuation proportional to $(p-p_0)$.[2] This means that for p_0 not close to p_0 an excited state of a system with strong interactions cannot be described in terms of quasi-particles. As $p \to p_0$ the state becomes describable in terms of quasi-particles even when the interaction is strong. We shall prove that the momentum distribution of the particles in the ground state has a discontinuity at $p = p_0$, for any kind of interaction. This result refers to the distribution of particles and not of quasi-particles.

The one-particle Green's function is defined by

$$G(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) = i\,\langle Te^{iHt_1}\Psi(\mathbf{r}_1)\,e^{-iH(t_1-t_2)}\,\Psi^+(\mathbf{r}_2)\,e^{iHt_2}\rangle, \qquad (1)$$

where the expectation value is taken in the ground state of the system $\Psi(\mathbf{r}) = \Sigma a_\mathbf{p}\, e^{i\mathbf{pr}}$, and $a_\mathbf{p}$ is the annihilation operator for a particle of momentum p. If there is no external field, G is a function only of $r = |\mathbf{r}_1 - \mathbf{r}_2|$ and $\tau = t_1 - t_2$. Expressing G as a Fourier series in coordinate space, we find

$$G(r,\tau) = \sum G(p,\tau)\,e^{i\mathbf{pr}}; \qquad (2)$$

$$G(p,\tau) = \begin{cases} ie^{iE_0\tau} < a_p e^{-iH\tau} a_p^+ >, & \tau > 0, \\ -ie^{-iE_0\tau} < a_p^+ e^{iH\tau} a_\mathbf{p} >, & \tau < 0. \end{cases}$$

This equation connects the function $G(p,\tau)$ with the momentum distribution of particles in the ground state, which is

$$n(p) = < a_p^+ a_\mathbf{p} > = iG(p,\tau)\,|_{\tau\to -0}.$$

Writing

$$G(p,\tau) = \int G(p,\varepsilon)\,e^{-i\varepsilon\tau}\,d\varepsilon\,/\,2\pi,$$

we obtain