SOVIET PHYSICS JETP VOLUME 4, NUMBER 6 JULY, 1957

The Theory of Galvanomagnetic Phenomena in Semiconductors

M. I. KLINGER
Chernovitskii State University
(Submitted to JETP editor May 27, 1954)
J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 1055-1061 (1956)

The technique of studying galvano-magnetic phenomena, described previously by the author, using crossed electric and magnetic fields and taking into account the quantization of the energy spectrum of the current carriers, is applied to calculations of the resistivity and the Hall constant in a strong transverse magnetic field in semiconductors in which the electron gas is not degenerate.

1. INTRODUCTION

IN a study of galvano-magnetic phenomena one does not usually take into account the quantization of the energy spectrum of the current carriers in crossed electrical field E and magnetic field H. However, the quantization of the spectrum changes the kinetic relations of the galvano-magnetic phenomenon.

In a previous work[1] an attempt was made to prove the method of calculation used by Titeica[2] to obtain the isothermal hole effect and the resistivity in a transverse magnetic field. This method allowed the inclusion of the quantization of energy spectrum of the current carriers. In the given work[1], this method was applied to the calculation of the resistivity of the semiconductor and the Hall constant in a strong transverse magnetic field.

It should be noted that Titeica made the first attempt to include the quantization of the energy spectrum of the current carriers in the fields $E \perp H$ in a calculation of the resistivity of metal in a magnetic field. However, it seems to us that the physical essence of the method was not sufficiently developed in his work and the meaning of the physical quantities used in it was not given. Furthermore, the derivation of the equations used for the calculation of resistivity ρ in the magnetic field and of the Hall constant R was not given, and therefore the conditions under which they are applicable were not well determined.

The object of study is the current carrier which moves in a solid body in the presence of crossed electrical and magnetic fields and interacts with phonons. As is well known[1,2], the crossed electrical and magnetic fields ($E \perp H$) change the spectrum of the current carrier, so that the energy of the current carrier is determined by an expression (including the spin of the electron)

$$\mathscr{E}_{Q\pm} = P_z^2/2\mu + \hbar\omega_0(n + 1/2) + eEx_0 \pm \mu_B H,$$
$$n = 0, 1, 2, \ldots; \quad \omega_0 = eH/\mu c; \tag{1}$$
$$x_0 = -P_y/\mu\omega_0 - eE/\mu\omega_0^2,$$

where x_0 is the center of the oscillator, μ_B is Bohr magneton, μ is the effective mass of the appropriate quasi-particle.

Since with the application of the fields the motion of the current carrier becomes anisotropic, one should introduce the conduction tensor $\sigma_{ik}(H)$ (i, $k = 1, 2, 3$) and $\rho_{ik}(H)$, in which $\sigma_{13} = \sigma_{23} = \rho_{13} = \rho_{23} = 0$. In Ref. 1 the equations were developed for the resistivity ρ in a transverse magnetic field H and for the Hall constant R:

$$\rho = j_x E_x/(j_x^2 + j_y^2); \qquad R = -j_y E_x/(j_x^2 + j_y^2) H, \tag{2}$$

where j_x, j_y are the total electrical macroscopic currents in an infinite gyrotropic medium. If one is considering an amphoteric semiconductor, then $j_x = j_x^+ + j_x^-$ and $j_y = j_y^+ + j_y^-$ where the plus and minus signs relate to conduction holes and electrons, respectively.

2. THE HALL EFFECT AND THE ELECTRICAL RESISTIVITY IN STRONG TRANSVERSE MAGNETIC FIELDS IN SEMICONDUCTORS

The current j_y in an infinite gyrotropic medium can be determined for the general case in the following way: a) one calculates $\hat{v}_y = [y\hat{\mathscr{H}}]$, where $\mathscr{H}$ is the Hamiltonian of the considered system in crossed fields, b) one determines the current $j_y = |eS_p(\hat{\rho}\hat{v}_y)|$ by means of the density matrix ρ corresponding to $\mathscr{H}$.

As has been shown previously[1], the average value of the operator of the y component of x velocity of a current carrier v_y is in the first approximation equal to

$$\bar{v}_y = (1/\mu)(P_y + \mu_0\omega_0 x_0) = -cE/H \equiv \gamma.$$

Therefore, in the simplest case, when the current carriers are quasi-particles with some effective mass (quadratic dispersion law),

$$j_y = -e\,\mathrm{Sp}\,(\hat{\rho}\,\hat{v}_y) = (ecE/H)\,N(H, T), \tag{3}$$

where $N(H, T)$ is the number of the current carriers in the semiconductor per cm^3. Since we are at first interested only in the case of electric fields E small enough so that the current j_x and j_y are proportional to E (and nonadequate electron gas),

$$N(H, T) = N_0(H, T)\, e^{\Phi/kT},$$
$$N_0(H, T) = 2\left(\frac{2\pi\mu kT}{h^2}\right)^{3/2} \qquad (4)$$
$$\times \quad \frac{\hbar\omega_0}{2kT\sinh(\hbar\omega_0/2kT)}\cosh\left(\frac{\mu_B H}{kT}\right),$$

are the statistical sums of the electron gas in magnetic fields including the spin of the electron (Φ is the chemical potential). It is clear that the current j_y fundamentally lacks the Ohmic character.

In the method of the stationary states, the current j_x is calculated in the following way.

1. In the Hamiltonian $\mathcal{H}$ of the system under study, we separate several main terms $\mathcal{H}_0$ which give the fundamental energy spectrum of the system and small interaction terms V that are considered as perturbations.

2. We solve the problem $\hat{\mathscr{H}}_0\Psi_Q = \varepsilon_Q\Psi_Q$ and calculate the average value of

$$\bar{x}_Q = \int \Psi_Q^* x \Psi_Q \, d\tau,$$

in which the zero approximation is such that $\bar{x}_q$ depends on the quantum number Q of the unperturbed problem.

3. We introduce the operator V, which takes into account the interactions of the quasi-particles of the system that were omitted in the unperturbed problem. This operator is considered as a perturbation which causes transitions between stationary states. Since $\bar{x}_Q$ depends on Q, then the transitions induced by the perturbing operator V change $\bar{x}_Q$, and thus specify displacement of the current carrier to some other point $\bar{x}_{Q'} \neq \bar{x}_Q$.

In the present formulation of the problem, the following difference should be noted compared to the usual formulation. It is usually assumed that the field E accelerates the current carrier, which gains energy from the field and scatters this acquired energy in interactions with phonons. In the point of view adopted here the process takes place as follows: the field E (together with $H \perp E$) participate in the formation of the stationary states of the current carrier, and the part of the energy of the carrier which depends on E, is equal to eEx_0 (cf. Eq. (1)).

The current carrier in displacing from x_0 to any x_0' exchanges energy with phonons $eE(x_0 - x_0') \equiv eE\Delta x_0$. As follows from the probability of the displacement, Δx_0 depends on E and has different magnitudes for $\Delta x_0 > 0$ (displacement of the current carrier along the field E) and for $\Delta x_0 < 0$ (displacement against the field). These conditions specify microscopic current j_x different from zero.

A number of the current carriers passing in one second through a plane $x = 0$ from left to right is equal to

$$N_1 = \sum_{\alpha=1,\,2}\ \sum_{(\bar{x}_Q>0,\ \bar{x}_{Q'}<0)} |V_{QQ'}|^2 \rho_{Q'\alpha},$$

where $\rho_{Q\alpha}$ is the equilibrium distribution function, $\alpha = 1$ corresponds to the spin of the electron parallel to the field H, and $\alpha = 2$ to the antiparallel spin. The number of current carriers passing through the plane $x = 0$ from right to left is equal to

$$N_2 = \sum_{\alpha=1,\,2}\ \sum_{(\bar{x}_Q>0,\ x_{Q'}<0)} |V_{QQ'}|^2 \rho_{Q\alpha}.$$

Therefore the resultant current density j_x is equal to

$$\bar{j}_x = -e(N_1 - N_2) \qquad (5)$$
$$= -e\sum_{\alpha=1,\,2}\ \sum_{(\bar{x}_Q>0,\ \bar{x}_{Q'}<0)} |V_{QQ'}|^2 (\rho_{Q\alpha'} - \rho_{Q\alpha}).$$

In the approximation used here as previously[1], $\bar{x}_Q = x_0$ is the oscillator center; as perturbation V we use the interaction of the current carrier with the acoustical phonons. We shall limit ourselves to the single phonon scattering. The perturbation operator has the usual form:

$$V(\mathbf{r}) = \sum (\mathbf{e}_f \nabla V_p)\{b_f e^{i(\mathbf{fr})} \qquad (6)$$
$$+ b_f^+ e^{-i(\mathbf{fr})}\}\sqrt{\hbar/2Msf},$$

where $\mathbf{e}_f$ is equal to $\mathbf{f}/f$ (the longitudinal waves), s is the sound velocity, b_f^+, b_f are the creation and annihilation operators of the phonon momentum $\hbar f$; $V_p(\mathbf{r})$ is the periodic lattice potential. As is customary, we impose the periodic conditions by introducing the unit cell of the linear dimensions L, $f \leq f_0$.

Since $V_p(\mathbf{r})$ varies substantially over the distances of the order of the lattice constant, then on calculation of $W_{QQ'} = |V_{QQ'}|^2$ the operator $\mathcal{H}_0$ should be taken in the following form

$$\hat{\mathscr{H}}_0 = \frac{p_x^2}{2m} + \frac{1}{2m}\left(p_y + \frac{e}{c}Hx\right)^2 \qquad (7)$$
$$+ \frac{p_z^2}{2m} + eEx + V'(\mathbf{r}) + V_p(\mathbf{r})$$

and the function ψ_Q should be determined from the equation

$$\hat{\mathscr{H}}_0\Psi_Q = \varepsilon_Q\Psi_Q,\ V'(\mathbf{r})$$

$V'(\mathbf{r})$ is the operator for the interaction energy of the electron with the optical phonons[3].

According to the theorem of Pekar[3], generalized by Luttinger[4], for the case $H \neq 0$, close to the bottom of the conduction band

$$\Psi_Q = \sum_{\mathbf{k}} \alpha_{Qk} = e^{i\mathbf{kr}} u_{\mathbf{k}}(\mathbf{r}); \quad \alpha_{Qk} = \int \varphi_Q e^{-i(\mathbf{kr})} d\tau,$$

where the wave function φ_Q is found from the equation

$$\tilde{\mathscr{H}}_0 \varphi_Q = \varepsilon_Q \varphi_Q; \quad \tilde{\mathscr{H}}_0 = \frac{p_x^2}{2\mu} + \frac{1}{2\mu}\left(p_y + \frac{e}{c} Hx\right)^2 + \frac{p_z^2}{2\mu} + eEx + V'(\mathbf{r}), \tag{8}$$

in which $\mu(H)$ is the effective mass of the current carrier in a magnetic field[5].

Using the result of Pekar[3] in the solution of Eq. (8) we obtain, after several transformations

$$W_{QQ'}^{\pm} = \frac{2fg^2}{9\hbar sM} \delta(\varepsilon_{Q'} - \varepsilon_{Q'} \pm \hbar\omega_f)\, \Delta \times (P_y - P_y' \pm \hbar f_y) \times \Delta(P_z - P_z' \pm \hbar f_z)\, W_{nn'} \begin{Bmatrix} N_f \\ N_f + 1 \end{Bmatrix} \tag{9}$$

(The upper factor is used in the absorption of phonons, the lower factor in the emission of phonons). Here $\Delta(x) = 0$ for $x \neq 0$, $\Delta x = 1$ with $x = 0$; $g = h^2/2m \times \int (\nabla u_0)^2 d\tau$ is the Bloch constant; N_f is the number of acoustical phonons with the momentum hf;

$$W_{nn'} = \left| \int_{-\infty}^{\infty} \exp\{\pm i f_x x\} H_n(x - x_0) H_{n'}(x - x_0')\, dx \right|^2, \tag{10}$$

$H_n(x - x_0)$ is the Hermite polynomial, M is the mass of ions per unit volume.

Introducing the expression for $W_{QQ'}$ into Eq. (7), we express P_y in terms of x_0 and use the properties of the symbol $\Delta(x)$ to exclude the summation over f_y, f_z:

$$j_x = \frac{2eg^2}{9M\hbar s(2\pi)^5}\left(\frac{\mu\omega_0}{\hbar}\right)^2 L^2 \sum_{n,n',\alpha} \int_{-\hbar f_0}^{+\hbar f_0}\int dP_z dP_z' \times \int_{(x_0>0)} dx_0 \int_{(x_0'<0)} dx_0' \sum_{f_x} f W_{nn'}\{[\rho_{Q\alpha}^0 N_f - \rho_{Q'\alpha}^0 (N_f + 1)]\, \delta(\varepsilon_Q - \varepsilon_{Q'} - \hbar\omega_f) + [\rho_{Q\alpha}^0 (N_f + 1) - \rho_{Q'\alpha}^0 N_f]\, \delta(\varepsilon_Q - \varepsilon_{Q'} + \hbar\omega_f)\}, \tag{11}$$

Here Q is a function of the quantities n, P_z and P_y, or $x_0(P_y)$; l is the linear dimension of the unit cell; $x_0' < 0$; $x_0 > 0$.

Since we are limiting ourselves to the consideration of the phenomena linear in E (Ohm's law), ρ is chosen in the following way

$$\rho_{Q\alpha} = \rho(\mathscr{E}_{Q+}) = \exp\left\{\frac{1}{kT}\left(\Phi - \frac{P_z^2}{2\mu} - \hbar\omega_0\left(n + \frac{1}{2}\right) \pm \mu_B H\right)\right\}. \tag{12}$$

Substituting $\rho_{q\alpha}$ from Eq. (13) into Eq. (11), and assuming that the field of the acoustical phonons is in a thermal equilibrium, we obtain

$$N_f \to \bar{N}_f = \{e^{\hbar s f/kT} - 1\}^{-1}. \tag{13}$$

Now we make in the Eq. (11) the following changes of variables and the following transformations[2]:

1. $x_0 - x_0' = r$; $x_0 + x_0' = s$;

$$0 \leqslant s \leqslant r; \quad 0 \leqslant r \leqslant r_0 = (\hbar/\mu\omega_0) f_0$$

since $x_0 > 0, x_0' < 0)$; $P_z' - P_z = v$;

$$P_z' + P_z = u; -\hbar f_0 \leqslant u \leqslant \hbar f_0, \; -\hbar f_0 \leqslant v \leqslant \hbar f_0.$$

2. We integrate over s and u, using the properties of the δ-function and pass from a sum over f_x to integral over df_x.

3. Taking into account that according to Eq. (9) $P_{y,z} - P_{y,z}' \pm \hbar f_{y,z} = 0$,

we introduce new variables

$$r \to f_y = (\mu\omega_0/\hbar)\, r; \quad 0 \leqslant f_y \leqslant f_0;$$

$$v \to f_z = v/\hbar; \quad -f_0 \leqslant f_z \leqslant f_0,$$

and finally in the plane (f_x, f_y) we introduce the polar coordinates f' and α'. After making all the above described transformations we obtain:

$$j_x = \frac{j_0}{(2\pi)^5} \cosh\frac{\mu_B H}{kT} \times \sum_{n,n'} \int_0^{\pi} \sin\alpha' d\alpha' \int_0^{f_0} df' \int_{-f_0}^{f_0} df_z \frac{f'^2 f}{f_z} \times W_{nn'} \frac{\exp(-\hbar^2 f_z^2/8\mu kT)}{\sinh(\hbar s f/2kT)} \times \sinh\left(\frac{\hbar\gamma f' \sin\alpha'}{2kT}\right)\left\{e^{-(X_0^+ + X_1^+)} + e^{-(X_0^- + X_1^-)}\right\}, \tag{14}$$

$$j_0 = (4e\mu g^2/9M\hbar^2 s)\, e^{\Phi/kT}, \qquad X_1^{\mp} \sim \gamma;$$

We shall limit ourselves to those fields E, for which j_x is proportional to E, i.e., we replace in Eq. (14) sinh by its argument and X_1^+ by zero. Then integrating over $\alpha\alpha'$ we obtain

$$j_x = \frac{j_0\gamma}{4kT\omega_0(2\pi)^4}\int_0^{f_0} df'$$

$$\times \int_{-f_0}^{f_0} df_z \frac{f'^3 f}{|f_z|\sinh(\hbar s f/2kT)} e^{-\hbar^2 f_z^2/8\mu kT} P(f', f_z),$$

$$P = \sum_{n,n'} \hbar\omega_0 W_{nn'} \left\{e^{-X_0^{\pm}} + e^{-X_0^{\mp}}\right\}.$$

Introducing the polar coordinates in the plane (f', f_z) and the new variable

$$\xi = \hbar s f/kT \; (0 \leqslant \xi \leqslant \xi_{\max} = T_D/T),$$

for which

$$f_z = (\xi kT/\hbar s)\cos\varphi; \; f' = (\xi kT/\hbar s)\sin\varphi,$$

and expressing j_x in the form

$$j_x = \left(\frac{kT}{\hbar s}\right)^5 \frac{j_0\gamma\cosh(\mu_B H/kT)}{4kT\omega_0(2\pi)^4}\int_0^{\pi/2}\frac{\sin^3\varphi}{\cos\varphi}d\varphi \qquad (15)$$

$$\times \int_0^{T_D/T} \frac{\xi^4 e^{-\nu\xi^2\cos^2\varphi}}{\mathrm{sh}\,(\xi/2)} P(\xi,\varphi)\, d\xi,$$

where T_D is the Debye temperature, $\nu = kT/8\mu s^2$.

According to Rumer[5], P can be expressed in the form of a contour integral which in the case considered above has the following form:

$$P(\xi,\varphi) = \frac{\hbar\omega_0}{i\sinh(\hbar\omega_0/2kT)}$$

$$\times \sqrt{\frac{\alpha_1 kT}{\pi}} \int_{\tau - i\infty}^{\tau + i\infty} \exp\left\{\alpha_1 kT y^2\right. \qquad (16)$$

$$\left. + \hbar\omega_f y \mp \frac{\beta}{\hbar\omega_0}\frac{\cosh(\hbar\omega_0 y) - \mathrm{ch}\,(\hbar\omega_0/2kT)}{\sinh(\hbar\omega_0/2kT)}\right\} dy,$$

$$\beta = kT\nu\xi^2\sin^2\varphi; \quad \alpha_1 = 4kT\nu\xi^2\cos^2\varphi,$$

in which $\tau < 1/2kT$, and may one set $\tau = 0$. We are interested only in the case of large magnetic fields, when $\hbar\omega_0 \gg kT$, since in the opposite case the quantization of the energies has a small effect and the usual kinetic theory apparently is more satisfactory than the method used in the present work.

For $\hbar\omega_0 \gg kT$, , and setting $\tau = 0$ and retaining only the main term

$$P(\xi,\varphi) \approx \hbar\omega_0 \exp\left\{-\frac{\hbar\omega_0}{2kT} - 1/16\,\nu\cos^2\varphi\right\}. \quad (17)$$

In electronic semiconductor $|j_y| \gg |j_x|$, so that the resistivity ρ is determined from Eqs. (2), (15) and (17) and by the expression

$$\rho \approx \frac{j_x E}{j_y^2} = \rho_0 I \left(\frac{kT}{\hbar s}\right)^5 \frac{\hbar\omega_0}{kT}\exp\left[-\frac{\Phi + \hbar\omega_0/2}{kT}\right], \quad (18)$$

$$\rho_0 = 8\mu^2 g^2/9M\hbar^2 s e^2 N_0^2,$$

$$I = \int_0^{\pi/2}\frac{\sin^3\varphi}{\cos\varphi}d\varphi \qquad (19)$$

$$\times \int_0^{T_D/T}\frac{\xi^4 d\xi}{\sinh(\xi/2)}\exp\left[-\nu\xi^2\cos^2\varphi - 1/16\,\nu\cos^2\varphi\right].$$

Since we are considering low temperatures only, we can replace the limit T_D/T by ∞. Considering that the greatest contribution into the integral is from small ξ and investigating the expression inside the integral in Eq. (19) it is not difficult to see that for $\nu \gg 1$, $I \approx 128\pi$ neglecting the terms of the order of $1/\nu$ etc.

To make a final calculation of $\rho(H,T)$ one must find $\Phi(H,T)$.

In the case of an electronic semiconductor we use the corresponding equation of neutrality and take into account the spin of the admixed electron in the absence of degeneracy of the electron gas:

$$N_0(H,T)e^{\Phi^*} + n_{\mathrm{loc}} = n_1, \qquad (20)$$

where n_1 is the concentration of the (univalent)[1] impurity

$$n_{\mathrm{loc}}/n_1 = (1 + {}^1/_2 \times \exp\{-\Delta E^* - \Phi^* - \mu_B^* H\})^{-1} + (1 + {}^1/_2\exp\{-\Delta E^* + \mu_B^* H - \Phi^*\})^{-1}. \qquad (21)$$

In Eqs. (20) and (21) and below, the asterisk denotes the division by kT; ΔE is the energy gap between the impurity level and the bottom of the conductor band. From Eqs. (20) and (21) it follows that for $\hbar\omega_0^* \gg 1$, $\mu_B^* H \gg 1$ approximately

$$e^{\Phi^*} \approx \sqrt{\frac{n_1}{2Z_0\hbar\omega_0^*}} \times \exp\left(\frac{\hbar\omega_0^*}{4} - \mu_B^* H - \frac{\Delta E^*}{2}\right). \qquad (22)$$

Substituting Eq. (22) into Eqs. (4), (2) and (18) we obtain, for H large,

$$N(H_1 T) = 2^{-1/2}\sqrt{n_1 Z_0 \hbar\omega_0^*} \times \exp(-\Delta E^*/2 - \hbar\omega_0^*/4), \qquad (23)$$

$$R \approx -1/Nec = (-\sqrt{2}/ec)(n_1 Z_0 \hbar\omega_0^*)^{-1/2} \quad (24)$$

$$\times \exp(\Delta E^*/2 + \hbar\omega_0^*/4),$$

$$\rho = \frac{16 g^2}{9\pi^3 M e^2 \hbar^2 s \sqrt{2n_1}} \left(\frac{kT}{\hbar s}\right)^5 \frac{\exp(\Delta E^*/2 + \hbar\omega_0^*/4)}{e^2 Z_0^{3/2} \sqrt{\hbar\omega_0^*}}. \quad (25)$$

From Eqs. (23) through (25) it is evident that the dependence of ρ and R on H,T is determined by the dependence of $N(H,T)$. However, $N(H,T)$ increases with increasing H because the ground level of the charged oscillator $h\omega_0/2$ (and also all its other levels) increase with the increasing H; at the same time the decrease in the interaction energy between the spin magnetic moments of the current carriers parallel to the field H with the increase in H is approximately compensated by analogous phenomenon for electrons in the ground impurity level. Therefore with increasing H for a given temperature both ρ and R increase very rapidly ($\sim \exp(\hbar\omega_0^*/4)$). We note that in the ordinary kinetic theory, which does not include the quantization of the energy in a magnetic field, both ρ and R approach a saturation value in semiconductors for large H. This result is indeed obtained because the dependence of N (H, T) is not taken into account, and this quantity has an appreciable value for $\hbar\omega_0 \gg kT$.

It should be noted, however, that the character of the change of $N(H,T)$ with an increasing H depends on the type of semiconductor. For example, for an intrinsic semiconductor, the behavior of $N(H,T)$ with the change of H is different from the one described above. In that case, in the effective mass approximation[1] $j_y = j_y^+ + j_y^- = 0$, since $N_+(H,T) = N_-(H,T)$ (the plus and minus signs relate to the holes and the electrons, respectively).

In this case, in the absence of degeneracy, one may use the relationship ($\hbar\omega_{0\pm}^* > 1$):

$$\sqrt{\mu_+}\exp(-\delta\varepsilon^* - \Phi^* - {}^1/_2\hbar\omega_{0+}^*)$$

$$= \sqrt{\mu_-}\exp(\Phi^* - {}^1/_2\hbar\omega_{0-}^*),$$

to evaluate Φ, from which

$$e^{\Phi^*} = (\mu_+/\mu_-)^{1/4} e^{-\delta\varepsilon^*/2} \exp[\mu_B^* H(s_- - s_+)], \quad (26)$$

where $s_\pm = m/\mu_\pm$, $\delta\varepsilon$ is the width of the forbidden zone. We obtain

$$N_+(H,T) = N_-(H,T)$$
$$= {}^1/_2 Z_{0+}\hbar\omega_{0+}^* \exp\{\mu_B^* H(1 - s_+) + \Phi^*\}. \quad (27)$$

Consequently,

$$\rho = \frac{E}{j_x^+ + j_x^-} = \frac{9\pi^3 M\hbar^6 s^6 H}{8g^2 ce(kT)^4} \cdot e^{\delta\varepsilon^*/2} \quad (28)$$

$$\times \left(\frac{\mu_+}{\mu_-}\right)^{-1/4} \frac{\exp[\mu_B^* H(s_+ - 1)]}{\mu_+^{-1}\exp[\mu_B^* H(s_- - s_+)] + \mu_-^{-1}}.$$

In this case, ρ can both increase and decrease with an increase in H, depending on the relationship between n, μ_+ and μ_-. If, for example,

$$\mu_+ > \mu_-, \quad s_+ < s_-, \quad s_+ < 1,$$

then with increasing H ρ will decrease.

In conclusion, the author expresses gratitude to Prof. A. G. Samoilovich, for suggesting this work and for valuable discussions and help in carrying out the present work.

---

1 M. I. Klinger, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 439 (1955); Soviet Phys. JETP **4**, 251 (1957).

2 S. Titeica, Ann. Physik **22**, 129 (1935).

B. I. Davydov and I. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) **9**, 1294 (1939).

3 S. I. Pekar, *Investigation of the electron theory of crystals*, GTTI, Moscow-Leningrad, 1952.

4 R. Peierls, Z. Physik **80**, 763 (1933); J. Luttinger, Phys. Rev. **84**, 814 (1951)

5 Iu. B. Rumer, J. Exptl. Theoret. Phys. (U.S.S.R.) **22**, 214 (1952).

6 M. I. Klinger, J. Exptl. Theoret. Phys. (U.S.S.R.) **26**, 159 (1954).

Translated by M. J. Stevenson
220