The transition probability between states with magnetic quantum number m_0 and m (in the laboratory, not the rotating system of reference) is given by

$$R_{m,m} = \Big| \sum_{m'} G_{m'm_0} \{0, \beta, \pi\} G_{mm'} \{0, \beta, \pi\} e^{im'st} \Big|^2. \quad (8)$$

For $J = 1/2$, this expression becomes

$$R_{1/2, -1/2} = \frac{\omega_0^2 \sin^2\vartheta}{\omega_0^2 + \omega^2 - 2\omega\omega_0 \cos\vartheta} \quad (9)$$

$$\times \sin^2 \frac{t}{2} (\omega_0^2 + \omega - 2\omega\omega_0 \cos \omega t)^{1/2}.$$

In conclusion, I express my gratitude to E. Rivin for aid in the calculations.

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Translated by E. J. Saletan
163

Concerning the Spin of the Λ^0-Particle

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WE present several general formulas obtained according to methods described elsewhere[1,2], with relation to the problem of determining the spin Λ^0-particle from the angular distribution of its decay product.

We shall characterize the spin state of an ensemble of Λ^0-particles by giving the magnitudes of the angular momentum tensors T_ν^q (defined in Ref. 1), which makes it possible to describe an arbitrary (most general) spin state of the particles in this ensemble. The angular distribution of the decay product of the Λ^0-particles ($\Lambda^0 \to p + \pi$) is of the form

$$F(\vartheta, \varphi) = \frac{w}{2\sqrt{\pi}} \sum_{q=0,2\ldots}^{2s-1} (2q+1)^{-1/2} Q(s, q) \quad (1)$$

$$\sum_{\nu=-q}^{q} (-1)^\nu Y_{q,-\nu}(\vartheta, \varphi) T_\nu^q,$$

where w is the total decay probability for the Λ^0 according to the reaction $\Lambda^0 \to p + \pi^-$, s is the spin of the Λ^0, and

$$Q(s, q) = (-1)^{s-1/2-q/2} (2s+1)^{-1/2} Z(l'sl's; \tfrac{1}{2}q).$$

The coefficients Z are tabulated by Biedenharn[3]. It can be shown that

$$Z(l'sl's; \tfrac{1}{2}q)$$

$$= (-1)^{q/2} (2l'+1)(2s+1) W(l'sl's; \tfrac{1}{2}q) C^{q0}_{l'0l'0}$$

does not depend on l' (equal to $s + \frac{1}{2}$ or $s - \frac{1}{2}$) and therefore $F(\vartheta, \varphi)$ does not depend on the parity of the Λ^0-particle. Equation (1) is written in the center-of-mass system of the Λ^0-particle, and the z axis of this system will henceforth be taken parallel to the direction of motion $\mathbf{n}_\Lambda$ of the Λ^0-particle in the laboratory system [of course, Eq. (1) is valid for arbitrary choice of the z axis].

Integrating (1) over the interval of solid angle $(\varphi, \varphi + \Delta\varphi)$, $0 \le \vartheta \le \pi$ we can obtain the distribution in φ. The angle φ ($0 \le \varphi \le 2\pi$) can be defined as the angle between the normal $\mathbf{N}$ to the production-plane of the Λ^0-particle (more exactly, $\mathbf{N} = \mathbf{n}_0 \times \mathbf{n}_\Lambda$, where $\mathbf{n}_0$ is the unit vector in the direction of the incident particles in the production reaction) and the vector $\mathbf{n} = \mathbf{n}_p \times \mathbf{n}_\Lambda$, where $\mathbf{n}_p$ is the direction of the decay proton.

$$F(\varphi) = \frac{w}{4\pi} T_0^0 \Big\{ 1 + \sqrt{2} \sum_{m=2}^{2s-1} \quad (2)$$

$$\times \sum_{q=m}^{2s-1} [\cos m\varphi (\mathrm{Re}\, t_m^q) + \sin m\varphi (\mathrm{Im}\, t_m^q)] Q(s, q) J_{qm} \Big\},$$

$$J_{qm} = \left[\frac{(q+m)!}{(q-m)!} \frac{2q+1}{2} \right]^{1/2}$$

$$\frac{m}{2} 2^{1-m/2} \frac{(q/2-1)!\,(q-m-1)!!}{(q/2+m/2)!\,(q+1)!!},$$

$$t_m^q = (2q+1)^{-1/2} T_m^q / T_0^0,$$

where m and q take on only even values. This formula differs from similar ones[4,5] in that (2) contains an explicit expression for the coefficients A_M and B_M[4,5] in terms of the initial spin state of the Λ^0.

If T^q_m is real for even values of q, i.e., if $\mathrm{Im}\, t^q_m = 0$ (as can be shown[2], this will occur, for example, when the Λ^0 is produced in a reaction in which the incident particle and target are completely polarized), then $F(\varphi)$ becomes a polynomial of $\cos m\varphi$ only. The same polynomial in $\cos \eta m$ gives the distribution over the angle η between the production-plane of the Λ^0 and its decay-plane for complex values of T^q_m [compare Ref. 5, Eq. (2)].

A similar integration of Eq. (1) gives the distribution $F(\theta)$ of the number of particles emitted per unit solid angle at an angle θ to the z axis (parallel to $\mathbf{n}$), averaged over all azimuth angles φ (see Walker and Shephard, Ref. 6):

$$F(\theta) = \frac{w}{4\pi} T_0^0 \left\{1 + \sum_{q=2,4,,}^{2s-1} Q(s,q) P_q(\cos\theta) T_0^q / T_0^0\right\}. \tag{3}$$

Comparing (2) and (3) we that see $F(\varphi)$ [or $F(\eta)$] is given by those components of the tensor T^q_m for which $q = 2, 4, \ldots, 2s-1$ and $m = 2, 4, \ldots q$, whereas $F(\theta)$ depends on entirely different components of the tensor, namely, T^q_0. The components T^q_0 and T^q_m do not determine the spin state of the Λ^0 entirely independently, but so long as they do not take on their maximum values the distribution over η can be somewhat arbitrary (within certain limits) for fixed $F(\theta)$ [cf. Ref. 7, Eqs. (17) and (18)]. Therefore, the distributions in η and $\cos\theta$ obtained by Walker and Shephard[6] are not in contradiction, which has already been noted by Morpurgo[7] who examined the cases $s = 3/2$ and $5/2$. These distributions may (the statistics are quite poor!) indicate that the Λ^0-particles observed are not entirely polarized perpendicular to the plane of the reaction $\pi^- + p \to \Lambda^0 + \theta^0$ (compare this with the note at the end of this letter**).

The observed cases of the reactions $\pi^- + p \to \Lambda^0 + \theta^0$, $\Lambda^0 \to p + \pi^-$ are at energies of about 1 and 1.5 bev. Even if the statistics at these energies were better, we could only hope to obtain a more accurate value for the lower bound of the Λ^0 spin. We shall show that a measurement of the angular distribution of the decay products of the Λ^0 produced at the threshold of the $\pi^- + p \to \Lambda^0 + \theta^0$ reaction (about 755 mev) gives the value of the Λ^0 spin itself if we make two natural assumptions, namely, that the spin of the θ^0-particle is zero (which is, at any rate, not in contradiction with the data available) and that the interaction between the Λ^0 and the θ^0 is a short range one. The latter means that we may neglect all elements of the R-matrix $(i_\Lambda i_\theta s' l' \alpha' | R^{JE} | \tfrac{1}{2} 0 \tfrac{1}{2} l \alpha)$ with $l' > 0$ in comparison with the element for which $l' = 0$ close to the threshold* (the notation is defined elsewhere[1]; the matrix R is related to the well-known S-matrix as follows: $R = S - 1$). With these assumptions we may rewrite Eqs. (7)-(9) of Ref. 1 (the target is assumed nonpolarized). Making use of the properties of the coefficients $G_{\varkappa'}$ and G_0 and the parity conservation law, introducing the notation s instead of i_Λ and suppressing the index Λ on q, we obtain the following expression for the angular momentum tensors of the Λ^0-particle with even values of q (those with odd values of q vanish):

$$T^{q0}_{\tau 0}(\mathbf{n}_\Lambda, p_\Lambda) = A(2s+1)(2q+1)^{-1/2} Q(s,q) Y_{q\tau}(\vartheta_\Lambda, \pi), \tag{4}$$

where A is a constant proportional to the total cross section for the reaction $\pi^- + p \to \Lambda^0 + \theta^0$ with $E_\pi \sim 755$-780 mev. Let us note that the index τ refers to $\mathbf{n}_\Lambda$ as the axis of quantization, and that the angle ϑ_Λ is the angle between the direction of the Λ^0-particle and the π-meson beam.

From the expression (4) for the angular momentum tensor of the Λ^0-particle, Eqs. (2) and (3) can now be used to obtain the distribution in η and θ, which will depend only on the Λ^0 spin (and on ϑ_Λ). We shall not write these general formulas here**. It is interesting, however, to note that if we integrate them over all angles ϑ_Λ, the distribution over θ is isotropic, whereas that over φ (or η) becomes

$$I_s(\eta) = C\left\{1 + \sum_{m=2}^{2s-1} \cos m\eta \sum_{q=m}^{2s-1} [Q(s,q) J_{qm}]^2 (2q+1)^{-1}\right\}. \tag{5}$$

For instance,

$$I_{3/2}(\eta) \sim 1 + {}^1/_3 \cos 2\eta.$$

If the tensors of Eq. (4) are inserted into (1), we obtain a general expression for the distribution over the angle γ

$$F(\vartheta, \varphi) \tag{6}$$

$$= \frac{Aw}{2\sqrt{\pi}} \sum_q (2q+1)^{-1} [Q(s, q)]^2 \sum_\nu Y^*_{q,\nu}(\vartheta, \varphi)\, Y_{q,\nu}(\vartheta_\Lambda, \pi)$$

$$= Aw\,[8\pi\sqrt{\pi}]^{-1} \sum_{q=0}^{2s-1} [Q(s, q)]^2 P_q(\cos\gamma) \equiv F_s(\gamma);$$

where γ is the angle between the directions (ϑ_Λ, π) and (ϑ, φ), or, as can be shown, the angle in the rest system between the direction of motion of the decay proton and that of the incident π-meson beam. The formula $F_{3/2}(\gamma) \sim 1 + 3\cos^2\gamma$ was given first by Wolfenstein[8];

$$F_{5/2}(\gamma) \sim 1 - 2\cos^2\gamma$$

$$+ 5\cos^4\gamma \sim 1 + {}^4/_5 \cos^2\gamma + {}^1/_3 \cos 4\gamma.$$

I express my gratitude to Professor M. A. Markov, who suggested the present work.

* The angular distribution of the Λ^0 and θ^0 for 1 bev indicates the presence[6] of at least $l' = 2$. It follows from this that the range of the $\Lambda^0 - \theta^0$ forces is about 2×10^{-13} cm and that our assumption is valid for incident π-meson energies in the laboratory system from 755 to about 780-800 mev. It is, of course, possible that the matrix element $(i_\Lambda 0 i_\Lambda 0 \alpha' | R^{JE} | \tfrac{1}{2} 0 \tfrac{1}{2} l \alpha)$ is small due to some particular property of the reaction. In that case, the angular distribution of the Λ^0 and θ^0 will be nonisotropic.

** For $s = 3/2$ we have

$$F_{3/2}(\eta) \sim 1 + 0{,}5 \sin^2\vartheta_\Lambda \cos 2\eta,$$

$$F_{3/2}(\theta) \sim 1 - (1 - 3\cos^2\vartheta_\Lambda)(5 - 3\cos^2\vartheta_\Lambda)^{-1}\cos^2\theta.$$

If the angle $\vartheta_\Lambda = 90°$ we obtain $F_{3/2}(\eta) \sim 1 + 0.5 \times \cos 2\eta$, whereas $F_{3/2}(\theta) \sim 1 - 0.2\cos^2\theta$; i.e., the probability is not increased for $\cos\theta = \pm 1$, although the distribution over η is the same as that for entirely polarized Λ^0-particles.

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164