In particular, along with the previously known equation

$$
Q_{2}=\delta\left(q_{1}-q_{2}\right) \delta\left(p_{1}-p_{2}\right)
$$

we have

$$
\begin{equation*}
Q_{3}=(\pi \hbar)^{-2 n} e^{2 i \Delta} \hbar ; \tag{55}
\end{equation*}
$$

$$
\begin{aligned}
& Q_{3}^{s}=(\pi \hbar)^{-2 n} \cos (2 \Delta / \hbar) \\
& \Delta=\left|\begin{array}{ccc}
1 & 1 & 1 \\
q_{1} & q_{2} & q_{3} \\
p_{1} & p_{2} & p_{3}
\end{array}\right| .
\end{aligned}
$$

For the spin kernel (15) ( $s=1 / 2$ ) we find

$$
\begin{align*}
& Q_{2}=(1 / 4 \pi)\left(1+3 n_{1} n_{2}\right)  \tag{56}\\
& Q_{3}=1 / 4(2 \pi)^{-3 / 2}\left(1+3 n_{1} n_{2}+3 n_{2} n_{3}\right. \\
& \left.\quad+3 n_{3} n_{1}+3 \sqrt{3} i n_{1}\left[n_{2} n_{3}\right]\right) \\
& Q_{3}^{s}=1 / 4(2 \pi)^{-3 / 2}\left(1+3 n_{1} n_{2}+3 n_{2} n_{3}+3 n_{3} n_{1}\right)
\end{align*}
$$

Equations (54) and (55) are characterized by a lesser degree of quantum degeneracy (correlation) than Eq. (56), since $Q_{2}$, and also the reduced functions $\int Q_{r+1} d q_{1} \ldots d q_{r}$ and $\int Q_{r+1} d p_{1} \ldots$ $d p_{r}$, have the "classical" form (are equal to $\delta$ functions), which is not true of the functions with spin.

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# On the Derivation of the Fokker-Planck Equation for a Plasma 

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The Fokker-Planck equation for a many-component plasma is derived by the method of N. N. Bogoliubov, and the coefficients are calculated in explicit form.

THE Fokker-Planck equation is usually derived from Smoluchowski's equation for stochastic processes, ${ }^{1}$ and thus the dependence of the coefficients in the Fokker-Planck equation on the law of interaction between the particles is left undetermined. For a plasma the Fokker-Planck equation can be obtained from a known kinetic equation of a form given by Landau. ${ }^{2}$ In this case divergences appear for large and small distances, owing to the long-range nature of the Coulomb forces, so that in Ref. 2 the integrals are cut off at the limits of small and large distances.

The method of Bogoliubov ${ }^{3}$ makes it possible to derive the Fokker-Planck equation on thebasis of the mechanics of an assembly of molecules and to calculate the coefficients in explicit form for a given
interaction law. In the case of a plasma the divergence of the Fokker-Planck coefficients at large distances is disposed of by cutting off at the Debye radius, which is not introduced from outside, as in Ref. 2, but follows automatically from Bogoliubov's method. In the present paper we give a derivation of the Fokker-Planck equation for a many-component plasma with uniform spatial distribution, and study the asymptotic cases of the behavior of plasma particles at large and small energies of motion.

We consider the plasma in a state of statistical equilibrium and investigate the behavior of a certain individual particle belonging to the plasma ( or a foreign charged particle projected into theplasma). In the derivation of the equation for the distribution function of such a particle we assume that its in-
teraction withthe plasma does not disturb the statistical equilibrium of the plasma. Let the plasma, contained in the volume $V$, consist of $N$ charged particles, which $\vartheta$ belong to $M \geq 2$ different kinds. Let $N_{a}$ be the number of particles with charge $e_{a}$ and mass $\mu_{a}, a=1,2, \ldots, M$. The Hamiltonian of such a system has the form ${ }^{3}$

$$
\begin{align*}
& H= \sum_{\substack{1 \leqslant a \leqslant M \\
1 \leqslant i \leqslant N_{a}}} H_{a}\left(x_{i}\right)  \tag{1}\\
&+\sum_{\substack{1 \leqslant a, b \leqslant M \\
1 \leqslant i<j \leqslant N_{a}}}^{\sum} \Phi_{a b}\left(\left|q_{i}-q_{j}\right|\right) \\
& H_{a}\left(x_{i}\right)= \\
& \sum_{(1 \leqslant \alpha \leqslant 3)} \frac{\left(p_{i}^{\alpha}\right)^{2}}{2 \mu_{a}} \\
& \Phi_{a b}\left(\left|q_{i}-q_{j}\right|\right)=\frac{e_{a} e_{b}}{\left|q_{i}-q_{j}\right|}
\end{align*}
$$

$x_{i}=\left(q_{i}, p_{i}\right)$ are the coordinates and momenta of the $i$ th particle of the plasma.
Now let

$$
\begin{aligned}
& F_{a, b, \ldots, b_{s}}\left(X, x, \ldots, x_{s}\right. \\
&\left.F_{a}(X, t), F_{b}(x, t), \ldots, F_{b_{s}}\left(x_{s}, t\right)\right)
\end{aligned}
$$

be the distribution functions of $\vartheta$ systems of particles composed of the chosen particle of kind $a$ and $s$ other particles of the plasma of kinds $b_{\vartheta}, \ldots b_{s}$. These functions at an arbitrary instant of time $t>0$ are assumed to depend on the single particle distribution functions for the kinds of particles in question, $F_{a}, F_{b} \ldots$, at the same instant. Here $X=(Q, P)$ are the coordinates and momenta of the chosen particle (or of the charged particle projected into the plasma).

The equation for the distribution function of any single particle of the plasma has the form ${ }^{3}$

$$
\begin{align*}
\frac{\partial F_{a}}{\partial t}=\left[\sum_{\alpha} \frac{\left(P^{\alpha}\right)^{2}}{2 \mu_{a}}\right. & \left.+U_{a}\left(F_{a} ; Q\right) ; F_{a}\right]  \tag{2}\\
& +\sum_{b} n_{b} \int_{\Omega}\left[\Phi_{a b}(|Q-q|) ; g_{a b}\right] d x
\end{align*}
$$

Moreover, for the correlation functions $g a b$ we have (cf. Ref. 3)

$$
\begin{align*}
& D_{0} g_{a b}=\left[\sum_{\alpha}\left(P^{\alpha}\right)^{2} / 2 \mu_{\alpha}+\sum_{\alpha}\left(P^{\alpha}\right)^{2} / 2 \mu_{b}\right.  \tag{3}\\
&\left.+U_{a}\left(F_{a} ; Q\right)+U_{b}\left(F_{b} ; q\right) ; g_{a b}\right] \\
&+\sum_{c}\left(n_{c} / v\right) \int\left[\Phi_{a c}\left(\left|Q-q^{\prime}\right|\right) ; \quad g_{b c} F_{a}\right] d x^{\prime} \\
&+\sum_{c}\left(n_{c} / v\right) \int\left[\Phi_{b c}\left(\left|q-q^{\prime}\right|\right)\right. \\
&\left.g_{a c} F_{b}\right] d x^{\prime}+(1 / v)\left[\Phi_{a b}\left(\left|Q-q^{\prime}\right|\right) ; F_{a} F_{b}\right]
\end{align*}
$$

Here theintegration istaken over the entire phase space $\Omega ; U_{a}$ and $U_{b}$ are self-consistent potentials, $v=V / N$ is the mean volume per particle, and $n_{c}$ $=N_{c} / N$ are the concentrations of particles of kind $c$, $c=1,2, \ldots, M$. By substituting into Eq. (2) the solutions of the equations (3), one can obtain the equation of motion of charged particles in theplasma

In the special case of a spatially homogeneous distribution of the plasma Bogoliubov's equation has the form: ${ }^{3}$

$$
\begin{align*}
& \frac{\partial w_{a}(t, P)}{c t}  \tag{4}\\
& =\sum_{b, \alpha} n_{b} \frac{\partial}{\partial P^{\alpha}} \int \frac{\partial \Phi_{a b}(|\zeta|)}{\partial \zeta^{\alpha}} h_{a b}\left(\zeta, P ; w_{a}(t, P)\right) d \zeta \\
& h_{a b}\left(\zeta, P ; w_{a}\right)=\int g_{a b}\left(\zeta, P, p ; w_{a}, w_{b}\right) d p  \tag{5}\\
& =-\sum_{c, \alpha} n_{c} \frac{1}{v} \iint_{0}^{\infty} \int_{0} \frac{\partial w_{b}(p)}{\partial p^{\alpha}} \\
& \times \frac{\partial \Phi_{b c}\left(\left|\zeta-q^{\prime}-\left(P / \mu_{a}-p / \mu_{b}\right) \tau\right|\right)}{\partial \zeta^{\alpha}} \\
& \quad \times h_{a c}\left(q^{\prime}, P ; w_{a}\right) d p d \tau d q^{\prime} \\
& \times \frac{1}{v} \iint_{0}^{\infty} \int_{c, \alpha}^{\infty} \frac{\partial \Phi_{a c}\left(\left|\zeta-q^{\prime}-\left(P / \mu_{a}-p / \mu_{b}\right) \tau\right|\right)}{\partial P^{\alpha}} \\
& \quad \times h_{b c}\left(-q^{\prime}, p ; w_{b}\right) d p d \tau d q^{\prime} \\
& \quad \\
& \quad-\sum_{\alpha}^{\alpha} \frac{1}{v} \int_{0}^{\infty} \int_{0} \frac{\partial \Phi_{a b}\left(\left|\zeta-\left(P / \mu_{a}-p / \mu_{b}\right) \tau\right|\right)}{\partial \zeta^{\alpha}} \\
& \times\left\{\frac{\partial w_{a}(t, P)}{\partial P^{\alpha}} w_{b}(p)-\frac{\partial w_{b}(p)}{\partial p^{\alpha}} w_{a}(t, P)\right\} d p d \tau
\end{align*}
$$

Here $\zeta=Q-q$, and $w_{a}(t ; P)$ and $w_{b}(p)$ are the distribution functions of the chosen particle of kind $a$ and of any other particle in the plasma for a uniform distribution in space. The first two integral forms in the equations (5) produce the effect of the Debye screening, which cuts off the correlation functions $h_{a b}$ at large distances. ${ }^{3}$

To simplify the calculations we apply to the correlation functions $h_{c b}$ an approximation of the form: ${ }^{3}$ -

$$
\begin{equation*}
h_{b c}\left(-q^{\prime}, p ; w_{b}\right)=g_{b c}^{0}\left(-q^{\prime}\right) w_{b}(p), \tag{6}
\end{equation*}
$$

where $g_{b}{ }_{c}^{0}$ are the Debye functions:

$$
\begin{aligned}
g_{b c}^{0}\left(-q^{\prime}\right)= & -\left(\lambda_{b} \lambda_{c} / r_{D}^{2}\right) \exp \left\{-x\left|q^{\prime}\right|\right\} /\left|q^{\prime}\right| \\
\lambda_{b}= & e_{b} / \sqrt{4 \pi \Sigma n_{s} e_{s}^{2}} ; \\
& 1 / r_{D}^{2}=4 \pi \Sigma n_{s} e_{s}^{2} / \theta v ; \quad x=1 / r_{D}
\end{aligned}
$$

For the solution of the problem we employ the Fourier integral ${ }^{3}$

$$
\begin{aligned}
& h_{a b}\left(\zeta, P ; w_{a}\right)=\int e^{i(\nu \zeta)} H_{a b}\left(\nu, P ; w_{a}\right) d \nu \\
& \Phi_{a b}(|\zeta|)=\frac{v}{4 \pi} \int e^{i(\nu \zeta)} Y_{a b}(|\nu|) d \nu ; \\
& g_{b c}\left(q, q_{1}\right)=\int e^{i\left(\nu\left(q-q_{1}\right)\right)} K_{b c}(|\nu|) d \nu \\
& Y_{a b}(|\nu|)=\Theta 22 \lambda_{a} \lambda_{b} / \pi r_{D^{2}}^{2} ; \\
& K_{b c}(|\nu|)=-\left(\lambda_{b} \lambda_{c} / 2 \pi^{2}\right) /\left(\nu^{2} r_{D}^{2}+1\right)
\end{aligned}
$$

Applying the inverse transformation to the equations (5), we find, in virtue of Eq. (6):

$$
\begin{aligned}
& H_{a b}\left(\nu, P ; w_{a}\right) \\
& \begin{aligned}
&+2 \pi^{2} \sum n_{c} Y_{b c}(|\nu|) B_{b}(\nu, P) H_{a c}\left(\nu, P ; w_{a}\right) \\
&=L_{a b}\left(\nu, P ; w_{a}\right)
\end{aligned} \\
& L_{a b}=-\frac{i}{4 \pi} Y_{a b} A_{b}\left(\nu, P ; w_{a}\right) \\
& -\frac{i}{4 \pi} \sum_{c, \alpha} \frac{\partial w_{a}(t, P)}{\sigma P^{\alpha}} \nu^{\alpha} n_{c} B_{b} Y_{a c} K_{b c} ; \\
& A_{b}\left(\nu, P ; w_{a}\right)=\sum_{c} \frac{\partial w_{a}(t, P)}{c P^{\alpha}} \lambda^{\alpha} B_{b}^{\prime}(\nu, P) \\
& \quad-w_{a}(t, P) B_{b}(\nu, P)
\end{aligned}
$$

$$
\begin{align*}
& B_{b}(\nu, P)=\sum_{\alpha} \nu^{\alpha} \int_{0}^{\infty} \int_{0}^{\partial} \frac{\partial w_{b}(p)}{\partial p^{\alpha}}  \tag{8}\\
& \times \exp \left\{i \tau\left(\nu\left(\frac{p}{\mu_{b}}-\frac{P}{\mu_{a}}\right)\right)\right\} d p d \tau \\
& B_{b}^{\prime}(\nu, P)=\int_{0}^{\infty} \int w_{b}(p) \\
& \quad \times \exp \left\{i \tau\left(\nu\left(\frac{p}{\mu_{b}}-\frac{P}{\mu_{a}}\right)\right)\right\} d p d \tau
\end{align*}
$$

The solution of the system (7) for fixed $a$ and $b$ taking all possible values from: $l$ to $M$ is given by the ratio

$$
H_{a b}=D_{a b} / \Delta,
$$

where $\Delta$ is the determinant of the system

$$
\Delta=1+2 \pi^{2} i \sum_{c} n_{c} Y_{c c}(|\nu|) B_{b}(\nu, P)
$$

and $D_{a b}$ is thedeterminant in which the $b$ th column has been replaced by the right members of the equations of the system

$$
\begin{aligned}
& D_{a b}=L_{a b}\left(1+2 \pi^{2} i \sum_{c} n_{c} Y_{c c} B_{b}\right) \\
& \quad-2 \pi^{2} i \sum_{c} n_{c} Y_{b c} B_{b} L_{a c}
\end{aligned}
$$

The solution of the system (7) can now be written in the form

$$
\begin{equation*}
H_{a b}\left(\nu, P ; w_{a}\right)=H_{a b}^{0}\left(\nu, P ; w_{a}\right) \tag{9}
\end{equation*}
$$

$$
+\grave{\delta} H_{a b}\left(\nu, P ; w_{a}\right),
$$

where $H_{a b}^{0}$ describes the influence of the whole assembly of charged particles of the plasma on the behavior of the chosen particle of kind $a$ :

$$
H_{a b}=-(i / 4 \pi) Y_{a b} A_{b} /\left(1+2 \pi^{2} i \sum_{\epsilon} n_{c} Y_{c c} B_{c}\right)
$$

and $\delta H_{a b}$ describes the influence of theplasma particles on each other:

$$
\delta H_{a b}=-\frac{i}{4 \pi} \sum_{b^{\prime}, d} \frac{\partial w_{a}(t, P)}{\partial P^{\alpha}} \nu^{\alpha} n^{b^{\prime}} Y_{a b^{\prime}} K_{b b^{\prime}} B_{b}^{\prime} / \Delta .
$$

Applying the inverse transformation to Eq. (4), we find

$$
\begin{aligned}
& \frac{\partial w_{a}(t, P)}{\partial t} \\
& =i \frac{v}{4 \pi} \sum_{b, \alpha} n_{b} \frac{\partial}{\partial P^{\alpha}} \int_{(\nu)} v^{\alpha} Y_{a b}(\mid v!) H_{a b}\left(v, P ; w_{a}\right) d \nu,
\end{aligned}
$$

from which there follows the Fokker-Planck equation for a multicomponent plasma

$$
\begin{align*}
\frac{\partial w_{a}(t, P)}{\partial t} & =\sum_{\alpha, \beta=1}^{3} \frac{\partial}{\partial P^{\alpha}} B^{\alpha \beta}(P) \frac{\partial}{\partial P^{\beta}} w_{a}(t, P)(]  \tag{10}\\
& -\sum_{\alpha=1}^{3} \frac{\partial}{\partial P^{\alpha}} A^{\alpha}(P) w_{a}(t, P)
\end{align*}
$$

Here $A^{\alpha}(P)$ and $B^{\alpha \beta}(P)$, the coefficients in the Fokker-Planck equation, are given by

$$
\begin{gathered}
A^{\alpha}(P)=-\frac{v}{16 \pi^{2}} \sum_{b} n_{b} \int_{(\nu)} \frac{\nu^{\alpha}}{\Delta} Y_{a b}^{2}(|v|) B_{b}(\nu, P) d \nu ; \\
=\frac{v}{16 \pi^{2}} \sum_{b} n_{b} \int_{(\nu)} \frac{\nu^{\alpha} \nu^{\beta}}{\Delta} Y_{a b}^{2}(|v|) B_{b}^{\prime}(\nu, P) d \nu \\
\quad+\frac{v}{16 \pi^{2}} \sum_{b, b^{\prime}} n_{b} n_{b^{\prime}} \\
\times \int_{(v)} \frac{\nu^{\alpha} \nu^{\beta}}{\Delta} Y_{a b}(|v|) Y_{a b^{\prime}}(|v|) K_{b b^{\prime}}(|v|) B_{b}^{\prime}(\nu, P) d \nu .
\end{gathered}
$$

For the determination of these coefficients it is necessary to calculate $B_{b}(\nu, P)$ and $B_{b}^{\prime}(\nu, P)$. In statistical equilibrium of the plasma the distribution function for each of its particles, with the exception of the chosen particle, whose motion is assumed nonstationary, can be taken to be a Maxwell distribution
$w_{b}(p)=\left(2 \pi \mu_{b} \theta\right)^{-3 / 2} \exp \left\{-\sum_{\alpha}\left(p^{\alpha}\right)^{2} / 2 \mu_{b} \Theta\right\}$.
Substituting this into Eq. (8) and integrating over p , we find
$B_{b}(\nu, P)=-i \frac{\nu^{2}}{\mu_{b}} \int_{0}^{\infty} \tau \exp \left\{-\frac{\theta \nu^{2}}{2 \mu_{b}} \tau^{2}-i \frac{\tau}{\mu_{a}}(\nu P)\right\} d \tau$,

$$
B_{b}^{\prime}(\nu, P)=\int_{0}^{\infty} \exp \left\{-\frac{\Theta \nu^{2}}{2 \mu_{b}} \tau^{2}-i \frac{\tau}{\mu_{a}}(\nu P)\right\} d \tau
$$

Here it is convenient to make a change of the variable of integration

$$
\tau^{\prime}=\nu \tau\left(\Theta / \mu_{b}\right)^{1 / 2}
$$

and introduce the notation

$$
u_{a b}(\nu, P)=\left(\mu_{b} / \mu_{a}\right)^{1 / 2}\left(\nu \mathbf{P} / \nu \sqrt{\mu_{a}(\theta)}\right)
$$

It is now easy to calculate* $B_{b}$ and $B_{b}$, and the resulting expressions for $A^{\alpha}$ and $B^{\alpha} \beta$ are

$$
\begin{gather*}
A^{\alpha}(P)=-v \frac{\left(\mu_{a} \Theta\right)^{1 / 2}}{16 \pi^{2} r_{D}^{3} t_{0} \Theta^{2}} \sum_{b} n_{b}  \tag{12}\\
\times \int_{(\mathbf{k})} \frac{k^{\alpha}}{f} Y_{a b}^{2}(|k|) u_{a b}(k, P) \\
\times \exp \left\{-1 / 2 u_{a b}^{2}(k, P)\right\} d k \\
B^{\alpha \beta}(P)=v \frac{\mu_{a}}{16 \pi^{2} r_{D}^{3} t_{0} \Theta} \sum_{b} n_{b}\left(\frac{\left.\mu_{b}{ }^{\prime}\right)^{1 / 2}}{\mu_{a}}\right)^{(13)} \\
\times \int_{(\mathbf{k})} \frac{k^{\alpha}}{f} \frac{k^{\beta}}{k} Y_{a b}^{2}(|k|) \exp \left\{-1 / 2 u_{a b}^{2}(k, P)\right\} d k \\
\quad-v \frac{\mu_{a}}{16 \pi^{2} r_{D}^{3} t_{0} \Theta} \sum_{b, b^{\prime}} n_{b} n_{b^{\prime}}\left(\frac{\mu_{b}}{\mu_{a}}\right)^{1 / 2} \\
\times \int_{(\mathbf{k})} \frac{k^{\alpha} k^{\beta}}{f k} Y_{a b}(|k|) Y_{a b^{\prime}}(|k|) K_{b b^{\prime}}(|k|) \\
\times \exp \left\{-1_{2}^{1 / 2} u_{a b}^{2}(k, P)\right\} d k .
\end{gather*}
$$

Here

$$
\begin{gathered}
f=1+2 \pi^{2} \sum_{c} n_{c}\left(1+\eta_{a c}\right) Y_{c c}(|k|) / \theta \\
\eta_{a c}(k, P)=u_{a c}(k, P) \int_{0}^{\infty} e^{-\tau^{2} / 2} \sin u_{a c}(k, P) \tau d \tau
\end{gathered}
$$

We have taken as the unit of time $t_{0}=r_{D}\left(2 \mu_{a} / \pi \Theta^{l / 2}\right.$; $k$ is a dimensionless wave-number: $k=\nu r_{D}$.

Let us examine the coefficients of the FokkerPlanck equation for a plasma. We note than $B \alpha \beta$ $=0$ for $\alpha \neq \beta$. If the energy of the selected particle is small, much smaller than the average energy of the thermal motion of the plasma particles, i.e., for

$$
P^{2} / 2 \mu_{a} \ll \Theta,
$$

expansion of $A^{\alpha}$ and $B^{\alpha} \beta$ in series gives (since here $\left.\eta_{a c}(k, P) \ll 1\right)$

[^1]\[

$$
\begin{gather*}
A^{\alpha}(P)=-1 / 3 \rho\left(P / \mu_{a} \Theta\right) B+\mathrm{o}\left(1 / \Theta^{2}\right) ;  \tag{14}\\
B^{\alpha \beta}(P)=1 / 3 \rho B-\left(1 / 48 \pi^{3}\right) \rho B L^{-1}+\mathrm{o}(1 / \Theta),  \tag{15}\\
B=4 \pi \sum_{c}\left(n_{c} e_{c}^{2} e_{a}^{2} / u_{c}\right) L,
\end{gather*}
$$
\]

$u_{c}$ is the reduced speed of particles of kind $c$ in the plasma,

$$
u_{c}=2 \pi^{4}\left(\dot{2} \Theta / \pi \mu_{c}\right)^{1 / 2}
$$

$\rho$ is the mean density of the plasma, $\rho=N / V$, and $L$ is a logarithmic factor:

$$
\begin{equation*}
L=\int_{0}^{k_{\max }} \frac{k d k}{k^{2}+1} \tag{16}
\end{equation*}
$$

The upper limit of the integration, $k_{\text {max }}=r_{D} / r_{1}$. is introduced because of the divergence of $L$ at small distances $r<r_{1}$, which correspond to large wave-numbers. At large distances and at small wave-numbers the Debye screening, which follows naturally from Bogoliubov's method, assures the good convergence of the coefficients $A^{\alpha}$ and $B^{\alpha} \beta$ Indeed, choosing $r_{1}$ in the form${ }^{2} r_{1}=e^{2} / \Theta$, we find

$$
L=\ln \sqrt{1+\left(r_{D}^{2} \theta^{2} / e^{4}\right)}
$$

Expanding the integral (16) in series, we find a formula agreeing with that of Landau ${ }^{2}$

$$
\begin{equation*}
L=\ln \left(k_{\max } / k_{0}\right), \tag{17}
\end{equation*}
$$

where the lower integration limit $k_{0}=r_{D} / r_{2}$ is introduced because of the logarithmic divergence of $L$ at large distances $r>r_{2}$ and small wavenumbers.
From a comparison of Eqs. (16) and (17), we find $k_{0}$ :

$$
k_{0}=\left(1+k_{\max }^{-2}\right)^{-1 / 2} .
$$

If the temperature $\Theta$ of the plasma is sufficiently high the screening radius $r_{2}$ is equal to the Debye radius $r_{D}$ :

$$
\begin{aligned}
r_{2} \doteq r_{\mathrm{D}} / k_{0}=r_{\mathrm{D}} & \sqrt{1+e^{4} / r_{\mathrm{D}}^{2} \theta^{2}} \\
& =r_{\mathrm{D}}\left(1+e^{4} / 2 r_{\mathrm{D}}^{2} \Theta^{2}-\cdots\right) \approx r_{\mathrm{D}}
\end{aligned}
$$

The factor $L$ can now be expressed by theordinary Landau formula. At high plasma temperature the second term in Eq. (15), which takes account of the mutual influences of the plasma particles, can be neglected.

For the very slow particles of the plasma we have from Eqs. (14) and (15)

$$
\left|A^{\alpha}(P)\right| \ll\left|B^{\alpha \beta}(P)\right|,
$$

so that the equation for the asymptotic behavior of the distribution function of such particlescan be taken in the form

$$
\frac{\partial w_{a}(t, P)}{d t}=\sum_{\alpha, \beta} \frac{\partial}{\partial P^{\alpha}} B^{\alpha \beta}(P) \frac{\partial}{\partial P^{\beta}} w_{a}(t, P) .
$$

For large energy of the selected particle of the plasma, i.e., for

$$
P^{2} / 2 \mu_{a} \gg\left(\mu_{a} / \mu\right) \Theta
$$

we find from Eqs. (12) and (13), since $\eta_{a c}(k, P) \approx 1$,

$$
\begin{aligned}
& A^{\alpha}(P)=-\sqrt{2 \pi} \frac{\left(\mu_{a} \Theta\right)^{1 / 2}}{P^{2}}\left(\frac{\mu_{a}}{\mu}\right)^{3 / 2} B ; \\
& B^{\alpha \beta}(P)=\sqrt{2 \pi} \frac{\left(\mu_{a} \Theta\right)^{3 / 2}}{P^{s}}\left(\frac{\mu_{a}}{\mu}\right)^{\mathrm{s} / 2} B,
\end{aligned}
$$

where $\mu$ is the average mass of the plasma particles.
For the very fast particles of theplasma

$$
\left|B^{\alpha \beta}(P)\right| \ll\left|A^{\alpha}(P)\right|
$$

in consequence of which the equation for the asymptotic behavior of the distribution function of the high-energy charged particles takesthe following form:

$$
\frac{\partial w_{a}(t, P)}{\partial t}=-\sum_{\alpha, \beta} \frac{\partial}{\partial P^{\beta}} A^{\alpha}(P) w_{a l}^{\prime}(t, P) .
$$

For the stationary motion of the selected particle of the plasma we get the Maxwell distribution in each of the cases considered.

In conclusion I express my deep gratitude to

Academician N. N. Bogoliubov for suggesting the problem and directing the work, and to D. N. Zubarev for a discussion of the work.

1 A. N. Kolmogorov, Uspekhi Matem. Nauk 5, 5 (1938).
2 L. D. Landau, Phys. Z. Sowjetunion 10, 154 (1936).

3 N. N. Bogoliubov, Problems of Dynamical Theory in Statistical Physics, State Tech. Press, 1946.

Translated by W. H. Furry
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## Other Errata

| Page | Column | Line | Reads | Should Read |
| :---: | :---: | :---: | :---: | :---: |
| Volume 4 |  |  |  |  |
| 38 | 1 | Eq. (3) | $\ldots \frac{\pi r^{2} \rho^{2} \rho^{2}{ }_{n}}{\rho_{s}^{2}}$ | $\frac{\pi r^{2} \rho^{2} \rho_{n}}{\rho_{s}^{2}},$ |
| 196 |  | Date of submittal | May 7, 1956 | May 7, 1955 |
| 377 | 1 | Caption for Fig. 1 | $\delta_{35}=\eta-21 \cdot \eta^{5}$ | $\delta_{35}=-21^{\circ} \eta^{5}$. |
| 377 | 2 | Caption for Fig. 2 | $\alpha_{3}=6.3^{\circ} \eta$ | $\alpha_{3}=-6.3^{\circ} n$ |
| 516 | 1 | Eq. (29) | $s^{2}, c^{2} \ldots$ | $s / c$ |
| 516 | 2 | Eqs. (31) and (32) | Replace | $2{ }^{2}$ by $A_{1}$ |
| 497 |  | Date of submittal | July 26, 1956 | July 26, 1955 |
| 900 | 1 | Eq. (7) | $\cdots \frac{i}{4 \pi} \sum_{c, \alpha} \frac{\partial w_{a}(t, P)}{\partial P^{\alpha}} \ldots$ <br> (This causes a corre numerical coefficients result from the calculat the plasma particles o | $\ldots 2 \pi^{2} i \sum_{c, \alpha} \ldots$ <br> nding change in the he expressions that of the effects of ch other). |
| 804 | 2 | Eq. (1) | $\ldots \exp \left\{-\left(\bar{T}-V^{\prime}\right)\right\}$ | $\ldots \exp \left\{-\left(\bar{T}-V^{\prime}\right) \tau^{-1}\right\}$ |

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| 59 | 1 | Eq. (6) | $v_{l}\left(1 \partial F_{0} / \partial \mathbf{x}\right)+\ldots$ <br> where $E_{l}$ is the projection of the electric field $E$ on the direction 1 | $\left.\overline{\left(v \partial F_{0} / \partial x\right.}\right)+\ldots$ <br> where the bar indicates averaging over the angle $\theta$ and $E_{l}$ is the projection of the electric field E along the direction 1 |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 91 \\ & 253 \\ & 318 \\ & 398 \end{aligned}$ | 2 1 | Eq. (26) <br> First line of summary <br> Figure caption <br> Figure caption | $\begin{gathered} \Lambda=0.84(1+22 / A) \\ \mathrm{Tl}^{204}, 206 \\ \ldots e^{2} m c^{2}=2.8 \cdot 10^{-23} \mathrm{~cm}, \\ \ldots \text { to a cubic relation. } \\ \text { A series of points etc. } \end{gathered}$ | $\begin{aligned} \Lambda= & 0.84 /(1+22 / A) \\ & \mathrm{Tl}^{203,205} \end{aligned}$ $\ldots e^{2} / m c^{2}=2.8 \cdot 10^{-13} \mathrm{~cm}$ <br> ...to a cubic relation, and in the region 10 $-20^{\circ} \mathrm{K}$ to a quadratic relation. A series of points $\bullet$, coinciding with points $O$, have been omitted in the region above $10^{\circ} \mathrm{K}$. |


[^0]:    1 J. Moyal, Quantum mechanics as a statistical theory, in the collection Questions of Causality in Quantum Mechanics, edited by Ia. P. Terletskii and A. A. Gusev, Foreign Lit. Press, Moscow, 1955.

    2 G. Wick, Calculation of the collision matrix, in the collection Newest Developments of Quantum Electrodynamics, edited by D. D. Ivanenko, Foreign Lit. Press, Moscow, 1954.
    3. P. I. Kuznetsov and R. L. Stratonovich, Izv. Akad. Nauk SSSR, Ser. Mat. 20, 167 (1956).

[^1]:    ${ }^{*} B$ and $B^{\prime}$, respectively, are given by the absolute values of the real parts of the expressions (11).

