$$b_{nlm\sigma\tau} \equiv b(q),\ b^+(q)$$

destroy and create nucleons in the state q. In the new representation, the vector of state is given by the sets of functions

$$C_A(q_1, q_2 \cdots q_A),$$

depending on a sets of indices; the Hamiltonian is of the form

$$\hat{H} = \sum_{q_1 q_2 q_1' q_2'} -\frac{1}{2} b^+(q_1)\, b^+(q_2) \langle q_1 q_2 | \hat{H} | q_1' q_2' \rangle\, b(q_1')\, b(q_2').$$

No assumptions are made about the convergence of the matrix elements

$$\langle q_1 q_2 | \hat{H} | q_1' q_2' \rangle$$

in terms of the oscillator quantum number n. Only a small part of these elements will be independent and non–vanishing; in an interaction between a pair of particles, the total momentum of both particles, the coordinates of their center of gravity, the total moment and the isotopic spin, their projections, and parity are conserved. In order to make use of the above conservation laws, one can express the matrix elements through the matrix elements

$$\langle Q | \hat{H} | Q' \rangle$$

in terms of the oscillator wave functions of the relative motion of the two particles (Q and Q' are the sets of quantum numbers describing the relative motion of the two particles).

The operator $\hat{H}$ (or $\hat{V}$) does not act upon the coordinate of the center–of–mass of two particles

$$(r_1 + r_2)/2.$$

The matrix element

$$\langle Q | \hat{V} | Q' \rangle$$

is diagonal in respect to the total moment and the isotopic spin of the two particles and is independent of their projections. If, in the expansion of the wave functions, we limit ourselves to the first two oscillator states with $n = 0$ and $n = 1$, then there will be only 16 different matrix elements

$$\langle Q | V | Q' \rangle$$

We can hope that it will be possible to describe, by means of these 16 values, the ground and the lower excited states of nuclei up to oxygen. The actual computations in this approximation require the use of computers.

We shall present the results of computation in the most crude approximation ($n = 0$, i.e., all nucleons in the ls state) for the H^3, He^3 and He^4 nuclei. In these approximations, there are only two matrix elements

$$\langle {}^3S_1, T = 0 | V | {}^3S_1, T = 0 \rangle = A_1;$$

$$\langle {}^1S_0, T = 1 | V | {}^1S_0, T = 1 \rangle = A_0,$$

for which we obtained the following system of equations:

$$(3\hbar^2 / mr_0^2) - 3(A_1 + A_0) = -8.49 \text{ mev}, \qquad (H^3)$$

$$(3\hbar^2 / mr_0^2) + (2e^2 / \sqrt{\pi} r_0) - 3(A_1 + A_0) = -7.73 \text{ mev}, \qquad (He^3)$$

$$(9\hbar^2 / 2mr_0^2) + (2e^2 / \sqrt{\pi} r_0) - 6(A_1 + A_0) = -28{,}27 \text{ mev}, \qquad (He^4)$$

This system of three equations contains two unknowns r_0 and $A_1 + A_0$. The equations are satisfied for

$$A_1 + A_2 = 10.83 \text{ mev, and } r_0 = 2.27 \cdot 10^{-13} \text{ cm.}$$

The Coulomb energy

$$2e^2 / \sqrt{\pi} r_0$$

equals to 0.716 mev, while the experimental value is 0.764 mev.

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1 J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics.*

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Scale Transformation and the Virial Theorem in Quantum Field Theory

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UNDER the term "scale transformation" we shall understand the transformation of the scale of coordinates, accompanied by an inverse change of the mass scale

$$x_\mu \to \lambda x_\mu; \quad m \to m/\lambda; \quad M \to M/\lambda, \qquad (1)$$

where λ is a real positive number, m is the mesonic mass, and M the nucleonic mass. Neutral meson and nucleon fields will be considered for the sake of simplicity. Two facts pertaining to the group of scale transformations will be noted below: a)the invariance of field equations with respect to this group of transformations and the relations following from this, and b) the virial theorem, deduced by means of scale variations. The variation of the length scale was earlier studied by Fock[1] and then by Demkov[2] in connection with the virial theorem in quantum mechanics. The variation of the scales of length, time and mass for the Dirac equation were studied by Infeld and Schild[3] in gravitational theory.

The field equations for the meson and the nucleon fields are of the form

$$D(x, M)\psi(x, M) \equiv i(\gamma_\mu \partial/\partial x_\mu + M)\psi(x, M) \qquad (2)$$
$$= g\gamma_5\varphi(x, m)\psi(x, M),$$

$$(\square - m^2)\varphi(x, m) = -g\overline{\psi}(x, M)\gamma_5\psi(x, M) - 4k\varphi^3(x, m).$$

It is easily seen that Eqs. (2) are invariant under scale transformation (1), if the field operators are transformed according to the formulas

$$\psi'(x, M) = \lambda^{3/2}\psi(\lambda x, M/\lambda);$$

$$\varphi'(x, m) = \lambda\varphi(\lambda x, m/\lambda). \qquad (3)$$

It should be noted that the term containing φ^3 in Eq. (2) is the only non–linear term which can be added to the meson equation without impairing the scale invariance of the field equations. The creation operators

$$a^+, \ b^+, \ c^+$$

and destruction operators a, b, c for free fields, which satisfy the commutation relations

$$\{a^+(\mathbf{p}, M), a(\mathbf{p}', M)\} = \{b^+(\mathbf{p}, M), b(\mathbf{p}', M)\}$$

$$= [c(\mathbf{p}, m), c^+(\mathbf{p}', m)] = \delta(\mathbf{p} - \mathbf{p}')$$

are transformed, according to (1), in the following way:

$$a'^+(\mathbf{p}, M) = \lambda^{-3/2}a^+(\mathbf{p}/\lambda, M/\lambda), \qquad (4)$$

$$c'^+(\mathbf{k}, m) = \lambda^{-3/2}c^+(\mathbf{k}/\lambda, m/\lambda).$$

It follows from the invariance of the field equations with respect to the homogeneous space–time "expansion" $x \to \lambda x$ and the proportional mass construction that the transformation function

$$U_{12} = (\Psi(\sigma_2), \Psi(\sigma_1)),$$

is invariant. $\Psi(\sigma)$ denotes the Heisenberg vector of state, determined by means of the field operators on the space–like surface σ. Indeed, according to the Schwinger action principle

$$\delta U_{12} = i(\Psi(\sigma_2), \ \delta W_{12}\Psi(\sigma_1)) \qquad (5)$$

the variation δU_{12} is determined by the variation of the action operator δW_{12} which, however, does not change under transformation (1). The variated and unvariated actions

$$W'_{12} = \int_{\lambda\sigma_1}^{\lambda\sigma_2} L\left(x', \frac{m}{\lambda}, \frac{M}{\lambda}; \ \psi, \overline{\psi}, \varphi\right) dx',$$

$$W_{12} = \int_{\sigma_1}^{\sigma_2} L(x', m, M; \ \psi, \overline{\psi}, \varphi)\, dx',$$

are equal, if relations (3) are satisfied. In the expression for W'_{12}, $\lambda\sigma$ denotes the space–like surface obtained from σ as the result of replacing x by λx.

In the special case when $\sigma_2 \to +\infty$ and $\sigma_1 - \infty$, the transformation function U_{12} becomes the scattering matrix $S_{12} = (\Psi_{out}^{(2)}, \Psi_{in}^{(1)})$. If we limit ourselves to scattering problems in which Ψ_{out} and Ψ_{in} describe free particles, we can make use of the relations (4). The relation $S'_{12} = S_{12}$ is then equivalent to

$$S_{12}(\mathbf{p}_1 \ldots \mathbf{p}_n; \ \mathbf{p}'_1 \ldots \mathbf{p}'_{n'}; \ m, M) \qquad (6)$$

$$= \lambda^{-3/2(n+n')} S_{12}\left(\frac{\mathbf{p}_1}{\lambda} \ldots \frac{\mathbf{p}_n}{\lambda}; \ \frac{\mathbf{p}'_1}{\lambda} \ldots \frac{\mathbf{p}'_{n'}}{\lambda}; \ \frac{m}{\lambda}, \frac{M}{\lambda}\right),$$

where the unprimed values correspond to the initial state, and the primed values to the final one; $\mathbf{p}_i$ is the momentum of the i th particle, and n is the number of particles (nucleons and mesons). For a uniform change of all the masses and momenta of the particles, the transition matrix elements is therefore multipled by the normalization factor

$$\lambda^{-3(n+n')/2}.$$

In the case of the Bethe–Salpeter equation, a relation concerning the kernel Q of the equation follows from the invariance of the field equations with respect to the scale transformation (1). We write the Bethe–Salpeter equation in the form

$$D(x, M_a)\, D(y, M_b)\, f(x, y) = \int Q(x, y;\; x', y';\; M_a, M_b, m)\, f(x', y')\, dx'\, dy',$$

where M_a and M_b are the experimental masses of the particles, and the kernel Q can be regarded as already normalized.

The relation in question is then of the form

$$Q(x, y;\; x', y';\; \lambda m, \lambda M_a, \lambda M_b) = \lambda^{10} Q(\lambda x, \lambda y;\; \lambda x', \lambda y';\; m, M_a, M_b). \quad (7)$$

The Virial Theorem. The invariance of the field equations (2) with respect to the scale transformations (1) is disturbed in the presence of external fields. The variation of the scattering matrix δS_{12} does not vanish in the presence of an external field. In order to find its value we shall make use of the action principle (5). It is evident that, in the absence of an external nucleon field, the variation of a matrix element of the scattering matrix

$$\delta S_{12} = i\,(\Psi'^{(2)}_{\text{out}},\ \delta W_e \Psi'^{(1)}_{\text{in}})$$

will be determined by the variation of the part of the action operation W_e, depending on the external meson field φe:

$$W_e = -ig \int \overline{\psi}(x)\, \gamma_5 \varphi_e(x)\, \psi(x)\, dx.$$

The varied action W'_e is obtained from W_e by putting $x = \lambda y$ and the introduction of variated operators $\psi'(y) = \lambda^{3/2} \psi(\lambda y)$ and $\psi(y) = \lambda^{3/2} \psi(\lambda y)$, which yield

$$W'_e = -ig\lambda \int \overline{\psi'}(y)\, \gamma_5 \varphi_e(\lambda y)\, \psi'(y)\, dy.$$

Putting $\lambda = 1 + \epsilon$, where ϵ is infinitesimally small, we find the relation

$$\sum_i^n p_i \frac{\partial S_{12}}{\partial p_i} + \sum_j^{n'} p'_j \frac{\partial S_{12}}{\partial p'_j} + m\frac{\partial S_{12}}{\partial m} + M\frac{\partial S_{12}}{\partial M} + \frac{3}{2}(n + n')\, S_{12} = \left(\Psi'^{(2)}_{\text{out}}, -g\int \varphi\, \gamma_5\, \psi \left[\varphi_e + x_\mu \frac{\partial \varphi_e}{\partial x_\mu}\right] dx\, \Psi'^{(1)}_{\text{in}}\right), \quad (8)$$

where the summation is extended on the incident particles (total number n) and the scattered particles (total number n'). The expression under the sign of the integral in the right–hand side of Eq. (8) is of the form typical for the virial theorem. The relation (8) can be therefore regarded as the virial theorem of the quantum field theory.

If we consider the case when we have an external nucleon field $\psi_e(x)$ instead of the external meson field, the nucleon field being characterized by the current

$$j_e = -ig\,\overline{\psi}_e(x)\, \gamma_5 \psi_e(x),$$

we find in an analogous manner that the right–hand side of the equation (8) will be equal to

$$i\left(\Psi'^{(2)}_{\text{out}},\ \int \varphi(x)\left[3 j_e(x) + x_\mu \frac{\partial j_e(x)}{\partial x_\mu}\right] dx\, \Psi'^{(1)}_{\text{in}}\right). \quad (9)$$

The generalization of the above ideas for the case of other fields is straightforward.

Note added in proof: After the paper had been submitted to the editor, I. M. Shmushkevitch drew the attention of the author to the fact that the scale invariance of the equations of quantum electrodynamics is mentioned in the book of Jauch and Rohrlich.[4] The consequences of this fact and the virial theorem are not, however, studied there.

1 V. A. Fock, Z. Physik 63, 855 (1930).
2 M. N. Demkov, Dokl. Akad. Nauk SSSR 89, 249 (1953).
3 L. Infeld and R. Schild, Phys. Rev. 70, 410 (1946).
4 J. Jauch and F. Rohrlich, *Theory of Photons and Electrons*, Cambridge, Mass, 1955.

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Some New Electrets from Inorganic Dielectrics

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IT is known that the electret represents an electric analog of the magnet. It is a dielectric characterized by a "constant" electrification with opposite charges at its ends. Usually electrets are obtained by cooling a heated dielectric in an electric field.