TABLE

No. of Shower	1	2	3	4	5	6	7	8
Total length in t-units . .	2.4	3,4	1.7	3,2	3	4.6	1.6	1.5
$y=\ln(E_0/E_1)$	7	6	7	5	7	7	9	10

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Dispersion Relations for Pions Scattered by Deuterons

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CONSIDER zero-angle elastic scattering of pions by deuterons. The dispersion relations corresponding to this process differ substantially from the dispersion relations for the scattering of pions by free nucleons: first, the dispersion relations depend on the polarization of the deuterons; second, if the Coulomb interaction is neglected, there is only one dispersion relation for the sum of the scattering amplitudes of the positive and negative pions.

Let us denote by $D_m(\omega)$ and $A_m(\omega)$ the real and imaginary parts of the amplitude of the scattering of pions with energy ω by deuterons having a spin projection m along the direction of motion of the pions. Using the relationship between the imaginary part of the zero-angle scattering amplitude and the total cross section, $A_m(\omega) = (k/4\pi)\,\sigma_m(\omega)$ (where $k^2 = \omega^2 - \mu^2$, and μ is the pion mass), and using the usual procedure for obtaining the dispersion relations[1,2] we obtain

$$D_m(\omega) - D_m(\mu) = \frac{2k^2}{\pi}\int_0^{\mu} \frac{\omega' A_m(\omega')\,d\omega'}{k'^2(\omega'^2-\omega^2)} + \frac{k^2}{2\pi^2}\int_0^{\infty}\frac{\omega'\sigma_m(\omega')\,d\omega'}{(\omega'^2-\omega^2)\,k'}. \quad (1)$$

To determine the contribution from the region $0 \le \omega \le \mu$ we employ the following expressions for $A_m(\omega')$, obtained readily from Ref. 1:

$$A_m(\omega') = \pi \sum_f |M_m(\omega', \mathbf{f})|^2\, \delta\left[\omega' - \varepsilon_0 - \frac{k'^2}{4M} - \frac{f^2}{M}\right], \quad (2)$$

where $M_m(\omega', \mathbf{f})$ is the matrix element corresponding to the capture of a pion by a deuteron in state m and the formation of two identical nucleons with a relative momentum $\mathbf{f}$:

$$M_m(\omega', \mathbf{f}) = (\sqrt{2}\,g/M)\,\langle \Phi_m^* \sigma_1 \mathbf{k}' F_i(\mathbf{k}', \rho)\,\Psi_{\mathbf{f}}\rangle. \quad (2')$$

here Φ_m is the deuteron wave function, $\Psi_{\mathbf{f}}$ the wave-function of two identical nucleons in the final state, g the pion to nucleon coupling constant, M the nucleon mass, and ϵ_0 the coupling energy of the deuteron.

The function $F_i(\mathbf{k}', \rho)$ equals $\sin(\mathbf{k}'\rho/2)$ if the two forming nucleons are in the triplet state and $\cos(\mathbf{k}'\rho/2)$ in the case of a singlet state.

Integrating with respect to ω', we get

$$\frac{2k^2}{\pi}\int_0^{\mu}\frac{\omega' A_m(\omega')\,d\omega'}{k'^2(\omega'^2-\omega^2)} = 2k^2 \sum_{\mathbf{f}} \frac{\mu^2/4M - (f^2/M + \varepsilon_0)}{\tilde{k}^2(\omega^2 - \tilde{\omega}^2)}\,|M_m|^2. \quad (3)$$

Generally speaking, $\tilde{\omega}$ is a function of $\mathbf{f}$, determined by the conservation laws. However, taking into account the fact that the matrix element differs substantially from zero only in the region $\mathbf{f} \sim \mathbf{k}/2$, we shall assume hereinafter $\tilde{\omega} \approx \mu^2/2M - \epsilon_0$. Next, since small changes in ω' correspond to large changes in $\mathbf{f}$, we extend the summation with respect to $\mathbf{f}$ to infinity. Using the completeness of the set of functions $\Psi_{\mathbf{f}}$, we have

$$\sum_f |M_m|^2 = \frac{2g^2}{M^2}\tilde{k}^2 \left\{\delta_{m1}\int \varphi_0^2 F_t^2 d\rho + \delta_{m0}\int \varphi_0^2 F_s^2 d\rho\right\}, \quad (4)$$

where φ_0 (ρ) is the coordinate portion of the deuteron wave function.

The second term in expression (3) is calculated in the following manner:

$$\sum_{\mathbf{f}} (f^2/M + \varepsilon_0)\,|M_m|^2 \qquad (5)$$

$$= \frac{2g^2}{M^2} \sum_{\mathbf{f}} \langle \Phi_m^* \sigma_1 \mathbf{k} F_i \Psi_{\mathbf{f}} \rangle \langle \Psi_{\mathbf{f}}^* (H_1 \sigma \mathbf{k} F_i - \sigma_1 \mathbf{k} F_i H)\, \Phi_m \rangle,$$

where H is the Hamiltonian of the interaction of the two nucleons. Its general form (without taking the tensor forces into account) is as follows:

$$H = \mathbf{f}^2/M + {}^1\!/_4\, U_t(\rho)(\sigma_1\sigma_2 + 3) \qquad (6)$$

$$- {}^1\!/_4\, U_s(\rho)(\sigma_1\sigma_2 - 1)$$

$$+ \{{}^1\!/_4 \tilde{U}_t(\rho)(\sigma_1\sigma_2 + 3) - {}^1\!/_4 \tilde{U}_s(\rho)(\sigma_1\sigma_2 - 1)\}\, P_{12}.$$

Here f^2/M is the kinetic-energy operator, U_t and U_s are the potential energies in the triplet and singlet states, $\tilde{U}_t$ and $\tilde{U}_s$ the exchange energies, and P_{12} is the particle commutation operator.

Calculating the sum (5) with the aid of the Hamiltonian (6), and inserting the result into (1), leads to the following dispersion relationships.*

For deuterons polarized parallel (anti-parallel to the incident beam:

$$D_{\pm 1}(\omega) - D_{\pm 1}(\mu) \qquad (7')$$

$$= g^2 \left(\frac{\mu}{2M}\right)^2 \frac{2}{M} \frac{k^2}{\omega^2 - \tilde{\omega}^2} \left[1 + \frac{8M}{\mu^2} \int \varphi_0^2 \sin^2 \frac{\tilde{k}\rho}{2}\, \tilde{U}_t\, d\rho\right]$$

$$+ \frac{k^2}{2\pi^2} \int_\mu^\infty \frac{\omega' \sigma_{\pm 1}(\omega')\, d\omega'}{k'(\omega'^2 - \omega^2)}$$

For deuterons polarized perpendicular to the incident beam:

$$D_0(\omega) - D_0(\mu) \qquad (7'')$$

$$= g^2 \left(\frac{\mu}{2M}\right)^2 \frac{2}{M} \frac{k^2}{\omega^2 - \tilde{\omega}^2} \left[1 + \frac{4M}{\mu^2} \int \varphi_0^2 \cos^2 \frac{\tilde{k}\rho}{2} \{\tilde{U}_s - U_s + \tilde{U}_t + U_t\}\, d\rho\right]$$

$$+ \frac{k^2}{2\pi^2} \int_\mu^\infty \frac{\omega' \sigma_0(\omega')\, d\omega'}{k'(\omega'^2 - \omega^2)}.$$

The dispersion relations obtained for the scattering of pions by deuterons contain, in addition to the constant g, also certain effective values of the potential interaction energy of two nucleons in different states and these values affect substantially the value of the singularity term for deuterons, polarized perpendicularly to the incident beam.

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*Let us call attention to the fact that we obtained (7′) and (7″) without using the actual form of the coordinate part of the wave function of the deuteron.

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Influence of the Earth's Magnetic Field on the Space-Distribution of Particles in Extensive Air Showers

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THE space-distribution of charged particles in the extensive air showers of cosmic rays has been studied in a number of experiments[1-4]. In none of the computations used, despite their high statistical accuracy, has the influence of the earth's magnetic field been taken into account. The theoretical estimates made[5] have shown that the distortion of axial symmetry in the space-distribution of the electron component of the extensive air shower, produced by the action of the earth's magnetic field, does not exceed the statistical limits of experimental errors. Nevertheless, the results of the experimental investigation given in Ref. 6 aroused doubts as to whether the disregard of the influence of the earth's magnetic field on the space-distribution of shower particles is justified. This disregard, by the way, was considered permissible in Ref. 2.

Subsequently, we made a supplementary analysis of the experimental data given in Ref. 2. These related to a study completed in the summer of 1952 at an elevation of 3860 m (Pamir). The relative position of a part of the experimental set-up with