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Multiple Formation of Particles in 5.3 bev Nucleon-Nucleon Collisions

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WE calculated theoretically the distribution of nucleon-nucleon collisions at 5.3 bev from the number of secondary particles, using the statistical theory of multiple-particle formation[1] with and without the isobar states[2]. In the calculations we employed the method suggested in Ref. 3, with which statistical weights can be accurately calculated.

The percentage statistical weights of the various processes are given in Table I. A classification by charged state, as required for conservation of the isotopic spin, is given in Table II for(p-p)-collisions and in Table III for (n-p)-collisions (N---nucleon, N'---isobar state, M---number of pions). Thus, for example, for (p-p)-collisions the process $NN\ 2\pi$ (the statistical weight of which is indicated in Table I) gives a probability of 0.300 for the charged state ($pp + -$), a probability of 0.100 for the charge state ($pp00$), etc. (see Table II).

From the data cited it is easy to obtain the distribution of the inelastic collisions from the number of charged particles ("prongs") which, in the case of (p-p)-collisions, can be compared with the experimental data by Fowler and others[4]. Such a comparison is shown in Table IV. It is seen from this Table that allowing for the resonant interaction between the nucleons and mesons by introducing the isobar states leads to a better agreement with experiment.

In conclusion, I thank I. L. Rozental' for useful advice.

We note with gratitude the constant interest of the late Professor S. Z. Belen'kii, who stimulated the performance of the calculations.

TABLE I

Number of mesons	Type of process	Statistical Weight (%)		Number of mesons	Type of process	Statistical Weight (%)	
		$p-p$	$n-p$			$p-p$	$n-p$
0	NN	0.3	0.4	3	$NN3\pi$	4.5	4.5
1	$NN\pi$	6.5	6.8		$NN'2\pi$	31.8	31.0
	NN'	1.0	0.7		$N'N'\pi$	11.7	11.1
2	$NN2\pi$	11.5	12.0	4	$NN4\pi$	2.7	2.7
	$NN'\pi$	16.7	17.4		$NN'3\pi$	1.2	1.2
	$N'N'$	0.9	1.2		$N'N'2\pi$	11.2	11.1

TABLE II

Number of Mesons	Charged State	Probabilities of Charged States of Various Processes $NNm\pi$	$NN'(m-1)\pi$	$N'N'(m-1)\pi$
0	*pp*	1.000		
1	*pp* 0	0.250	0.167	
	pn +	0.750	0.833	
2	*pp* + —	0.300	0.350	0.200
	pp 00	0.100	0.117	0.178
	pn + 0	0.450	0.483	0.578
	nn ++	0.150	0.050	0.044
3	*pp* + — 0	0.267	0.280	0.244
	pn ++—	0.333	0.360	0.422
	pp 000	0.033	0.033	0.030
	pn + 00	0.233	0.247	0.252
	nn ++ 0	0.134	0.080	0.052
4	*pp* ++——	0.122	0.131	0.119
	pp +— 00	0.180	0.190	0.186
	pn ++—0	0.408	0.431	0.480
	nn +++—	0.082	0.060	0.036
	pp 0000	0.012	0.013	0.014
	pn + 000	0.106	0.110	0.112
	nn ++ 00	0.090	0.065	0.053

TABLE III

Number of Mesons m	Charged State	Probabilities of Charged States of Various Processes $NNm\pi$	$NN'(m-1)\pi$	$N'N'(m-2)\pi$
0	*pn*	1.000		
1	*pp* —	0.278	0.167	
	pn 0	0.444	0.666	
	nn +	0.278	0.167	
2	*pp* — 0	0.189	0.137	0.067
	pn + —	0.466	0.563	0.733
	pn 00	0.156	0.163	0.133
	nn +—	0.189	0.137	0.067
3	*pp* +——	0.138	0.124	0.076
	pp — 00	0.100	0.087	0.078
	pn +— 0	0.462	0.508	0.611
	nn ++—	0.138	0.124	0.076
	pn 000	0.062	0.070	0.081
	nn + 00	0.100	0.087	0.078
4	*pp* +—— 0	0.179	0.163	0.133
	pn ++——	0.209	0.229	0.267
	pp — 000	0.048	0.043	0.035
	pn +— 00	0.316	0.338	0.380
	nn ++— 0	0.179	0.163	0.133
	pn 0000	0.021	0.021	0.017
	nn + 000	0.048	0.043	0.035

TABLE IV

Number of "prongs" (inelastic interactions)	Number of Events		
	Experiment of Ref. 4	Theoretical (isobar states included)	Theoretical (isobar states not included)
2	14	15.1	21,5
4	16	16.3	10.5
6	2	0.6	0.5

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Consequences of the Renormalizability of Quantum Electrodynamics and Meson Theory

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THE consequences of the renormalizability of quantum electrodynamics and meson theory which have been obtained by Gell-Mann and Low[1] and Bogoliubov[2] are most easily formulated, in our opinion, in the following way. We shall start from the following equations of Gell-Mann and Low[1]

$$\alpha(g_0^2, \xi - L) = \frac{\alpha_c(g_c^2, \xi)}{\alpha_c(g_c^2, L)}, \tag{1}$$

$$\beta(g_0^2, \xi - L) = \frac{\beta_c(g_c^2, \xi)}{\beta_c(g_c^2, L)}, \quad d(g_0^2, \xi - L) = \frac{d_c(g_c^2, \xi)}{d_c(g_c^2, L)}.$$

Here α_C, β_C and d_C are the asymptotic expressions for the slowly-varying factors of the renormalized vertex parts and Green's functions for the nucleon and meson*, g_C is the renormalized meson coupling constant, $\xi = \ln(-k^2/m^2)$, $L = \ln(\Lambda^2/m^2)$ (Λ is the momentum "cutoff"). The quantities α, β, d, g_0 are the nonrenormalized quantities corresponding to the cutoff momentum.

For convenience, we have introduced the logarithmic variables ξ and L from the beginning. In addition to the trivial inference that α, β and d become unity for $\xi = L$, Eq. (1) includes the statement, fundamental in what follows, that for $\xi \gg 1$, α, β and d asymptotically approach functions only if the difference $\xi - L = \ln(-k^2/\Lambda^2)$, i.e., no longer depend on the nucleon mass m.

We then introduce a quantity which may be called "the effective coupling constant"

$$g^2(\xi) = g_0^2 \alpha^2(g_0^2, \xi - L)\, \beta^2(g_0^2, \xi - L)\, d(g_0^2, \xi - L) \tag{2}$$

$$= g_c^2 \alpha_c^2(g_c^2, \xi)\, \beta_c^2(g_c^3, \xi)\, d_c(g_c^2, \xi).$$

The second of Eqs. (2) is obtained from (1) and from the relation between the renormalized and nonrenormalized coupling constants. From Eq. (2) it is seen that the effective coupling constant g may be considered either a function of g_0^2 and $\xi - L$, or of g_C^2 and ξ.

The final formulation consists of the assertion that the logarithmic derivatives of α and α_C, etc., with respect to ξ, which are equal according to Eq. (1), depend on one variable, namely, on the effective coupling constant

$$\alpha'/\alpha = \alpha_c'/\alpha_c = F_1(g^2); \quad \beta'/\beta = \beta_c'/\beta_c = F_2(g^2); \tag{3}$$

$$d'/d = d_c'/d_c = F_3(g^2);$$

$$(g^2)'/g^2 = 2F_1(g^2) + 2F_2(g^2) + F_3(g^2)$$

The primes here denote differentiation with respect to the arguments $\xi - L$ or ξ, whichever is appropriate. The last of Eqs. (3) follows from the first three and Eq. (2). As an example, let us prove the first of the equations. According to Eq. (2), ξ