

has been shown in Ref. 1). This allows the magnitude of the total cross section for formation of π^0 -mesons in nucleon collisions to be determined from measurements of the emission of γ -quanta only at one angle. If the distribution of the initial particles contains odd powers of the cosine then to obtain the magnitude of the total cross section it is necessary to measure the emission of secondary particles at two angles θ^* and $\pi-\theta^*$. The indicated "isotropic" properties of the angular distributions of the secondary particles considerably simplify the problem of measuring the energy dependence of the total cross section particularly in the case where the angular distribution of the initial particles differs in the investigated interval of energy.

¹ A. A. Tiapkin, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 1150 (1956); Soviet Phys. JETP 3, 179 (1956).

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Behavior of Particles with Nonzero Spin in Crossed Constant and Varying Magnetic Fields

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WE give an exact solution for the behavior of particles of arbitrary spin in crossed constant and varying magnetic fields¹.

The wave function of the particle in a magnetic field can be written in the form

$$i\hbar d\psi/dt = -(\mathbf{H}\hat{\mathbf{M}})\psi, \quad (1)$$

where ψ is the $(2J+1)$ -component wave function of the particle and $\hat{\mathbf{M}}$ is the magnetic moment vector operator, proportional to the angular momentum. We shall consider the case when the external magnetic field acting on the particle is composed of a constant field H_0 (along the z axis) and a varying field which has components $H_x = H_1 \cos \omega t$ and $H_y = H_1 \sin \omega t$. In this case the wave equation

$$(1) \text{ becomes } i\hbar \frac{d\psi}{dt} = -\frac{\mu}{2} H_1 [e^{i\omega t} (\hat{J}_x - i\hat{J}_y) + e^{-i\omega t} (\hat{J}_x + i\hat{J}_y)] \psi - \mu H_0 \hat{J}_z \psi, \quad (2)$$

where \hat{J} is the angular momentum operator and μ is the magnetic moment of the particle.

Let us transform to a reference system rotating about the original z axis at a frequency ω . The components of the wave function ψ'_m in the new reference system are related to the corresponding ones ψ_m in the original system by the expression

$$\psi_m = e^{im\omega t} \psi'_m \quad (-J \leq m \leq J). \quad (3)$$

Inserting expression (3) into Eq. (2), and making use of the well-known properties of the operators $\hat{J}_x \pm i\hat{J}_y$, \hat{J}_z , we arrive at the following equation:

$$i\hbar d\psi'/dt = (-\mathbf{H}\hat{\mathbf{M}} + \Omega\hat{\mathbf{J}})\psi. \quad (4)$$

The components of the magnetic field vector \mathbf{H} which enter into this equation are the following: $H_x = H_1$, $H_y = 0$, and $H_z = H_0$; Ω is the angular velocity vector $\omega\mathbf{k}$ (where \mathbf{k} is the unit vector along the z axis). The operator on the right side of Eq. (4) does not depend on time and contains the term $\omega\hat{\mathbf{J}}$, which is the "centrifugal energy" operator, whose form corresponds to the expression for the centrifugal energy in classical mechanics. Thus, Eq. (4) may be considered a wave equation in a noninertial (rotating) system of reference. Equation (4) takes on its simplest form in the (non-inertial) reference system where the z axis is chosen along the vector $-\mu\mathbf{H} + \Omega$. The projection s onto angle β between this vector and the original z axis are easy to determine and are given by

$$s = \sqrt{\omega_0^2 + \omega^2 - 2\omega\omega_0 \cos \vartheta}, \quad (5)$$

$$\omega_0 = \mu H_0 / \hbar$$

$$\beta = \arcsin \omega_0 \sin \vartheta / \sqrt{\omega_0^2 + \omega^2 - 2\omega\omega_0 \cos \vartheta}, \quad (6)$$

where $\tan \vartheta = H_1/H_0$. Clearly, the solution of Eq. (4), whose initial component $\psi_m = \delta_{mm_0}$, can be written in the form²

$$\psi_{m_0}(t) = \sum G_{m'm_0} \{\alpha, \beta, \gamma\} G_{m''m'} \{\alpha, \beta, \gamma\} e^{im'st} \psi_{m'}. \quad (7)$$

The quantities $G_{m'm} \{\alpha, \beta, \gamma\}$ entering into this equation are the matrix elements of the $(2J+1)$ -dimensional irreducible representation of the three-dimensional rotation group, corresponding to rotations through the Euler angles α, β, γ (see, for instance, Ref. 2).

The transition probability between states with magnetic quantum number m_0 and m (in the laboratory, not the rotating system of reference) is given by

$$R_{m_0, m} = \left| \sum_{m'} G_{m'm_0} \{0, \beta, \pi\} G_{mm'} \{0, \beta, \pi\} e^{im'st} \right|^2 \tag{8}$$

For $J = 1/2$, this expression becomes

$$R_{1/2, -1/2} = \frac{\omega_0^2 \sin^2 \vartheta}{\omega_0^2 + \omega^2 - 2\omega\omega_0 \cos \vartheta} \times \sin^2 \frac{t}{2} (\omega_0^2 + \omega - 2\omega\omega_0 \cos \omega t)^{1/2} \tag{9}$$

In conclusion, I express my gratitude to E. Rivin for aid in the calculations.

¹ E. Majorana, *Nuovo Cimento* **9**, 43 (1932).

² V. I. Smirnov, *A Course in Higher Mathematics*, Vol. 3, Moscow, 1946.

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Concerning the Spin of the Λ^0 -Particle

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WE present several general formulas obtained according to methods described elsewhere^{1,2}, with relation to the problem of determining the spin Λ^0 -particle from the angular distribution of its decay product.

We shall characterize the spin state of an ensemble of Λ^0 -particles by giving the magnitudes of the angular momentum tensors T_ν^q (defined in Ref. 1), which makes it possible to describe an arbitrary (most general) spin state of the particles in this ensemble. The angular distribution of the decay product of the Λ^0 -particles ($\Lambda^0 \rightarrow p + \pi$) is of the form

$$F(\vartheta, \varphi) = \frac{w}{2V\pi} \sum_{q=0,2,\dots}^{2s-1} (2q+1)^{-1/2} Q(s, q) \tag{1}$$

$$\sum_{\nu=-q}^q (-1)^\nu Y_{q,-\nu}(\vartheta, \varphi) T_\nu^q,$$

where w is the total decay probability for the Λ^0 according to the reaction $\Lambda^0 \rightarrow p + \pi^-$, s is the spin of the Λ^0 , and

$$Q(s, q) = (-1)^{s-1/2-q/2} (2s+1)^{-1/2} Z(l's'l's; 1/2q).$$

The coefficients Z are tabulated by Biedenharn³. It can be shown that

$$Z(l's'l's; 1/2q) = (-1)^{q/2} (2l'+1)(2s+1) W(l's'l's; 1/2q) C_{l'0}^{q0}$$

does not depend on l' (equal to $s + 1/2$ or $s - 1/2$) and therefore $F(\vartheta, \varphi)$ does not depend on the parity of the Λ^0 -particle. Equation (1) is written in the center-of-mass system of the Λ^0 -particle, and the z axis of this system will henceforth be taken parallel to the direction of motion \mathbf{n}_Λ of the Λ^0 -particle in the laboratory system [of course, Eq. (1) is valid for arbitrary choice of the z axis].

Integrating (1) over the interval of solid angle $(\varphi, \varphi + \Delta\varphi)$, $0 \leq \vartheta \leq \pi$ we can obtain the distribution in φ . The angle φ ($0 \leq \varphi \leq 2\pi$) can be defined as the angle between the normal \mathbf{N} to the production-plane of the Λ^0 -particle (more exactly, $\mathbf{N} = \mathbf{n}_0 \times \mathbf{n}_\Lambda$, where \mathbf{n}_0 is the unit vector in the direction of the incident particles in the production reaction) and the vector $\mathbf{n} = \mathbf{n}_p \times \mathbf{n}_\Lambda$, where \mathbf{n}_p is the direction of the decay proton.

$$F(\varphi) = \frac{w}{4\pi} T_0^0 \left\{ 1 + V\sqrt{2} \sum_{m=2}^{2s-1} \times \sum_{q=m}^{2s-1} [\cos m\varphi (\text{Re } t_m^q) + \sin m\varphi (\text{Im } t_m^q)] Q(s, q) J_{qm} \right\}, \tag{2}$$

$$J_{qm} = \left[\frac{(q+m)!}{(q-m)!} \frac{2q+1}{2} \right]^{1/2}$$

$$\frac{m}{2} 2^{1-m/2} \frac{(q/2-1)! (q-m-1)!!}{(q/2+m/2)! (q+1)!!}$$

$$t_m^q = (2q+1)^{-1/2} T_m^q / T_0^0,$$

where m and q take on only even values. This formula differs from similar ones^{4,5} in that (2)