T / N	4.5 / 300	4.5 / 600	9.0 / 300	9.0 / 400
α	$-E_0(\alpha)$			
0.5	0.5051	0.5048	0.5047	0.5043
1.0	1.0208	1.0195	1.0195	1.0192
2.0	2.0880	2.0819	2.0839	2.0851
4.0	4.3900	4.3[illegible]87	4.3469	4.3538
5.0	5.6271	5.5787	5.5205	5.5305

Carlo approximation) 190- and 280-fold integrals in place of the path integral. The results, after the production of statistical corrections, are given in Tables 1 and 2.

As a control, we computed the mean value of random quantities and their chief moments, according to which we found the coefficients of expansion of the function $E_0(\alpha)$ in the series $E_0(\alpha) = (\varkappa_1/1!)\,\alpha + (\varkappa_2/2!)\,\alpha^2 + \ldots$

T	4.5	4.5	9.0	9.0	exact value according to Ref. 3
N	300	600	300	400	
$-\varkappa_1$	1.0063	0.9999	0.9912	0.9940	1.0000
$-\varkappa_2$	0.0397	0.0370	0.0354	0.0370	0.0252

Both A. S. Frolov and V. Iu. Krylov took part in this research. V. N. Toroptseva and T. I. Frolova programmed the problem. The authors express their gratitude to M. R. Shura-Bura for valued advice on the problems of programming the problem.

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On Quantities with Anomalous Parity and on a Possible Explanation of Parity Degeneracy of K-Mesons

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IT is well known that K-particles may decay into two or three π-mesons. The corresponding functions must have different parity with respect to the transformation of spatial reflection (i.e., the transformation $x_i \to -x_i$, $i = 1, 2, 3$, $t \to t$), in particular: the function corresponding to the θ-decay must be even, while the function corresponding to the τ-decay must be odd. At the same time it has been established that within experimental error the rest-masses of the θ- and τ-mesons coincide. This coincidence of masses is referred to as the parity-degeneracy of K-mesons. Since the situation in connection with this problem is not at all clear, it seems to be useful to investigate the behavior of quantities with respect to reflections (we use here the word quantity to denote a representation of the complete Lorentz group including reflections of both the space and the time coordinates). Such an investigation has a certain intrinsic interest of its own. It turns out that in addition to the well known possible parities with respect to reflections of the space or the time coordinates there exists also one additional possibility which we shall here refer to as anomalous parity.

In quantum mechanics wave functions are determined up to a factor. It is therefore natural to define transformations of quantities with respect to some given group also up to a factor. Well known examples of the appearance of such factors are provided by spinors which change sign as a result of a rotation by 360°, or wave functions of a system of particles obeying Fermi statistics which change sign as a result of an interchange of particles. The corresponding mathematical concepts are well known — they are the so-called projective representations of the group. We say that a projective representation of the group G is given if to each element $g \epsilon G$ there corresponds a linear transformation T_g such that $T_{g_1 g_2} = \alpha(g_1, g_2)\, T_{g_1} T_{g_2}$, where $\alpha(g_1, g_2)$ is a scalar. It may be shown that the quantities $\alpha(g_1, g_2)$ satisfy the following functional equation:

$$\alpha(g_1 g_2, g_3)\, \alpha(g_1, g_2) = \alpha(g_1, g_2 g_3)\, \alpha(g_2, g_3).$$

We shall consider the representations of a group of reflections which consist of four elements – the unit element and the operators of space, time, and space-time reflections, which we shall respectively denote by 1, t_{10}, t_{01}, t_{11}. The elements t are commutative and satisfy the following relations:

$$t_{10}t_{01} = t_{11};\quad t_{01}t_{11} = t_{10};\quad t_{10}t_{11} = t_{01}. \qquad (1)$$

We denote by T_t the operator of a certain projective representation $t \to T_t$ of the group of reflections. Then the operators T_t should satisfy the equation

$$T_{tt'} = \alpha(t, t')\, T_t T_{t'}.^{*} \qquad (2)$$

In the case that the operators T_t commute, i.e., if $\alpha(t, t') = \alpha(t', t)$, we can assume that $\alpha(t, t') = 1$ for all t, t'. Then the quantities which are transformed by the operators of the representation have with respect to the group of reflections the usual parity properties, giving rise to the four possibilities: (1, 1, 1), (1, –1, –1), (–1, 1, –1) (–1, –1, 1), where in brackets we have given the coefficient by which the quantities are multiplied as a result of being acted upon respectively by the operators T_{01}, T_{10}, T_{11}. The only additional possibility arises if we drop the requirement of the commutativity of the operators, i.e., when $\alpha(t, t') \neq \alpha(t', t)$. In this case relations (2) for the operators may be reduced to the following table:

(3)

	1	T_{01}	T_{10}	T_{11}
1	1	T_{01}	T_{10}	T_{11}
T_{01}	T_{01}	1	$-T_{11}$	$-T_{10}$
T_{10}	T_{10}	T_{11}	1	T_{01}
T_{11}	T_{11}	T_{10}	$-T_{01}$	-1

in which the operator which corresponds to the product may be found at the intersection of the appropriate row and column. We shall refer to quantities transformed by the operators of such a representation as quantities with anomalous parity.

In the simplest case when the quantities being transformed are scalars the operators may be written out in the form of three anticommuting matrices of the second rank analogous to the well known Pauli matrices

$$T_{01} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};\quad T_{10} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\quad T_{11} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad (4)$$

while the transformed quantities themselves ("scalars of anomalous parity") are pairs of numbers (ξ_1, ξ_2), which are unchanged by proper Lorentz transformations and which are transformed in accordance with the matrices (4) under reflections.

The representations of the Lorentz group are described in detail, for example, by Gel'fand and Iaglom[7] whose notation we shall employ in the following. From the commutation relations between the reflections and the Lorentz transformations we may obtain the following description. The irreducible representation of the Lorentz group together with the reflections nay be decomposed into two representations of the proper Lorentz group determined by the numbers (k_0, k_1) and ($-k_0$, k_1)*. If $k_0 \neq 0$ and ξ_p^k are basis vectors in the first representation ($k = |k_0|, |k_0| + 1, \ldots, k_1 - 1$) while η_p^k are basis vectors of the second representation, then the operators T_{10}, T_{01}, T_{11} are given by formulas (5), (6), (7), (7′), of which (5) and (6) correspond to normal parity, while (7) and (7′) correspond to anomalous parity:**

$$T_{01} = \begin{pmatrix} 0 & (-1)^k \\ (-1)^k & 0 \end{pmatrix};\quad T_{10} = \begin{pmatrix} 0 & (-1)^k \\ (-1)^k & 0 \end{pmatrix};\quad T_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad (5)$$

$$T_{01} = \begin{pmatrix} 0 & (-1)^k \\ (-1)^k & 0 \end{pmatrix}; \qquad (6)$$

$$T_{10} = \begin{pmatrix} 0 & (-1)^{k+1} \\ (-1)^{k+1} & 0 \end{pmatrix};\quad T_{11} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$T_{01} = \begin{pmatrix} 0 & (-1)^k \\ (-1)^k & 0 \end{pmatrix}; \qquad (7)$$

$$T_{10} = \begin{pmatrix} 0 & i(-1)^k \\ -i(-1)^k & 0 \end{pmatrix};\quad T_{11} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix},$$

$$T_{01} = \begin{pmatrix} 0 & (-1)^k \\ (-1)^k & 0 \end{pmatrix}; \qquad (7')$$

$$T_{10} = \begin{pmatrix} 0 & -i(-1)^k \\ i(-1)^k & 0 \end{pmatrix};\quad T_{11} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}.$$

In the above, for example, the first of formulas (5) should be interpreted as follows:

$$T_{01}\,\xi_p^k = (-1)^k \eta_p^k;\ T_{01}\,\eta_p^k = (-1)^k \xi_p^k.$$

Similar formulas also hold for the case $k_0 = 0$. In this case four normal and one anomalous possibilities arise.

It may be shown that the product of two quantities of anomalous parity is a normal quantity which decomposes into four quantities corresponding to the four possibilities with respect to space and time reflections. In the case of a product of a normal and an anomalous representation the corresponding quantities have anomalous parity.

Since the interaction Lagrangian must be a scalar (i.e., it must be invariant with respect to the transformations T_{01}, T_{10}, T_{11}), each term of the interaction operator must contain an even number of wave functions of particles with anomalous parity. Therefore in the case of invariance under space and time reflections the following statement must hold: if in a reaction only particles with normal parity occur then conservation of parity must hold; however, if particles with anomalous parity also take part in the reaction then conservation of normality must hold, i.e., the left and the right hand parts of the reaction equation must both contain either an even or an odd number of anomalous particles.

This argument enables us to divide particles into classes with normal and anomalous parities. If we ascribe anomalous parity to K-mesons and normal parity to π-mesons we may conclude that the Λ, Σ particles have the same normality, and so do the particles n, Ξ. For this it is sufficient for example, to consider the strong interaction reaction

$$\pi^- + p \rightarrow \Lambda^0 + \theta^0; \quad K^- + p \rightarrow \Sigma^+ + \pi^-; \quad \Xi^- \rightarrow K^- + \Lambda^0.$$

We have ascribed anomalous parity to the K-meson. The operator for the spatial reflection is given by the matrix T_{10} [Eq. (4)], and consequently the K-meson may exist in two states differing in their spatial parity. The masses corresponding to these two states must be the same.

From the fact that K-mesons which have anomalous parity decay into π-mesons it follows that normality is not conserved in slow reactions. Within the framework of the above explanation this is possible only in the case that the Lagrangian is not invariant under time reflections. The hypothesis proposed by Feynman with respect to the lack of invariance under spatial reflections appears to us to be improbable. On the other hand, sufficient foundations do not exist to assume that the Lagrangian is invariant under time reflections in the case of weak interactions. Such an explanation may also be related to the work of Lee and Yang[2].

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* The first representation contains $|k_0 + k_1|$ undotted and $|k_0 - k_1|$ dotted indices, while the second representation has these numbers reversed (in spinor notation).

** These formulas hold for integral values of k. For half-integral k in the above formulas k should be replaced by $k + \frac{1}{2}$.

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$\gamma - \gamma$ Angular Correlation in Transitions of Mesic Atoms

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PODGORETSKII[1] has pointed out that it would be possible to determine the spin of a muon by using data on the angular correlation between gamma rays emitted in successive transitions of mesic atoms. We wish to examine this problem in the present note and also to propose a method of verifying the spins of nuclei[2] for which $I = 0$ is doubtful or has been obtained only from theoretical considerations unsupported by experiment; the formulas which are derived will be used to determine the spins of all types of mesons.

We know[3] that for light mesic atoms ($Z < 15$), the probabilities of radiation transitions are small compared with the probabilities of conversion transitions. For heavier mesic atoms with small quantum numbers (n, l), the probabilities of conversion transitions can be neglected. The following is an expression for the correlation function $W(\vartheta)$ which is suitable for not very large Z ($15 < Z < 50$)*:

$$W(\vartheta) = \sum_{k=0}^{N} A'_{2k} P_{2k}(\cos\vartheta); \quad A'_{2k} = A_{2k} b_{2k}; \tag{1}$$

$$A_{2k} = A_{2k}(L_1 L_2 j_A j_B j_C) = (-1)^{j_A - j_B}(2L_1 + 1) \tag{1a}$$
$$(2L_2 + 1)(2j_B + 1) C^{2k\,0}_{L_1 1;\, L_1 -1} C^{2k\,0}_{L_2 1;\, L_2 -1} W(j_B j_B L_1 L_1;$$
$$\times\, 2k j_A) W(j_B j_B L_2 L_2;\ 2k j_C);$$