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## On the Motion of Inclusions in a Solid

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**C**ONSIDER a foreign inclusion in a solid body of infinite extent. Let the inclusion be spherical in shape, and filled with substance (liquid or gas) in which the material of the solid under existing conditions has a marked solubility. Let a constant but infinitesimal temperature gradient  $\nabla T$  be maintained in the solid; we will investigate the translational motion of the inclusion under the influence of this gradient.<sup>1,2</sup>

It is obvious that, in a solid, the translation of an inclusion can take place only by means of the transfer of matter into the inclusion, but a hydrodynamic mechanism similar to that involved in the rise of bubbles in a liquid is excluded (we do not consider viscous flow of the crystal). In the presence of only a thermal field, the indicated transfer is connected with the difference in saturation concentrations of the solution at the cold and hot ends of the inclusion, and takes place purely by diffusion. The presence of other fields leads, generally speaking, to the appearance of other flows, (for instance, the presence of a gravitational field of intensity  $g$  can lead to convection\*). Thus each element of surface surrounding the inclusion will have a velocity

$$\mathbf{v} = (D/\rho) \nabla c, \quad (1)$$

where  $\rho$  is the density of the substance comprising the solid,  $\nabla c$  is the concentration gradient of this substance in the material filling the inclusion, taken near the portion of the surface under consideration, and  $d$  is the diffusion coefficient.

The concentration  $c$ , which depends, generally speaking, on the coordinates and on the time, is determined from the equations of diffusion and thermal conduction, with suitable boundary conditions:

$$\begin{aligned} \partial c / \partial t &= D [\Delta c + (k_T / T) \Delta T]; \\ \partial T_1 / \partial t - (k_T / c_p) (\partial \mu / \partial c)_{p,T} \partial c / \partial t &= \chi_1 \Delta T_1; \\ \partial T_2 / \partial t &= \chi_2 \Delta T_2, \end{aligned} \quad (2)$$

where  $k_T$  is the coefficient of thermal diffusion,  $\mu$  the chemical potential of the contents of the inclusion,  $c_p$  their specific heat, and  $\chi_1$ ,  $T_1$  and  $\chi_2$ ,  $T_2$  respectively the thermal conductivity and temperature inside and outside the inclusion. Leaving the boundary conditions out of the picture for the moment, we go over in these equations to a coordinate system in which the inclusion is at rest. Terms proportional to  $\mathbf{v} \nabla T$  and  $\mathbf{v} \nabla c$ , appearing as a result of this transformation, will be of second order in  $\nabla T$  (since  $v \sim \nabla T$ ). For  $\nabla T = \text{const}$ ,  $\mathbf{v}$  does not depend explicitly on the time, and the partial derivatives of the temperature and concentration with respect to time will be at most of second order. Consequently, to the approximation being considered here, both the temperature and the concentration satisfy Laplace's equation. We now return to the conditions at the surface of separation. These will have the form:

$$\begin{aligned} T_1 = T_2, \quad \kappa_1 \partial T_1 / \partial n - \kappa_2 \partial T_2 / \partial n \\ = -qD \partial c / \partial n, \quad c = c_0, \end{aligned} \quad (3)$$

$\kappa_1$  and  $\kappa_2$  are the respective coefficients of thermal conductivity, and  $\partial / \partial n$  is an operator denoting the derivative along the normal to the surface. The right-hand side of the second condition makes allowance for the evolution (or absorption) of latent heat of crystallization  $q$  at the boundary, and the third condition requires that the solid solution be saturated at this surface. Since the gradient  $\nabla T$  is small, the change in temperature along the surface of separation will not be great, but if one reckons the temperature and concentration with respect to their values at the center of the inclusion, the last condition in (3) will be written thus:

$$c = (dc_0 / dT) T_1. \quad (4)$$

Here the derivative  $dc_0 / dT$  is taken along the appropriate equilibrium curve. Thus at the limits of the region, the concentration is proportional to the temperature, and since both the temperature and the concentration satisfy the same equation, there will be a similar relation between them everywhere inside the inclusion (irrespective of the form of the bounding surface). Therefore at the surface of separation

$$-qD \partial c / \partial n = -qD (dc_0 / dT) (\partial T_1 / \partial n).$$

If we now replace  $\kappa_1$  by  $\kappa_1 + qdc/dT$  in condition (3), we obtain the well-known problem concerning the distribution of temperature around a stationary spherical object for a constant temperature gradient at infinity (see, for example, Ref. 3). Multiplying its solution over the interior of the region by  $dc_0 / dT$ , we obtain the distribution of concentration over the inclusion. Making use of Eq. (1), we obtain finally

$$v = \frac{3\kappa_2 D}{\kappa_1 + 2\kappa_2 + qD dc_0 / dT} \frac{dc_0}{dT} \frac{\nabla T}{\rho}. \quad (5)$$

From this equation it is obvious that the velocity of translation of the inclusion does not depend on its dimensions.

The value of  $v$  is determined by the quantity  $dc_0 / dT$ , which under conditions of constant pressure is equal to  $qc_0 / kT^2$ . In substances whose solubility in the material filling the inclusion does not increase with temperature, the inclusion moves in the direction of the temperature gradient. If however  $dc_0 / dT < 0$ , the inclusion has to translate in the opposite direction. It is clear that the process under investigation is governed by Le Chatelier's principle. Indeed, for  $q > 0$ , the diffusion at the hot end of the inclusion and the condensation at the cold end both represent the same process, striving to bring the system back into equilibrium; this equilibrium is constantly being destroyed, however, by the source maintaining the temperature gradient.

It is not difficult to see that an analogous process takes place for the transfer of matter in a liquid medium from a crystal having a high temperature to a crystal with a lower one; that is, for the growth of one crystal at the expense of another.

The coefficient of diffusion has the order of magnitude  $10^{-5}$  cm<sup>2</sup>/sec;  $dc_0 / dT \sim 5 \times 10^{-2}$  g/cm<sup>3</sup>-sec-deg;  $q \sim 10^2$  cal/g;  $\kappa_2 \sim 5 \times 10^{-3}$  cal/cm-sec-deg.

Consequently,  $qD dc_0 / dT \ll \kappa_2$ , and

$$v = (3D / 2\rho) (dc_0 / dT) \nabla T,$$

that is,  $v$  does not depend on the thermal conductivity of the solid or of the contents of the inclusion.

If the conditions under which an inclusion exists are critical conditions for its contents, then the coefficient of diffusion will fall to zero and the inclusion will come to rest. For a temperature gradient of the order of 1 deg/cm, the velocity of translation of the inclusion amounts to about 0.1 mm/day. If conditions are created whereby matter can be transferred convectively, the process ought to be noticeably accelerated.

\*It is clear, for example, that there will be convection if the vectors  $g$  and  $\nabla T$  are parallel, but not if they are antiparallel.

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<sup>3</sup>L. D. Landau and E. M. Lifshitz, *Mechanics of Continuous Media*, GITTL, Moscow, 1953.

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### Investigation of High Energy Electron Showers by the "Emulsion Chamber" Technique

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**I**N the emulsion chamber suggested in Ref. 1, we have studied the development of electron showers of high energy ( $E = 10^{12}$  ev) at small depths (2-3  $\tau$ -units). The results obtained are compared with cascade theory.<sup>2,3</sup>

The emulsion chamber consisted of 24 emulsion plates (NIKFI) type R with emulsion thickness of 100  $\mu$  put together into a stack and held in a special frame. Iron plates of thickness 3.5 mm were placed between emulsions. The distance between adjacent layers of emulsion was 5 mm. The glass on which the emulsion was poured was previously polished to a size of 86  $\times$  116 mm<sup>2</sup> with an accuracy of 0.05 mm. This allowed the exact placement of adjacent plates which is essential in