write out the components of the force for two limiting cases. In the case $U_1 << v << U_2$,

$$F_y = (J^2/c^2 l)\,[1 - 2(v/U_2)^{1/2}],$$

$$F_z = -(2J^2/c^2 l)\,(v/U_2)^{1/2},$$

and in the case $v >> U_2$

$$F_y = (J^2/c^2 l)\,[1 - {}^1/_2\sqrt{\pi}\,(U_2/v)^{1/2}];$$

$$F_z = -(J^2\sqrt{\pi}/c^2 l2)\,(U_2/v)^{1/2}.$$

From these equations it is seen that for $v \sim U_2$, the component F_y changes sign and F_z changes from an increasing function of velocity to a decreasing one.

In conclusion I would like to express my gratitude to Professor A. A. Sokolov for his attention to the work.

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On the Damping Theory of Particle Scattering by a Fixed Center

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DAMPING theory, which is a stage beyond perturbation theory, enables us to calculate a cross section σ not only for long de Broglie waves ($\sigma < \lambda^2$) but also for small wave lengths ($\sigma >> \lambda^2$).

Damping theory was developed in Refs. 1–3 in the investigation of meson scattering by nucleons; Sokolov[1] established the relation

$$C^+C + \sum_{\mathbf{k}'} C'^+C' = 1, \qquad (1)$$

which states that the sum of incident and scattered particles at any instant of time remains unchanged. Subsequently we applied damping theory to an investigation of particle scattering by a fixed center.[4–5]

It is well known that the exact formula for the cross section of elastic scattering of particles with momentum hk ($k = 2\pi/\lambda$) is

$$\sigma = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\eta_l.$$

Perturbation theory enables us to determine the phase η_l when $\eta_l << 1$. Damping theory gives the following more exact approximation for the phase shift[5]

$$\operatorname{tg}\eta_l = -\frac{\pi K}{c\hbar}\int_0^{\infty} rV(r)\,J^2_{l+1/2}(kr)\,dr, \qquad (2)$$

(hk is the momentum and chK is the energy of the particle), which for $\eta_l << 1$ becomes the expression established for the phase by perturbation theory.

For the elastic scattering differential cross section , damping theory gives the following expression:

$$\frac{d\sigma}{d\Omega'} = \frac{1}{k^2}\sum_{l,l'}\frac{\operatorname{tg}\eta_l\operatorname{tg}\eta_{l'}(1+\operatorname{tg}\eta_l\operatorname{tg}\eta_{l'})(2l+1)(2l'+1)P_l(\cos\theta')P_{l'}(\cos\theta')}{(1+\operatorname{tg}^2\eta_l)(1+\operatorname{tg}^2\eta_{l'})} \qquad (3)$$

These formulas enable us to investigate the scattering of spinless particles acted on by short-range forces. In particular, we made a detailed study [5] of particle scattering by Yukawa forces. We have been able to extend our results to Dirac particles (i.e., with spin) only for a δ-function interaction.[4]

We shall now investigate particle scattering by damping theory when the interaction potential is of the form

$$V(r) = \begin{cases} B = 3V_0/4\pi a^3, & r < a, \\ 0, & r > a. \end{cases}$$

Then according to (2) we have for the phase η_l

$$\operatorname{tg}\eta_l = -\frac{\pi BK}{c\hbar k^2}\int_0^{ka} J^2_{l+1/2}(y)\,y\,dy.$$

In the one limit $ka >> 1$ scattering is practically determined by the s phase:

$$\sigma = \frac{4\pi}{k^2}\begin{cases} 4B^2K^2a^6k^2/9c^2\hbar^2 & \text{for } 2BKa^3k/3c\hbar \ll 1, \\ 1 & \text{for } 2BKa^3k/3c\hbar \gg 1, \end{cases}$$

i.e., damping will play a decisive part only for

$2bK \gg 1$. In the other limiting case $ka \gg 1$ the high energy region), we obtain for σ

$$\sigma = \frac{4\pi}{k^2}\int_0^\infty \frac{2l\,dl}{1+\operatorname{ctg}^2\eta_l}$$

$$\approx 4\pi a^2 \begin{cases} {}^1/_6\gamma^2 & \text{for } {}^2/_3\gamma^{2/3}BaK/c\hbar k \ll 1, \\ 1 & \text{for } {}^2/_3\gamma \gg 1. \end{cases}$$

Thus the cross section will be four times greater than its classical value. As has been shown by Massey and Mohr[8], the discrepancy between the classical and quantum values is associated with diffraction. Let us compare our results with those given by Mott and Massey[9] (p. 56) for an analogous problem using the smooth joining of wave functions:

$$\sigma = \begin{cases} 4\pi a^2 & \text{for } ka \ll 1, \\ 2\pi a^2 & \text{for } ka \gg 1, \end{cases}$$

i.e., the theories will agree exactly except for numerical coefficients when $ka \gg 1$.

For scattering from a δ-center we were recently able to compare damping theory and other theories. In the nonrelativistic case, the wave equation is (hk_0 / c is the mass of a particle)

$$(\nabla^2 + k^2)\Psi = (2k_0/c\hbar)\,V_0\delta(\mathbf{r})\,\Psi.$$

The solution of this equation which satisfies the radiation condition for $r \to \infty$ is

$$\Psi = Ae^{ikz} + A'\left(\frac{1}{8\pi^3}\int\frac{\exp\{i\varkappa\mathbf{r}\}}{k^2-\varkappa^2}d^3\varkappa - \frac{i}{4\pi}\frac{\sin kr}{r}\right).$$

Keeping in mind the value of the integral

$$\frac{1}{8\pi^3}\int\frac{\exp\{i\varkappa\mathbf{r}\}}{k^2-\varkappa^2}d^3\varkappa = \begin{cases} -(1/4\pi r)\cos kr, & r>0, \\ -(\alpha/\pi^2), & r=0, \end{cases}$$

where α defines the range in which the wave vector of the scattered particle changes ($k-\alpha \leqslant \varkappa \leqslant k+\alpha$), we obtain for the cross section

$$\sigma = \frac{|A'|^2}{4\pi|A|^2}$$

$$= k_0^2V_0^2/\pi c^2\hbar^2\left[\left(1+\frac{2k_0V_0\alpha}{c\hbar\pi^2}\right)^2 + \frac{k_0^2k^2V_0^2}{4\pi^2c^2\hbar^2}\right]^{-1}$$

In solving the problem by perturbation theory, we had to drop terms depending on V_0 in the denominator of (6). In damping theory we had to set α equal to zero in the denominator of (6). This represents the fact that we shall take only resonance terms into account for the scattering ($K \sim K$). Finally, solution by joining of wave functions corresponds to $\alpha \to \infty$. In this case, on complete agreement with (5), we find that for a δ-function interaction scattering should not occur at all.

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*Particle scattering with account taken of damping has also been discussed in Ref. 6 (p. 212).

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Total Inelastic Interaction Cross Sections of 225 ± 10 mev Negative π-mesons with C, Al, Cu, Sn, and Pb nuclei

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IN experiments carried out with the synchrocyclotron of the Laboratory of Nuclear Problems we have determined total cross-sections for the inelastic interaction of π-mesons with nuclei of carbon, aluminum, copper, tin and lead.

For these measurements we used the method of recording the events which result in the removal of the particle from the beam in passing through a scatterer made of the substance under investigation. The mean angle into which the particle was removed was 30°. A telescope consisting of three