Scattering of π–Mesons on Nucleons in Higher Approximations of the Tamm–Dancoff Method

Iu. M. Popov
P. N. Lebedev Physical Institute, Academy of Sciences, USSR
(Submitted to JETP editor September 29,1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 169–171 (January,1957)

In the Tamm–Dancoff method[1,2] the nucleon–meson equation for the state with isotopic spin $I = 3/2$ is studied in higher approximation than the one used until now. For this purpose, the equations are renormalized and the four–particle approximation is considered. In the equations (written according to the T–D method) for the nucleon–meson and nucleon–two mesons amplitudes (in the Dalitz–Dyson notation[3], for the

$$\langle a_{q\alpha} b_{-qu}\rangle \text{ and } \langle a_{q\alpha} a_{k\beta} b_{-q-kw}\rangle$$

amplitudes, respectively) only those amplitudes (with the number of particles $n \leq 4$ are taken into account) which are found on the right hand sides of the equations for these amplitudes (connected with them by one "step"). The kernels of the resulting equations contain infinities corresponding to the self–energy diagrams of mesons and nucleons, and we always drop these terms from the equations. The system of equations obtained in this manner in the "old" T–D method coincides with the system in the "new" method formulated by Dyson[4] when the amplitudes with "minus" particles are omitted from the latter. In the momentum representation, following Dirac,[5] we are looking for the first of the above–mentioned amplitudes in the following form:

$$\langle a_{q\alpha} b_{-qu}\rangle = \delta(q - q_0) + K_1(qq_0)\,\delta_+(\varepsilon - \omega_q - E_q). \quad (1)$$

Examining the higher approximations of the T–D method, we assume that

$$\varepsilon < m + 2\mu$$

that is to say, no additional mesons are produced and the second amplitude is of the form:

$$\langle a_{q\alpha} a_{k\beta} b_{-q-kw}\rangle = P(\varepsilon - \omega_q - \omega_k - E_{q+k})^{-1} K_2(qkq_0). \quad (2)$$

Taking all the above into account, we obtain the equations for K_1 and K_2

$$\begin{aligned} K_2(qkq_0) &= \int d^3p B(kqp)\, P(\varepsilon - \omega_p - \omega_k - E_{p+k})^{-1} K_2(kpq_0) \\ &+ \int d^3p B(qkp)\, P(\varepsilon - \omega_p - \omega_q - E_{p+q})^{-1} K_2(qpq_0) \\ &+ iA(qk)\,[\delta(k - q_0) + \delta_+(\varepsilon - \omega_k - E_k) K_1(kq_0)] \\ &+ iA(kq)\,[\delta(q - q_0) + \delta_+(\varepsilon - \omega_q - E_q) K_1(qq_0)]; \end{aligned} \quad (3)$$

$$\begin{aligned} K_1(qq_0) &= \int d^3k C(qk)\,[\delta(k - q_0) + \delta_+(\varepsilon - \omega_k - E_k) K_1(kq_0)] \\ &+ i \int d^3k L(qk)\, P(\varepsilon - \omega_q - \omega_k - E_{q+k})^{-1} K_2(qkq_0). \end{aligned} \quad (4)$$

$$B(kqp) = \frac{G^2}{2} Q \frac{1}{\sqrt{\omega_q \omega_p}} w_{-q-k}\gamma \times \left(\frac{\Omega^+(-q-k-p)}{\varepsilon - \omega_k - \omega_q - \omega_p - E_{k+q+p}} - \frac{\Omega^-(-q-k-p)}{\varepsilon - E_{q+k+p} - E_{p+k} - E_{q+k} - \omega_k} \right) \gamma v_{-k-p};$$

$$C(qk) = -\frac{G^2}{2} Q \frac{1}{\sqrt{\omega_q \omega_k}} w^*_{-q}\gamma \times \frac{\Omega^-(-q-k)}{\varepsilon - E_k - E_q - E_{q+k}} \gamma u_{-k}$$

$$A(qk) = -iGw^*_{-q-k}\gamma\tau_q v_{-k} / \sqrt{2\omega_q},$$

$$L(qk) = -iGu^*_q \gamma\tau_k w_{-q-k} / \sqrt{2\omega_k},$$

$$\Omega^{\pm}(p) = [E_p \pm (\alpha p + \beta m)] / 2E_p,$$

$$G = g/(2\pi)^{3/2}, \quad Q = (\tau_\gamma\tau_\alpha, \tau_\gamma\tau_\beta).$$

The system of equations (3) and (4) contains divergent terms even after removing infinities arising in nuclei. This fact represents the main difficulty in the study of higher approximations of the T–D method for the case of the meson–nucleon equation. We have found such a solution of the non–covariant (three–dimensional) equations (3) and (4) in which the infinities are contained in the functions for the vertex parts and can be removed by renormalization (Itabashi[6] proposed a similar method of renormalization for covariant equations). The amplitude

$$K_2(qkq_0)$$

can be expressed by the resolvent R_2 of Equation (3) in the following way:

$$K_2(qkq_0) = i \iint d^3q'\, d^3k' R_2(qkq'k')\, A(k'q')\, [\delta(q'-q_0) + \delta_+(\varepsilon - \omega_{q'} - E_{q'})\, K_1(q'q_0)]. \quad (5)$$

Performing several transformations upon the equation for R_2, analogous to those of Ref. 6, we obtained, after substituting (5) into (4)

$$K_1(qq_0) = \int d^3q' M(qq')\, [\delta(q'-q_0) + \delta_+(\varepsilon - \omega_{q'} - E_{q'})\, K_1(q'q_0)], \quad (6)$$

$$M(qq') = -[C(qq') + M_1(qq') + M_2(qq') + M_3(qq')]$$

(we omitted from the kernel the term corresponding to the self–energy of the nucleon),

$$M_1(qq') = \Gamma(qq')\Gamma'(qq'), \quad M_2(qq') = \int d^3p\, \Gamma(qp) B(pqq')\, \Gamma'(pq'),$$

$$M_3(qq') = \iiiint d^3p\, d^3p'\, dp^{3''}\, d^3p'''\, \Gamma(qp)\, P\frac{B(pqp')}{\varepsilon - \omega_{p'} - \omega_p - E_{p'+p}} R_2(pp'p''p''') \times P\frac{B(p'''p''q')}{\varepsilon - \omega_{p''} - \omega_{p'''} - E_{p''+p'''}}\, \Gamma'(p'''q').$$

The sense of the applied transformation lies in the fact that the infinities contained in the system of equations (3) and (4) are separated in the functions for the vertex parts $\Gamma(qq')$ and $\Gamma'(qq')$:

$$\Gamma(qq') = L(qq') / (\varepsilon - \omega_q - \omega_{q'} - E_{q+q'}) + \int d^3p\, \Gamma(qp)\, B(qpq'), \quad (7)$$

$$\Gamma'(qq') = A(qq') + \int d^3p B(q'qp)\, \Gamma'(pq').$$

The actual solution of Eqs. (3) and (4) is impossible without separating the angle variables. We shall separate the angles in the equation for

$$K_2(qkq_0)$$

in the case when the total moment of the system—nucleon—two mesons $J = 1/2$, and the angular moments of mesons lie between 0 and 1, which makes it possible to take into account higher approximations for the scattering of pions by nuclei in the S–state. In this way, in the case studied by us, the angular state of two mesons and a nucleon is characterized by the function

$$W_M^{J-1-1/2}(\Omega_1\Omega_2) = \frac{1}{\sqrt{3}} \times \begin{pmatrix} -\sqrt{3/2 - M}\; Y_1^{m-1/2}(\Omega_1)\, Y_0^0(\Omega_2) \\ \sqrt{3/2 + M}\; Y_1^{m+1/2}(\Omega_1)\, Y_0^0(\Omega_2) \end{pmatrix} \equiv \langle 1_{\Omega_1} 0_{\Omega_2} \rangle_M. \quad (8)$$

The state of a nucleon and a meson with $l = 0$ is characterized by the function

$$W_M^{1/2}(\Omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{M + 1/2}\; Y_0^{M-1/2}(\Omega) \\ \sqrt{-M + 1/2}\; Y_0^{M+1/2}(\Omega) \end{pmatrix} \equiv \langle 0_\Omega \rangle_M. \quad (9)$$

$$(qp)\, P\frac{B(pqp')}{\varepsilon - \omega_{p'} - \omega_p - E_{p'+p}} R_2(pp'p''p''')$$

We shall write the amplitudes of K_1 and K_2 as products of functions depending on the absolute value of vectors, and functions depending on the angles

$$K_2(qkq_0) = f_2(qkq_0) \sum_M [\langle 1_q 0_k \rangle_M + \langle 0_q 1_k \rangle_M]\, \langle 0_{q_0} \rangle_M,$$

$$K_1(qq_0) = f_1(qq_0) \sum_M \langle 0_q \rangle_M \langle 0_{q_0} \rangle_M^*. \quad (10)$$

Making use of the orthogonality of spherical functions with spin, we shall obtain from (3) and (4) a system of equations for the functions f_2 and f_1 which do not contain angle variables (the symbol $\sim$ denotes the independence of angles). The equation for the vertex parts is of the form

$$\tilde\Gamma(qq') = \tilde L(qq') + \int p^2 dp\, \tilde\Gamma(qp)\, \tilde B(qpq'), \quad (11)$$

$$\tilde\Gamma'(qq') = \tilde A(qq') + \int p^2 dp\, \tilde B(q'qp)\, \tilde\Gamma'(pq').$$

We shall introduce new functions:

$$V(zq') = \tilde\Gamma'(zq') / \tilde A(zq'), \quad \Lambda(qz) = \tilde\Gamma(qz)\; \tilde L(qz).$$

For the function

$$V^R(mqq')$$

regularized by the Dalitz–Dyson method,[3] we have

$$V^R(mq0) = 1 + \int dp\, [B_1(moqp) - B_1(moop)]\, V^R(mpo). \quad (12)$$

When $V^R(mqo)$ is found, we can further find

$$V^R(\varepsilon qq') = V^R(mqo) + W(\varepsilon qq'), \quad (13)$$

$W(\varepsilon qq')$ satisfies the equation

$$W(\varepsilon q q') = \int dp\,[B_1(\varepsilon q' q p) - B_1(m o q p)]\, V^R(m p o) + \int dp\, B_1(\varepsilon q' q p)\, W(\varepsilon p q' o). \quad (14)$$

In an analogous way, the regularized function $\Lambda^R(qq')$ is found. In order to eliminate the complex expressions from the equations for $f_1(qq_0)$, we shall introduce a new function

$$u(q) = f_1(q)\left[1 + \frac{i}{2}\frac{q_0\omega_{q_0}E_{q_0}}{E_{q_0}+\omega_{q_0}} f_1(q_0)\right]^{-1}. \quad (15)$$

After replacing $\tilde{\Gamma}(qq')$ and $\tilde{\Gamma}'(qq')$ by regularized functions $\tilde{\Gamma}^R(qq')$ and $\tilde{\Gamma}'^R(qq')$, the equation for the function $w(q)$ becomes

$$u(q) = \tilde{M}^R(q_0) + \int \frac{(q')^2\,dq'}{\varepsilon - E_{q'} - \omega_{q'}}\, \tilde{M}^R(qq')\, u(q'). \quad (16)$$

The function $w(q_0)$ determines the phase-shift of the scattering of π-mesons on nucleons in the state with isotopic spin $I = 3/2$ by means of the following formula:

$$\delta_S = -\operatorname{arc\,tg}\left[\frac{\pi q_0 \omega_{c_0} E_{c_0}}{E_{q_0}+\omega_{q_0}} u(q_0)\right]. \quad (17)$$

The author wishes to express his deep gratitude to Prof. I. E. Tamm for proposing the subject and for his help in its treatment, and also extends his thanks to V. Ia. Fainberg for his advice given in the discussion of the present work.

1 I. E. Tamm, J. Phys. (U.S.S.R.) 9, 445 (1945).
2 S. M. Dancoff, Phys. Rev. 78, 382 (1950).
3 R. H. Dalitz and F.I. Dyson, Phys. Rev. 99, 301 (1955).
4 F.I. Dyson, Phys. Rev. 91, 1543 (1953).
5 P. A. M. Dirac, *The Prinicples. of Quantum Mechanics*, 1933.
6 K. I. Itabashi, Progr. Theor. Phys. 12, 585 (1954).

Translated by H. Kasha
37

On the Absolute Value of the Stripping Cross Section and Cross Section for Diffraction Scattering of the Deuteron

I. I. IVANCHIK
P. N. Lebedev Physical Institute, Academy of Sciences, USSR

(Submitted to JETP editor September 21, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 164–165 (January, 1957)

A NUMBER of recent investigations were devoted to the interaction of deuterons with heavy nuclei.[1–5] The cross-section of the diffraction scattering of deuterons on nuclei was calculated in Refs. 3–5, while Ref. 4 treated the case of the diffractive scattering as well. The calculations (for an ideally black nucleus) were effected by seemingly different methods, which, however, can be shown to be essentially identical. Some difference in the final formulas is due to the fact that certain simplifying assumptions of Ref. 4 were too crude.

In Refs. 3 and 4, as well as in usual calculations of the stripping reactions cross-section[6], the wave function of the internal motion in the deuteron is taken as

$$\varphi \sim \exp(-\alpha r)/r, \quad \hbar\alpha = \sqrt{M\varepsilon},$$

where M is the nucleonic mass and ϵ is the deuteron binding energy which corresponds to the assumption of a zero radius of the p–n force. The question arises of how a finite force radius would influence the cross sections of the above processes. If we assume a square-well potential of the p–n interaction of radius a and depth U

(where $Ua^2 \sim \pi\hbar^2/2M$),

it is possible to calculate the stripping and diffraction disintegration cross-section by means of the well-known wave function obtained for this case.[7] It is convenient to make use of Glauber's method[5] [see his formulas (5) and (6)]. As the result, we obtain the following expressions for the stripping cross-section σ_{strip} and the diffraction disintegration cross-section σ_{diffr} of deuterons on a black nucleus of radius R: