

Letters to the Editor

Quantum Field Theoretical Solutions Without Perturbation Theory

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IN considering processes which take place among systems of bosons and fermions, one can, for instance, split the general transition matrix $U[\sigma]$ into a sum of particular transition matrices $U^{(\xi)}[\sigma]$.

The equation connecting the matrices

$$U^{(\xi)} = U^{(i,j;n,m;k,l)} U^{(i,j;n,m;k,l)}$$

describes a transition accompanied by the absorption of i bosons, n fermions and k antifermions and by the corresponding emission of j , m , l of these particles, given by us earlier¹, makes it possible, to obtain exact recurrence relations among the $U^{(\xi)}$; this in turn provides the possibility for analyzing the solution without making use of perturbation theory.

In order to avoid cumbersome expansions, we shall not consider antifermions here. We may then disregard k and l , and set n equal to m . In that case the equation for $U^{(\xi)}$ is considerably simplified (the number of terms on the right-hand of the equation is cut down by a factor of four) and takes the form

$$\begin{aligned} \delta U^{(i,j;n)} / \delta \sigma(x) = N \{ & \bar{u}(x) \gamma u(x) \varphi^{(+)}(x) [U^{(i,j-1;n-1)} + U^{(i,j-1;n)} + U^{(i,j-1;n)}] \\ & + U^{(i,j-1;n+1)} + U^{(i+1,j;n-1)} + U^{(i+1,j;n)} + U^{(i+1,j;n)} + U^{(i+1,j;n+1)} \} \\ & + N \{ \bar{u}(x) \gamma u(x) \varphi^{(-)}(x) [U^{(i-1,j;n-1)} + U^{(i-1,j;n)} + U^{(i-1,j;n)} + U^{(i-1,j;n+1)} \\ & + U^{(i,j+1;n-1)} + U^{(i,j+1;n)} + U^{(i,j+1;n)} + U^{(i,j+1;n+1)}]; \quad U^{(i,j;n)} = U^{(i,j;n,n)}. \end{aligned} \quad (1)$$

$u(x)$, $\Phi^{(-)}(x) = (\hbar c / i g) \varphi^{(-)}(x)$ are annihilation operators and $\bar{u}(x)$, $\Phi^{(+)}(x) = (\hbar c / i g) \varphi^{(+)}(x)$ are creation operators for free fermions and bosons. The expression for γ depends on the choice of interaction and meson type. Finally, the fact that $U^{(i,j-1,n)}$, enters twice in the equation, is linked with the various possible orders of emission and absorption of fermions.

We shall seek a solution of the system (1) for non-stationary problems in the form

$$\begin{aligned} U^{(i,j;n)} &= U^{(\xi)} \\ &= \frac{1}{i! j! (n!)^2} N \int_{\sigma_0}^{\sigma} d_v^{(\xi)} \Lambda^{(\xi)} \{ F^{(\xi)} \} e^{\{ K[\sigma] \}}, \end{aligned} \quad (2)$$

where

$$\Lambda^{(\xi)} = \bar{u}(\xi_1) u(\eta_1) \dots \bar{u}(\xi_n) u(\eta_n) \quad (3)$$

$$\times \varphi^{(+)}(x_1) \dots \varphi^{(+)}(x_j) \varphi^{(-)}(y_1) \dots \varphi^{(-)}(y_i),$$

$$d_v^{(\xi)} = d\xi_1 \dots d\xi_n d\eta_1 \dots d\eta_n dx_1 \dots dx_j dy_1 \dots dy_i,$$

$$\{ F^{(\xi)} \} = \left\{ \begin{array}{l} i, j, n \\ \xi_1 \dots \xi_n \eta_1 \dots \eta_n x_1 \dots x_j y_1 \dots y_i | \sigma \end{array} \right\}$$

is a function to be determined. The quantity in the exponential is

$$\begin{aligned} \{ K[\sigma] \} &= \int_{\sigma_0}^{\sigma} dz_1 \bar{u}(z_1) \gamma u(z_1) \varphi^{(+)}(z_1) \\ &\times \int_{\sigma_0}^{\sigma_1} d\xi d\eta dy \bar{u}(\xi) \gamma u(\eta) \varphi^{(-)}(y) \left\{ \begin{array}{l} 1, 0; 1 \\ \xi \eta y | \sigma \end{array} \right\} \\ &+ \int_{\sigma_0}^{\sigma} dz_1 \bar{u}(z_1) \gamma u(z_1) \varphi^{(-)}(z_1) \\ &\times \int_{\sigma_0}^{\sigma_1} d\xi d\eta dx \bar{u}(\xi) \gamma u(\eta) \varphi^{(+)}(x) \left\{ \begin{array}{l} 0, 1; 1 \\ \xi \eta x | \sigma \end{array} \right\}. \end{aligned} \quad (4)$$

The symbol N appearing in front of all terms in (2) denotes the normal product of the operators standing to the right of it. Apart from $\Lambda^{(\xi)}$ the field operators also enter in $\{ F^{(\xi)} \}$ and $\{ K[\sigma] \}$. Here, however, in contrast to $\Lambda^{(\xi)}$, there appears not the "final" operators, but combinations of non-commuting pairs of operators (0-pairs). Applying the symbol N to these expressions denotes that the operators which enter in them "quasi" commute (anti-commute) with all operators except for these which are included within the same curly brackets.

We shall omit the symbol N below, keeping only the curly brackets to denote 0-pairs.

An analysis of Eq. (1) shows that $\{ F^{(\xi)} \}$ must have the following form:

$$\begin{aligned}
\{F^{(\xi)}\} = & \sum_{q,p,r;l} \{[\delta_\tau(\xi_q \eta_p x_r) + \delta_\tau(\xi_q \eta_p y_l)] (F_1^{(\xi)})_{\alpha_q \beta_p} + [\delta_\tau(\xi_q x_r) \\
& + \delta_\tau(\xi_q y_l)] (F_2^{(\xi)})_{\alpha_q} + [\delta_\tau(\eta_p x_r) + \delta_\tau(\eta_p y_l)] (F_3^{(\xi)})_{\beta_p} + \delta_\tau(\xi_q \eta_p) (F_4^{(\xi)})_{\alpha_q \beta_p} \\
& + \delta_\tau(\xi_q) (F_5^{(\xi)})_{\alpha_q} + \delta_\tau(\eta_p) (F_6^{(\xi)})_{\beta_p} + [\delta_\tau(x_r) + \delta_\tau(y_l)] F_7^{(\xi)} + F_8^{(\xi)}\}. \quad (5)
\end{aligned}$$

In the preceding equation, the expression

$$\delta_\tau(\xi_q x_r); \delta_\tau(\xi_q \eta_p y_l) = \delta_\tau(\xi_q \eta_p) \delta_\tau(\eta_p y_l)$$

and other similar expressions denote the usual δ -functions, while the index τ is introduced to emphasize the fact that the operators in the integrand of (2) and which depend on the variables entering for example in $\delta_\tau(\xi_q x_r)$ refer to a time which follows all the remaining ones. σ is a hypersurface lying between σ and σ_0 , and which contains the points² $\xi_q \eta_p x_r$ or y_l .

The entire expression in the integrand must be symmetric with respect to the permutations $\xi_1 \dots \xi_n, \eta_1 \dots \eta_n$, and $x_1 \dots x_r y_1 \dots y_1$, which treat equally all particles of interest.

Substituting (2) into the original Eq. (1), and taking Eq. (5) into account, the recurrence relations for the unknown functions $F_1^{(\xi)} \dots F_8^{(\xi)}$ are easily obtained. These relations are as follows

$$[(F_1^{(\xi)})_{\alpha\beta} | \xi_n = \eta_n = x_j = z] = \gamma \{F^{(i,j-1; n-1)}\}, \quad (6)$$

$$[(F_2^{(\xi)})_\alpha | \xi_n = x_j = z] \quad (7)$$

$$= \gamma_{\alpha\beta} \int_{\sigma_0}^{\sigma} dq \{u_\beta(z) \bar{u}(q)\} \{F^{(i,j-1; n)}\} | \xi_n = q,$$

$$[(F_3^{(\xi)})_\beta | \eta_n = x_j = z] \quad (8)$$

$$= \gamma_{\alpha\gamma} \int_{\sigma_0}^{\sigma} dq \{\bar{u}_\alpha(z) u(q)\} \{F^{(i,j-1; n)}\} | \eta_n = q,$$

$$[(F_4^{(\xi)})_{\alpha\beta} | \xi_n = \eta_n = z], \quad (9)$$

$$\begin{aligned}
= & \gamma_{\alpha\beta} \int_{\sigma_0}^{\sigma} dq [\{\varphi^{(+)}(z) \varphi^{(-)}(q)\} \{F^{(i+1,j; n-1)}\} | y_{i+1} = q \\
& + \{\varphi^{(-)}(z) \varphi^{(+)}(q)\} \{F^{(i,j+1; n-1)}\} | x_{j+1} = q],
\end{aligned}$$

$$[(F_5^{(\xi)})_\alpha | \xi_n = z] = \quad (10)$$

$$= \gamma_{\alpha\beta} \int_{\sigma_0}^{\sigma} dq_1 dq_2 \{u_\beta(z) \bar{u}(q_1)\} [\{\varphi^{(+)}(z) \varphi^{(-)}(q_2)\}$$

$$\times \{F^{(i+1,j; n)}\} | \xi_n = q_1; y^{i+1} = q_2$$

$$+ \{\varphi^{(-)}(z) \varphi^{(+)}(q_2)\} \{F^{(i,i+1; n)}\} | \xi_n = q_1; x_{j+1} = q_2],$$

$$[(F_6^{(\xi)})_\beta | \eta_n = z] = \quad (11)$$

$$= \gamma_{\alpha\beta} \int_{\sigma_0}^{\sigma} dq_1 dq_2 \{\bar{u}_\alpha(z) u(q_1)\} [\{\varphi^{(+)}(z) \varphi^{(-)}(q_2)\}$$

$$\times \{F^{(i+1,j; n)}\} | \eta_n = q_1; y_{i+1} = q_2 +$$

$$+ \{\varphi^{(-)}(z) \varphi^{(+)}(q_2)\} \{F^{(i,i+1; n)}\} | \eta_n = q_1; x_{j+1} = q_2],$$

$$[(F_7^{(\xi)}) | x_j = z] = \gamma_{\alpha\beta} \int_{\sigma_0}^{\sigma} dq_1 dq_2 \{\bar{u}_\alpha(z) u(q_1)\} \quad (12)$$

$$\times \{u_\beta(z) \bar{u}(q_2)\} \{F^{(i,j-1; n+1)}\} | \xi_{n+1} = q_1; \eta_{n+1} = q_2,$$

$$F_8^{(\xi)}(\pm) = \int_{\sigma_0}^{\sigma} dz_1 \bar{u}(z_1) \gamma u(z_1) \varphi^{(\pm)}(z_1) \quad (13)$$

$$\times \int_{\sigma_0}^{\sigma_1} d\xi_0 d\eta_0 dy_0 \bar{u}(\xi_0) u(\eta_0) \varphi^{(\mp)}(y_0) \{f^{(\xi)}(\xi_0 \dots y_i | \sigma),$$

where

$$\begin{aligned}
\{f_{(+)}^{(\xi)}(\xi_0 \dots y_i | \sigma)\} = & \{F^{(i+1,j; n+1)}(\xi_0 \dots y_i | \sigma)\} \\
- & \{F^{(1,0; 1)}(\xi_0 \eta_0 y_0 | \sigma)\} \{F^{(\xi)}(\xi_1 \dots y_1 | \sigma)\}. \quad (14)
\end{aligned}$$

These relations yield an exact solution as a function of the interaction constant.

One might have the impression that the presence of the factor $e\{K[\sigma]\}$ leads to a solution in the form of transcendental functions of the operators, so that its practical usage requires a series expansion. This however is not the case, since $\{K[\sigma]\}$

and all other quantities in curly brackets form a single "package".

* In equation (5), $F_k(\xi) = F_k(\xi) (\xi_1 \dots \gamma_1 | \sigma')$ for $K=1, \dots, 7$, and $F_8(\xi) = F_8(\xi) (\xi_1 \dots \gamma_i | \sigma)$.

1 V. I. Grigor'ev, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 873 (1956); Soviet Phys. JETP **3**, 691 (1956).

Translated by M. A. Melkanoff
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**Concerning the Letter to the Editor
by V. A. Krasnokutskii
"Light from Aluminum Melts in an
Electrolytic Bath"**

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IN the above mentioned letter to the editor¹, V. A. Krasnokutskii discusses an interesting light effect observed during electrolytic oxidation of aluminum or of aluminum melts. However, the author erroneously states that this phenomenon was first discovered by him. Light emission from aluminum electrodes accompanying the formation of oxidation film in electrolytic solutions has been known for several decades, and is described in many monographs devoted to the technical application of anode oxidation of aluminum^{2,3}. In addition, Dufford⁴ investigated the light emission under discussion in the same electrolytes used by Krasnokutskii and partially obtained similar results. Krasnokutskii's statement that light emission is not observed in oxide solutions is applicable only to certain acids, while considerable light emission is observed in H_3PO_4 and in different organic acids⁴⁻⁶. The influence of different impurities in the aluminum melt on the spectrum of the light was the subject of detail investigations by Guminski⁶.

1 V. A. Krasnokutskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 192 (1956); Soviet Phys. JETP **3**, 120 (1956).

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6 K. Guminskii, Bull. Acad. Pol. Sci. Letters, Ser. A, **145**, 457 (1936).

Translated by J. L. Herson
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Paramagnetic Resonance in Alkali Metals

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PARAMAGNETIC absorption resonance due to electron conductivity in metals has been studied by a number of authors¹⁻⁴. We have investigated the dependence of the width of the electron resonance absorption curve on the particle size of metallic lithium containing about 5% impurities of different metals. Measurements were made at frequencies of the oscillating magnetic field of 9350, 400, 225 91 and 35 mcs/s using samples in which the average particle size of lithium varied within wide limits. The method of investigation was described in an earlier work⁴.

It was found that the width ΔH of the absorption resonance curve in metallic lithium at room temperature gradually decreases from 20 to 3 Oe with the decrease of the average metal particle size from ~ 50 to $\sim 0.1\mu$.

The width of the curve and the intensity of absorption in samples of lithium with average particle size $\sim 0.1\mu$ remain constant in the frequency range from 9500 to 35 mcs/s and at temperatures from 300 to 90°K. In samples of lithium with larger average size particles the width of the curve also does not depend on the frequency or temperature*, only a distortion in the form of the absorption resonance curve is observed^{3,6} which depends on the ratio of the particle size to the depth of the skin layer (see Fig.).

It was successfully shown at the 35 mcs/s frequency that lattice spin relaxation time is increasing with decrease of particle size.

The g -factor value was determined in samples of average particle size of 0.1μ at a 9500 mcs. The value was $g = 2.002 + 0.002$.

In the sodium sample of average particle size 0.1μ and containing 5% impurities, the absorption resonance curve at room temperature has $\Delta H = 110$ Oe, $g = 2.002$ at all investigated frequencies. The