If it happens that for $\xi_2 = \xi_4 = 0$ the difference $d\sigma_{\parallel} = d\sigma - d\sigma_{\perp} << d\sigma_{\perp}$, then the probability of emission of longitudinal and scalar γ-quanta, in the theory with non-local interaction, is negligibly small. We get by a standard method:

$$d\sigma_{\perp}(p_2; k_2 \mid p_1; k_1) \tag{5}$$
$$= \frac{r_0^2 k_2^2}{2k_1^2 m} d\Omega \left\{ \frac{1}{k_1} \Phi_1^2 (mk_2 + k_1^2 \sin^2 \vartheta) + \frac{1}{k_2} \Phi_2^2 \left(mk_1 - \frac{1}{2} k_2^2 \sin^2 \vartheta \right) + \Phi_1 \Phi_2 \left(\frac{1}{2} k_2 - k_1 - m \right) \sin^2 \vartheta \right\},$$

where $\phi_i = \phi_i(\theta)$ as informula (2).

The cross section (5) for Compton scattering differs considerably from the effective cross section got by Klein, Nishina and Tamm for energies $k_1 \sim \hbar \lambda / c$ and for small scattering angles. Comparing with (1), it follows that the γ-quanta with longitudinal and scalar polarizations contribute considerably to the cross section. As the energy k_1 of the scattered γ-quanta increases, the magnitude of the effective cross sections σ and $\sigma_{\perp}$ decreases rapidly.

Lawson[3] has measured the Compton scattering effective cross section for an energy $k_1 = 80$ mev of the γ-quanta. The experimental error is of 15% and the result agrees with the cross section computed by the Klein-Nishina-Tamm formula. It follows from these experiments that in any case $\lambda < 10^{-12}$ cm. More exact measurments are reported[4] for an energy $k_1 = 250$ mev; the experimental error is of 10%, and the result also agrres with the Klein-Nishina-Tamm calculation. It follows from Eqs. (1) and (5) that for $k_1 = 250$ mev and $\theta = 4°$ the magnitude of the constant λ cannot exceed 1.2×10^{-13} cm. This result is obtained for form-functions $\phi_1 = e^{-2\lambda^2 mk_1}$, $\phi_2 = e^{-2\lambda^2 mk_2}$, but it does not change appreciably when the form of these functions is varied.

To conclude, let us note that the study of the creation of electron-positron pairs by cosmic-rays electrons with energies of 0.1 to 10 bev, in photo-emulsions, also yields results in agreement with the calculations performed by the known methods of the theory of local interaction.[5] For electrons with energies of 100 bev there are some indications of disagreement with theory. However, the large experimental errors prohibit the drawing of any definite conculsions.

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Relation between Neutron Scattering in Polycrystals and Specific Heat

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THE cross section for the inelastic coherent scattering of thermal neutrons in polycrystals, relative to a single nucleus and for an arbitrary frequency spectrum, is expressed in the following way, correct to terms of order $1/M$ (that is, neglecting multiple phonon processes and thermal factors):*

$$\sigma_{\pm}(E, \omega)\, d\omega = \frac{\sigma_0}{3M} \sqrt{1 \mp \hbar\omega/E}\, (2 \mp \hbar\omega/E) \times \frac{(\pm 1)}{1 - e^{\mp \hbar\omega/T}} \frac{E}{\hbar\omega} \nu(\omega)\, d\omega. \tag{1}$$

Here $\sigma_{\pm}(E, \omega)\, d\omega$ is the scattering cross section for a neutron with energy E, as a result of which a phonon with a frequency in the interval $d\omega$ is excited (or absorbed); σ_0 is the cross section for the scattering of a neutron by a single massive nucleus, M is the mass number of a nucleus of the crystal, T is the temperature (in units of energy and $\nu(\omega)$ is the frequency spectrum of the crystal, relative to a single nucleus.

The crystalline frequency spectrum entering into Eq. (1) is in turn linked in a well-defined manner with the lattice specific heat at constant volume, $C(T)$, by the relation

$$C(T)=\frac{d}{dT}\int_0^\infty \frac{\hbar\omega\nu(\omega)}{e^{\hbar\omega/T}-1}\,d\omega\,, \tag{2}$$

and in principle can be determined from experimental values of the specific heat. Such a procedure is subject to errors of considerable magnitude which are difficult to control, however,**[2] particularly in the high-frequency region; consequently, calculations of the differential cross sections σ_+ from experimental specific heat data can scarcely lead to dependable results.

It is possible to do considerably better with those characterestics of neutron scattering which depend on integrals over the frequency spectrum, such as, for example, the total cross section for inelastic scattering

$$\sigma=\int_0^{E/\hbar}\sigma_+(E,\omega)\,d\omega+\int_0^\infty\sigma_-(E,\omega)\,d\omega\equiv\sigma_++\sigma_-, \tag{3}$$

the moments of the energy which can be transferred

$$\langle\Delta E^k\rangle=\frac{1}{\sigma}\Big[\int_0^{E/\hbar}\sigma_+(E,\omega)(\hbar\omega)^k d\omega+(-1)^k\int_0^\infty\sigma_-(E,\omega)(\hbar\omega)^k d\omega\Big] \tag{4}$$

and so forth, since these quantities are relatively insensitive to the details of the frequency spectrum. Furthermore, quantities in which the integral is taken over the entire frequency spectrum can be immediately expressed in terms of integrals over the specific heat along the real axis, so that it is possible to determine the error in the calculation directly. We will demonstrate this procedure in calculating the quantity σ_-, which for very cold neutrons coincides with the total coherent cross section, since the elastic cross section and σ_+ are equal to zero.[1]

First of all we note that from specific heat data it is possible to calculate the quantity***

$$\int_0^T C(t)\,dt=\int_0^\infty\frac{\hbar\omega\nu(\omega)}{e^{\hbar\omega/T}-1}\,d\omega \tag{5}$$

$$=\sum_{n=1}^\infty\int_0^\infty\hbar\omega\nu(\omega)\,e^{-n\hbar\omega/T}\,d\omega=\sum_{n=1}^\infty\psi\Big(\frac{T}{n}\Big)$$

$$=\sum_{n=1}^\infty\sum_{k=1}^\infty\mu_k\int_0^{T/kn}C(t)\,dt$$

$$=\sum_{q=1}^\infty\int_0^{T/q}C(t)\,dt\sum_{k|q}\mu_k=\int_0^T C(t)\,dt$$

$$\psi(T)=\int_0^\infty\hbar\omega e^{-\hbar\omega/T}\nu(\omega)\,d\omega=\sum_{k=1}^\infty\mu_k\int_0^{T/k}C(t)\,dt\,,$$

where the μ_k are the familiar coefficients appearing in the series expansion of the Dirichlet function $1/\xi(x)$[($\xi(x)$ is the Riemann ζ-function)], which, as is well known, are equal to 1 for $k=1$, to $(-1)^l$ if k is the product of l with different prime numbers, and are zero otherwise. Consequently, for any function $\nu(\omega)$ which falls to zero faster than ω, the integral over which converges, we have identically (for $p<2$):

$$\int_0^\infty\frac{\nu(\omega)d\omega}{\hbar\omega}e^{-\hbar\omega/T}\frac{(\hbar\omega)^p}{\sqrt{E+\hbar\omega}} \tag{6}$$

$$=\frac{e^{E/T}}{\Gamma(1/2)}\int_0^T\frac{ds}{s^2}\Big(\frac{1}{s}-\frac{1}{T}\Big)^{-1/2}e^{-E/s}\Big\{\frac{1}{\Gamma(2-p)}\times\int_0^s\Big(\frac{1}{t}-\frac{1}{s}\Big)^{1-p}\psi(t)\,\frac{dt}{t^2}\Big\}.$$

For $p=2$ it is necessary to replace the integral in the curly brackets by $\psi(s)$, and for $p>2$ by $d^{p-2}\psi(s)/d(1/s)^{p-2}$. Therefore, it is possible to obtain an expression for σ_- immediately, in view of the fact that

$$\sigma_-=\sum_{k-1}^\infty\frac{\sigma_0}{3M}\int_0^\infty\frac{E}{\hbar\omega}\frac{1}{\sqrt{1+\hbar\omega/E}}\times\Big(2+3\frac{\hbar\omega}{E}+\frac{\hbar^2\omega^2}{E^2}\Big)e^{-k\hbar\omega/T}\nu(\omega)\,d\omega\,. \tag{7}$$

In particular, for small energies,

$$\sigma_-=\frac{2}{3\sqrt{\pi}}\frac{\sigma_0}{M}\sqrt{\frac{T}{E}}\int_0^1\frac{\sqrt{1-x}}{\sqrt{x}}\,dx\times\sum_{q=1}^\infty\frac{1}{\sqrt{q}}\,C\Big(\frac{T}{q}x\Big)\sum_{k|q}\frac{\mu_k}{\sqrt{k}}\,. \tag{8}$$

In the Figure we present graphically the temperature dependence of σ_- for graphite, for neutrons with energies of 17.2° K, calculated from specific heat data[4] using the exact Eqs. (6) and (7) (solid curve), and from the approximate formula (8) (broken curve). Experimental values are shown for comparison.[5] The error in these calculations is < 2–3%. A curve calculated on the basis of the Debye approximation is also shown[5] (dot–dash line). As can be seen from these curves, the exact theoretical curve satisfactorily describes the temperature behavior of the experimental values of σ_-, but differs from them by an amount which indicates the presence of an additional temperature–independent scattering cross section of

~ 0.2 barn. This discrepancy can be linked with incoherent scattering by impurities in graphite, or with multiple scattering through small angles.

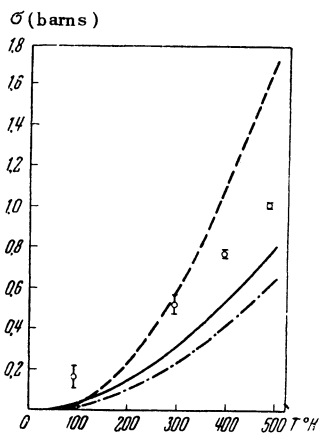
In an analogous manner it is possible to derive expressions for other integrals over the frequency spectrum, for example for σ_+ for large energies, when the first integral in (3) can be extended out to infinity, etc. For instance, the average value of the cross section $\bar{\sigma}_{eff} = \overline{\sigma v}/\bar{v}$ and the moments of the energy transferred to (or from) neutrons with Maxwellian distribution are:

$$\bar{\sigma}_{eff} = \frac{1}{T^2}\int_0^\infty \sigma(E) E e^{-E/T} dE, \quad \overline{\langle \Delta E^k \rangle} = \frac{1}{\bar{\sigma}_{eff} T^2}\int_0^\infty \sigma(E) \langle \Delta E^k \rangle E e^{-E/T} dE, \qquad (9)$$

which likewise can be described in relation to the specific heat. Thus, after substituting (3) into (9) and making simple transformations of the integrals we obtain for $\bar{\sigma}_{eff}$:

$$\bar{\sigma}_{eff} = \frac{\sigma_0}{3MT^2}\left\{\int_0^1 \sqrt{1-x}\,(2-x)\frac{dx}{x^4} \times \sum_{k=0}^{\infty} \chi\left[\frac{1}{T}\left(\frac{1}{x}+k\right)\right] + \int_0^\infty \sqrt{1+x}\,(2+x)\frac{dx}{x^4}\sum_{k=1}^{\infty}\chi\left[\frac{1}{T}\left(\frac{1}{x}+k\right)\right]\right\}; \qquad (10)$$

$$\chi\left(\frac{1}{t}\right) = -\frac{d}{d(1/t)}\psi(t).$$



In conclusion I express my profound gratitude to V. L. Ginzburg for valuable discussions while this work was being carried out.

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*These formulas appear in the elementary generalization of the corresponding expression for Debye polycrystals (see Ref. 1, for example), when it is possible to replace the sum over the vectors of the reciprocal lattice by an integration; this may always be done for σ_-, but, for σ_+, only when the neutron wavelength is much less than the dimensions of the elementary cell of the crystal.

**This difficulty is connected with the fact that the transformation which is the inverse of the transformation of the form $g(p) = \int_0^\infty f(x)(e^{px}-1)^{-1}dx$ requires knowledge of the function $g(p)$ in the complex plane of the variable p.

***This possibility results from the fact that the sum of μ_k over all divisors of any number q ($\sum_{k/q}\mu_k$) is equal to unity for $q = 1$ and to zero for $q \neq 1$. Consequently, we then have

$$\int_0^T C(t)\,dt = \int_0^\infty \frac{\hbar\omega\nu(\omega)}{e^{\hbar\omega/T}-1}d\omega = \sum_{n=1}^{\infty}\int_0^\infty \hbar\omega\nu(\omega)e^{-n\hbar\omega/T}d\omega = \sum_{n=1}^{\infty}\psi\left(\frac{T}{n}\right)$$
$$= \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\mu_k\int_0^{T/kn}C(t)\,dt = \sum_{q=1}^{\infty}\int_0^{T/q}C(t)\,dt\sum_{k/q}\mu_k = \int_0^T C(t)\,dt.$$

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