Relation between the Angular Distributions of Particles and Their Decay Products

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IN a number of cases where it is impossible to obtain direct information about the process of formation or interactions of particles, because of their short lifetime, it is necessary to limit investigations to the secondary particles, the decay products of the initial particles (for example, the γ-quanta from the decay of the π^0-meson). It is essential to know how the distributions of the initial and secondary particles are related.

Let us consider the practically important case where the speed of the secondary particles is that of light and the angular distribution of the initial particles $W(\cos\theta, \varphi)$ does not depend on the azimuthal angle φ and can be represented as a linear combination of terms of the type $W_n = \frac{1}{2}(n+1)\cos^n\theta$ $\times$ (n—integer). The distribution of the secondary particles $F(\cos\theta)$ is then an analogous linear combination consisting of terms of the type

$$F_n(\cos\vartheta) = \frac{n+1}{2\beta^n\psi(\vartheta)} \sum_{k=0}^{k=n+1} \nu_{nk}\,\delta_k, \tag{1}$$

where β is the speed of the initial particle,

$$1/\gamma^2 = 1-\beta^2, \quad \psi(\vartheta) = 1-\beta^2\cos^2\vartheta,$$

$$\nu_{nk} = C_n^k\left(\frac{n+1}{n-k+1}\cos^2\vartheta - 1\right)\cos^{n-k}\vartheta,$$

and the functions δ_k are connected by the recurrence relation:

$$\delta_{k+2} \tag{2}$$

$$= \frac{(1+\beta\cos\vartheta)^3(\beta-\cos\vartheta)^{k+1} - (1-\beta\cos\vartheta)^3(-\beta-\cos\vartheta)^{k+1}}{2k\gamma^2\beta\,\psi^2(\vartheta)} - \frac{\sin^2\vartheta}{\gamma^2}\frac{k+1}{k}\delta_k.$$

Eq. (2) is valid for $k \geq 1$. For $k \leq 2$ the δ_k functions have the following form

$$\delta_0 = -1, \quad \delta_1 = \cos\vartheta/\gamma^2,$$

$$\delta_2 = \sin^2\vartheta/\gamma^2 - \psi(\vartheta)\,\mathrm{Arth}(\beta)/\beta\gamma^2,$$

$$(\mathrm{Arth}\,\beta = \sqrt{1+\beta/1-\beta}).$$

Using the above relations it can be shown that for any arbitrary even (odd) n the angular distribution $F_n(\cos\theta)$ is a polynomial consisting of even (odd) powers of cosine.

Expressions for the function $F_n(\cos\theta)$ were obtained for $n \leq 6$. In view of their cumbersome form, in the present short communication we limit ourselves to the equation for the angular distribution for $n = 4$:

$$F_4(\cos\vartheta) \tag{3}$$

$$= \frac{5}{4\beta^4}\left[2 + \frac{89-16\beta^2}{3\gamma^2} - \frac{5}{\beta\gamma^2}(7-3\beta^2)\,\mathrm{Arth}\,\beta\right]\cos^4\vartheta$$

$$- \frac{5}{2\beta^4\gamma^2}\left[15-4\beta^2 - \frac{3}{\beta\gamma^2}(5-3\beta^2)\,\mathrm{Arth}\,\beta\right]$$

$$\times \cos^2\vartheta + \frac{5}{4\beta^4\gamma^2}\left[3-2\beta^2 - \frac{3}{\beta\gamma^2}\mathrm{Arth}\,\beta\right].$$

A general characteristic of the functions F_n is the very rapid variations arbitrarily close to the point $\beta = 1$. Only for $\beta \approx 1$ does the angular distribution of the initial and secondary particles become similar. With decrease in β the anisotropy of the angular distribution rapidly disappears. The higher the power of n the more clearly does this characteristic appear. Even for high values of the speed β the angular distribution of the secondary particles is still close to the isotropic and high precision of measurement is required in order to determine the angular distribution of the initial particles. This has application, for example, in the investigation of the angular distribution of π^0 mesons in the vicinity of threshold. Thus if the angular distribution of π^0—mesons is proportional to $\cos^2\theta$ then the share of the isotropic portion of the angular distribution of the γ-quanta for a proton energy of 660 mev is one-half, while for an energy of 340 mev it consists already of almost 90%.

Until rather large even values of the index n the roots of the equation $F_n(\cos\theta) = \frac{1}{2}$ are included in a small interval of angles around $\theta^* = \arccos(1\sqrt{3})$. An important consequence of this characteristic of the F_n functions is that the emission of secondary particles at "the isotropic" angle θ^* depends little on the speed of the initial particles (for $n = 2$ the emission does not depend on β as has been shown in Ref. 1). This allows the magnitude of the total cross section for formation of π^0-mesons in nucleon collisions to be determined from measurements of the emission of γ-quanta only at one angle. If the distribution of the initial particles contains odd powers of the cosine then to obtain the magnitude of the total cross section it is necessary to measure the emission of secondary particles at two angles θ^* and $\pi-\theta^*$. The indicated "isotropic" properties of the angular distributions of the secondary particles considerably simplify the problem of measuring the energy dependence of the total cross section particularly in the case where the angular distribution of the initial particles differs in the investigated interval of energy.

[1] A. A. Tiapkin, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1150 (1956); Soviet Phys. JETP **3**, 179 (1956).

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Behavior of Particles with Nonzero Spin in Crossed Constant and Varying Magnetic Fields

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WE give an exact solution for the behavior of particles of arbitrary spin in crossed constant and varying magnetic fields[1].

The wave function of the particle in a magnetic field can be written in the form

$$i\hbar d\psi / dt = -(\mathbf{H}\hat{\mathbf{M}})\,\psi, \tag{1}$$

where ψ is the $(2J+1)$-component wave function of the particle and $\hat{\mathbf{M}}$ is the magnetic moment vector operator, proportional to the angular momentum. We shall consider the case when the external magnetic field acting on the particle is composed of a constant field H_0 (along the z axis) and a varying field which has components $H_x = H_1 \cos\omega t$ and $H_y = H_1 \sin \omega t$. In this case the wave equation (1) becomes

$$i\hbar\frac{d\psi}{dt} = -\frac{\mu}{2}H_1[e^{i\omega t}(\hat{J}_x - i\hat{J}_y) + e^{-i\omega t}(\hat{J}_x + i\hat{J}_y)]\,\psi - \mu H_0 J_z\,\psi, \tag{2}$$

where $\hat{J}$ is the angular momentum operator and μ is the magnetic moment of the particle.

Let us transform to a reference system rotating about the original z axis at a frequency ω. The components of the wave function ψ'_m in the new reference system are related to the corresponding ones ψ_m in the original system by the expression

$$\psi_m = e^{im\omega t}\,\psi'_m \qquad (-J \leqslant m \leqslant J). \tag{3}$$

Inserting expression (3) into Eq. (2), and making use of the well-known properties of the operators $\hat{J}_x \pm i\hat{J}_y$, $\hat{J}_z$, we arrive at the following equation:

$$i\hbar d\psi'/dt = (-\mathbf{H}\hat{\mathbf{M}} + \mathbf{\Omega}\hat{\mathbf{J}})\,\psi. \tag{4}$$

The components of the magnetic field vector $\mathbf{H}$ which enter into this equation are the following: $H_x = H_1$, $H_y = 0$, and $H_z = H_0$; Ω is the angular velocity vector $\omega\mathbf{k}$ (where $\mathbf{k}$ is the unit vector along the z axis). The operator on the right side of Eq. (4) does not depend on time and contains the term $\omega\,\mathbf{J}$, which is the "centrifugal energy" operator, whose form corresponds to the expression for the centrifugal energy in classical mechanics.

Thus, Eq. (4) may be considered a wave equation in a noninertial (rotating) system of reference. Equation (4) takes on its simplest form in the (noninertial) reference system where the z axis is chosen along the vector $-\mu\mathbf{H}+\Omega$. The projection s onto angle β between this vector and the original z axis are easy to determine and are given by

$$s = \sqrt{\omega_0^2 + \omega^2 - 2\omega\omega_0\cos\vartheta}, \qquad \omega_0 = \mu H_0/h \tag{5}$$

$$\beta = \mathrm{acrsin}\,\omega_0 \sin\vartheta \,/\, \sqrt{\omega_0^2 + \omega^2 - 2\omega\omega_0\cos\vartheta}\,, \tag{6}$$

where $\tan\vartheta = H_1/H_0$. Clearly, the solution of Eq. (4), whose initial component $\psi_m = \delta_{m m_0}$, can be written in the form[2]

$$\psi_{m_0}(t) = \sum G_{m'm_0}\{\alpha,\beta,\gamma\}\, G_{m''m'}\{\alpha,\beta,\gamma\}\, e^{im'st}\,\psi_{m'}. \tag{7}$$

The quantities $G_{m'm}\{\alpha,\beta,\gamma\}$ entering into this equation are the matrix elements of the $(2J+1)$-dimensional irreducible representation of the three-dimensional rotation group, corresponding to rotations through the Euler angles α, β, γ (see, for instance, Ref. 2).