Polarization of Relativistic Protons in Coulomb Scattering

M. G. URIN AND V. N. MOKHOV
Moscow Institute of Engineering Physics

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Perturbation theory was used to determine the polarization of relativistic protons which experience Coulomb scattering.

WHEN we limit ourselves to weak and almost constant electromagnetic fields the equation which describes a particle with spin 1/2 and magnetic moment $\mathfrak{M}_p = \alpha e\hbar/2mc$ (m is the proton mass and $\alpha = 2.79$) for $v \ll c$ is as follows[1]

$$\{i\gamma_k(p_k - eA_k) + {}^1/_2 i\mathfrak{M}'\gamma_k\gamma_l F_{kl} + m\}\Psi = 0,$$

where A_k is the four-dimensional potential, F_{kl} is the electromagnetic field tensor and $\mathfrak{M}' = \mathfrak{M}_p - e/2m$ (here and in the following $\hbar = c = 1$). For stationary states we have $\mathscr{H}\psi = \varepsilon\psi$.

In a Coulomb field ($\mathbf{A} = 0$; $A_4 = i\varphi = iZe/r$):

$$\mathscr{H} = i\beta\gamma\mathbf{p} + \beta m + e\varphi + \mathfrak{M}'\gamma\nabla\varphi.$$

Since the vectors in the problem can be used to form a unique pseudovector, that is, $[\mathbf{p}_0\mathbf{p}_1]$ (where $\mathbf{p}_0$ and $\mathbf{p}_1$ are the proton momentum before and after scattering) polarization in the scattering plane is zero; therefore, we naturally analyze the spin perpendicular to that plane (in the z direction). We assume elestic scattering, i.e., $p_1 = p_0 = p$.

Whenever the external field can be regarded as a perturbation the following formula holds true in first approximation:

$$d\sigma_{\mu_0\mu_1} = (2\pi/v)\left|\int \psi_1^* V\psi_0 d\mathbf{r}\right|^2 \delta(\varepsilon_1 - \varepsilon_0) d\mathbf{p}_1/(2\pi)^3$$

$$= [\varepsilon^2/(2\pi)^2]\,|V_{0\mu_0}^{1\mu_1}|^2\, d\Omega_{\mathbf{p}_1},$$

where $d\sigma_{\mu_0\mu_1}$ is the differential scattering cross section; v is the velocity of the proton; $\psi_0 = u_{\mu_0\lambda_0}e^{i\mathbf{p}_0\mathbf{r}}$ and $\psi_1 = u_{\mu_1\lambda_1}e^{i\mathbf{p}_1\mathbf{r}}$ are the free wave functions of the proton before and after scattering; $u_{\mu_0\lambda_0}$, $u_{\mu_1\lambda_1}$ are the unit bispinors which characterize states with certain polarization in the z direction); $V = e\varphi + \mathfrak{M}'\gamma\nabla\varphi$ is the perturbation; $d\Omega_{\mathbf{p}_1}$ is the element of solid angle in the direction of the scattered proton momentum;

$$V_{0\mu_0}^{1\mu_1} = u_{\mu_1\lambda_1}^* \int e^{i\mathbf{q}_{01}\mathbf{r}}(e\varphi + \mathfrak{M}'\gamma\nabla\varphi)\,d\mathbf{r}\, u_{\mu_0\lambda_0}$$

$$= (4\pi Ze^2/q_{01}^2)\, u_{\mu_1\lambda_1}^*[1 - (\mathfrak{M}'/e)\, i\gamma\mathbf{q}_{01}]\, u_{\mu_0\lambda_0},$$

where $\mathbf{q}_{01} = \mathbf{p}_0 - \mathbf{p}_1$.

For an unpolarized primary proton beam

$$d\sigma_{\mu_1} = {}^1/_2 \sum_{\mu_0} [\varepsilon^2/(2\pi)^2]\,|V_{0\mu_0}^{1\mu_1}|^2\, d\Omega_{\mathbf{p}_1}, \quad (1)$$

where $\sum_{\mu_0}$ denotes summation over initial polarizations. Since $u_{\mu\lambda}$ is the eigenfunction of the operator $1/2\,\Sigma_z\beta$, $u_{\mu_1\lambda_1}$ can be replaced by

$$\left[\mu_1 + {}^1/_2\,\Sigma_z\beta\right] u_{\mu\lambda_1}/2\mu_1 = \delta_{\mu\mu_1} u_{\mu\lambda_1},$$

which enables us to sum over μ_1 in (1). Then, using Casimir's operator[2] $(H_j + \lambda_j)/2\lambda_j$, where $H_j = \beta(i\gamma\mathbf{p}_j + m)$ in order to sum over λ_0, λ_1, we obtain

$$d\sigma_{\mu_1} = {}^1/_2 (Ze^2/q_{01}^2)^2 \operatorname{Sp}\{[1 - (\mathfrak{M}'/e)\, i\gamma\mathbf{q}_{01}](H_0 + \varepsilon) \times [1 + (\mathfrak{M}'/e)\, i\gamma\mathbf{q}_{01}](H_1 + \varepsilon)\left[\mu_1 + {}^1/_2\,\Sigma_z\beta\right]/2\mu_1\}\, d\Omega_{\mathbf{p}_1}.$$

Then the total differential scattering cross section is

$$d\sigma_{1/2} + d\sigma_{-1/2}$$

$$= {}^1/_2 (Ze^2/q_{01}^2)^2 \operatorname{Sp}\{[1 - (\mathfrak{M}'/e)\, i\gamma\mathbf{q}_{01}](H_0 + \varepsilon) \times [1 + (\mathfrak{M}'/e)\, i\gamma\mathbf{q}_{01}](H_1 + \varepsilon)\}\, d\Omega_{\mathbf{p}_1}$$

$$= \frac{(Ze^2/2pv)^2}{\sin^4(\theta/2)}[1 - 2v^2 L \sin^2(\theta/2)]\, d\Omega_{\mathbf{p}_1},$$

$L = [(\alpha - 1/2) - 1/2\,\alpha^2 v^2]/(1 - v^2)$; θ is the angle between $\mathbf{p}_0$ and $\mathbf{p}_1$.

In this approximation we have for the polarization

$$d\sigma_{1/2} - d\sigma_{-1/2}$$
$$= {}^1/_2 (Ze^2 / 2pv)^2 \operatorname{Sp} \{[1 - (\mathfrak{M}'/e)\, i\gamma \mathbf{q}_{01}] (H_0 + \varepsilon) \times [1 + (\mathfrak{M}'/e)\, i\gamma \mathbf{q}_{01}] (H_1 + \varepsilon) \Sigma_z \beta\}\, d\Omega_{\mathbf{p}_1} = 0.$$

We therefore consider the scattering cross section in the second perturbation approximation:

$$d\sigma_{\mu_1} = \frac{1}{2} \sum_{\mu_0} \frac{\varepsilon^2}{(2\pi)^2} \Big| V^{1\mu_1}_{0\mu_0} + \int \sum_{\mu_i \lambda_i} \frac{V^{1\mu_1}_{i\mu_i} V^{i\mu_i}_{0\mu_0}}{\varepsilon - \lambda_i} \frac{d\mathbf{p}_i}{(2\pi)^3} \Big|^2 d\Omega_{\mathbf{p}_1},$$

where the integral with respect to p_i is taken along the real axis encircling the point $p_i = p$ from below[3].

Retaining the first nonvanishing terms in the expression for the polarization we obtain

$$d\sigma_{1/2} - d\sigma_{-1/2}$$
$$= \sum_{\mu_0} \frac{\varepsilon^2}{(2\pi)^2} \operatorname{Re} \Big[(V^{1\,1/2}_{0\mu_0})^* \int \sum_{\mu_i \lambda_i} \frac{V^{1\,1/2}_{i\mu_i} V^{i\mu_i}_{0\mu_0}}{\varepsilon - \lambda_i} \frac{d\mathbf{p}_i}{(2\pi)^3} - (V^{1-1/2}_{0\mu_0})^* \int \sum_{\mu_i \lambda_i} \frac{V^{1-1/2}_{i\mu_i} V^{i\mu_i}_{0\mu_0}}{\varepsilon - \lambda_i} \frac{d\mathbf{p}_i}{(2\pi)^3} \Big] d\Omega_{\mathbf{p}_1}.$$

We must here sum over μ_i, λ_i, μ_0, and just as was done above we introduce the summations over λ_0, λ_1, μ_1:

$$d\sigma_{1/2} - d\sigma_{-1/2} = [(Ze^2)^i / (2\pi)^2 q_{01}^2]$$
$$\operatorname{Re} \Big[\int \operatorname{Sp} F\, d\mathbf{p}_i / i q_{01}^2 q_{i1}^2 (\varepsilon^2 - \varepsilon_i^2) \Big] d\Omega_{\mathbf{p}_1},$$
$$F = \beta [1 - (\mathfrak{M}'/e)\, i\gamma \mathbf{q}_{i1}] (H + \varepsilon) \times [1 - (\mathfrak{M}'/e)^3 \gamma \mathbf{q}_{0i}] (H_0 + \varepsilon) \times [1 + (\mathfrak{M}'/e)\, i\gamma \mathbf{q}_{01}] (H_1 + \varepsilon) \gamma_1 \gamma_2.$$

Here use has been made of $\Sigma_z = \gamma_1\gamma_2 / i$. It is easily seen that

$$\int d\Omega_{\mathbf{p}_i} \int p_i^2 dp_i \operatorname{Sp} F / i q_{0i}^2 q_{i1}^2 (\varepsilon^2 - \varepsilon_i^2)$$
$$= -\pi \int [p_i \varepsilon_i \operatorname{Sp} F / q_{0i}^2 q_{i1}^2 (\varepsilon + \varepsilon_i)]_{p_i = p} d\Omega_{\mathbf{p}_i} - \int d\Omega_{\mathbf{p}_i} \oint p_i^2 dp_i \operatorname{Sp} F / i q_{0i}^2 q_{i1}^2 (\varepsilon^2 - \varepsilon_i^2).$$

The second integral with respect to dp_i is taken along the real axis in the sense of the principal value.

Since the imaginary part of F is the sum of products of an odd number of γ matrices, Sp F is a real quantity and, consequently,

$$\operatorname{Re} \int d\Omega_{\mathbf{p}_i} \oint p_i^2 dp_i \operatorname{Sp} F / i q_{0i}^2 q_{i1}^2 (\varepsilon^2 - \varepsilon_i^2) = 0.$$

Then

$$d\sigma_{1/2} - d\sigma_{-1/2}$$
$$= -\Big[p (Ze^2)^3 / 8\pi q_{01}^2 \Big] d\Omega_{\mathbf{p}_i} \int d\Omega_{\mathbf{p}_i} [\operatorname{Sp} F / q_{0i}^2 q_{i1}^2]_{p_i = p}$$
$$= [mpL \,|\, [\mathbf{p}_0 \mathbf{p}_1] \,|\, (Ze^2)^3 / \pi q_{01}^2] \times [1 - 2 (\mathfrak{M}'/me) p^2]\, d\Omega_{\mathbf{p}_i} \times \int \Big[\frac{1 - \mathbf{p}_i (\mathbf{p}_0 + \mathbf{p}_1) / (p^2 + \mathbf{p}_0 \mathbf{p}_1)}{q_{0i}^2 q_{i1}^2} \Big]_{p_i = p} d\Omega_{\mathbf{p}_i}.$$

Integrating over the angles of $\mathbf{p}_i$ and converting to ordinary units we obtain

$$d\sigma_{1/2} - d\sigma_{-1/2} = \Big(\frac{Z}{137}\Big)^3 \frac{\hbar^2 mc}{p^3} \frac{KL}{\sin\theta} \ln \frac{1}{\sin(\theta/2)} d\Omega_{\mathbf{p}_1},$$
$$K = 1 - \frac{\alpha\beta^2}{1 - \beta^2}, \quad \beta = \frac{v}{c}.$$

The integral cross section is

$$\sigma_{1/2} - \sigma_{-1/2} = 2\pi^2 \ln 2 \,(Z/137)^3 (\hbar^2 mc / p^3) KL.$$

It can be seen that for $\beta = 0.6$ and 0.77 the polarization vanishes.

The relative polarization is

$$\frac{d\sigma_{1/2} - d\sigma_{-1/2}}{d\sigma_{1/2} + d\sigma_{-1/2}} = \frac{(Z/137)\, 4\beta \sqrt{1 - \beta^2} KL}{1 - 2\beta^2 L \sin^2(\theta/2)} \frac{\sin^3(\theta/2)}{\cos(\theta/2)} \ln \frac{1}{\sin(\theta/2)}.$$

When $v \ll c$ we arrive at the previously known result[4]:

$$\frac{d\sigma_{1/2} - d\sigma_{-1/2}}{d\sigma_{1/2} + d\sigma_{-1/2}} = 4\Big(\alpha - \frac{1}{2}\Big) \frac{v}{c} \frac{Z}{137} \frac{\sin^3(\theta/2)}{\cos(\theta/2)} \ln \frac{1}{\sin(\theta/2)}.$$

The problem has been solved by perturbation theory. Since $|e\varphi| > |\mathfrak{M}'\gamma\nabla\varphi|$ the criterion for applying perturbation theory to the present case will be the inequality $Ze^2/\hbar v \ll 1$. For relativistic protons this is equivalent to $Z/137 \ll 1$.

The formulas which have been obtained are applicable to the angular range

$$\hbar / pR_{\text{nuc}} < \theta < Ze^2 / \varepsilon R_{\text{nuc}}$$

For angles greater than $Ze^2/\epsilon R_{\text{nuc}}$ nuclear scattering is important. Small angles may also be excluded because the Coulomb field of the nucleus is screened by atomic electrons.

These results can also be used as a correction to polarization in nuclear scattering. In this case

$$d\sigma_{1/2} - d\sigma_{-1/2} \sim | A_{\text{nuc}\,1/2} + A_{q\,1/2} |^2 - | A_{\text{яд}\,-1/2} + A_{q\,-1/2} |^2,$$

where A_{nuc} and A_q are the nuclear and Coulomb scattering amplitudes, respectively. In the interference term it is sufficient to take A_q in the first order perturbation approximation.

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Isotopic Invariance and the Creation of Particles

L. I. LAPIDUS
Institute for Nuclear Problems
Academy of Sciences, USSR

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The consequences of conservation of isotopic spin are investigated. Relations between different cross sections are found which are valid if in the meson nucleon interaction a state with a particular value of isotopic spin predominates. The relations of Smorodinskii and Jacobson for elastic nucleon-nucleon cross sections are generalized for the case of meson and nucleon–antinucleon pair production. Furthermore, the consequences of isotopic spin conservation are given for the following cases: meson production on nuclei, creation of heavy meson pairs and nucleon antinucleon pairs in π–nucleon collisions, and for some processes of nucleon annihilation in collisions with deuterons.

1. IN connection with the increase in the number of possible high energy experiments on nucleons and mesons it is interesting to investigate the consequences of the so called hypothesis of charge independence or isotopic invariance.

The meson creation processes should allow the most direct experimental verification of the conservation of isotopic spin. Besides the relation given by Yang[1]

$$d\sigma(p + p \rightarrow \pi^+ + d) = 2d\sigma(n + p \rightarrow \pi^0 + d),$$

one can show,[2,3] using just one condition derived from isotopic invariance, that the following relation also holds

$$d\sigma(p + d \rightarrow \mathrm{H}^3 + \pi^+) = 2d\sigma(p + d \rightarrow \mathrm{He}^3 + \pi^0).$$

Several reactions are forbidden by isotopic invariance. Among them is the following curious case: in d–d collisions leaving the deuterons intact, only even numbers of mesons can be created. The forbidden character of the reactions

$$d + d \rightarrow d + d + \pi^0; \quad d + d \rightarrow \mathrm{He}^4 + \pi^0$$

is clear. The case of three π^0–mesons can be proven as follows. Calling Ψ_{T,T_z} a function with definite T^2 and T_z, one has for the wave function of two π^0–mesons

$$(\pi^0\pi^0) = \{\sqrt{2}\psi_{2,0} - \psi_{0,0}\} / \sqrt{3},$$

which does not contain $T = 1$ components. Therefore a system of three π^0–mesons cannot have a part with $T = 0$. This shows that three π^0–mesons cannot be created since the nuclear system has $T = 0$ before and after the collision.

2. The investigation of the consequences of isotopic spin conservation furthermore allows one to obtain information on the meson–nucleon and nucleon–nucleon interaction in states of definite isotopic spin. For example, as is well known, the elastic and the charge exchange scattering cross section of mesons is given in terms of the amplitudes of the states with $T = 3/2$ and $T = 1/2$, a_3